J. L. Moll*
J. F. Gibbons*

Threshold Current for *p-n* Junction Lasers

In a recent paper in the IBM JOURNAL Lasher[1] gives the relation for threshold current in a *p-n* junction laser as

$$j = \frac{8\pi q n'^2 d\Delta\nu}{\alpha\lambda^2} [\alpha_0 + T/l + \alpha_{\mathrm{diff}}], \tag{1}$$

where

q = electronic charge
n' = index of refraction
d = width of active region
$\Delta\nu$ = width of spontaneous emission line
λ = wavelength
α = quantum efficiency
T = percent of light transmitted through end of laser
l = length of crystal along active region between reflectors
α_{diff} = diffraction loss
α_0 = absorption loss in the active region.

In his calculation, Lasher neglected the effect of α_0 on the threshold current because he felt that it was difficult to estimate. It is possible to include the absorptive effects associated with incomplete population inversion. The purpose of this Letter is to point out how this absorption should be included. For the case of recombination through an acceptor level, the rate processes to be included are:

(1) Spontaneous electron recombination on neutral acceptor

$$\left(v_t\, \sigma_n\, N_T{}^0\, n\, \frac{1}{N_0 + 1}\right)$$

(2) Stimulated electron recombination on neutral acceptor

$$\left(v_t\, \sigma_n\, N_T{}^0\, n\, \frac{N}{N_0 + 1}\right)$$

(3) Photon absorption by negative acceptor

$$\left(v_t\, \sigma_n\, N_T{}^-\, n_1\, \frac{N_C - n}{N_C - n_0} \frac{N}{N_0}\right)$$

(4) Hole capture on negative acceptor

$$v_t\, \sigma_p\, N_T{}^-\, p$$

(5) Hole emission from neutral acceptor

$$v_t\, \sigma_p\, N_T{}^0\, p_1\, \frac{N_V - p}{N_V - p_0},$$

where $N_T{}^0$ and $N_T{}^-$ are the neutral and negative acceptor densities. The rest of the symbols are defined below Eq. (2). The various rates are obtained by making each process balance the inverse process at thermal equilibrium. The rate at which the electrons emit photons (1) and (2) is taken as proportional to the initial and final states times $N + 1$. The part proportional to N is the stimulated recombination and the 1 is spontaneous recombination. The rate of photon absorption (3) is just proportional to the density of photons and initial and final electron states. Processes (4) and (5) are not photon coupled and so involve only initial and final states. The net rate of electron recombination (photon production) is obtained by solving for $N_T{}^0$ and $N_T{}^-$ (neutral and negative trap densities) in the two equations

$$N_T{}^0 + N_T{}^- = N_T$$

$$\frac{dN_T{}^0}{dt} = 0$$

and substituting into the equation,

$$-\frac{dn}{dt} = R - G = (1) + (2) - (3).$$

The net rate of recombination becomes

$$R - G = v_t{}^2 \sigma_n \sigma_p N_T \times \left\{ \left[pn\, \frac{N+1}{N_0+1} - \frac{N}{N_0} \frac{N_C - n}{N_C - n_0} \frac{N_V - p}{N_V - p_0}\, n_i{}^2 \right] \Big/ \left[v_t \sigma_p (p + p_1) \left(\frac{N_V - p}{N_V - p_0}\right) + v_t \sigma_n \left(n\, \frac{N+1}{N_0+1} + n_1 \frac{N}{N_0} \frac{N_C - n}{N_C - n_0} \right) \right] \right\}, \tag{2}$$

where

v_t = thermal velocity
σ_n, σ_p = capture cross sections for electrons and holes respectively

* Stanford Electronics Laboratories, Stanford University, Stanford, California.

N_T = number of traps (acceptors)
n, p = electron and hole densities
n_0, p_0 = equilibrium electron and hole densities
n_1, p_1 = electron densities if the fermi level is at the trap level
N = average population of the electromagnetic modes
N_0 = equilibrium electromagnetic population and is just the Bose-Einstein function
N_V, N_C = effective density of states in the valence and conduction bands
n_i = intrinsic electron density

The net stimulated emission is[2]

$$(R - G)_s = v_t^2\sigma_n\sigma_p N_T \times \left\{\left[\left(pn - \frac{N_C - n}{N_C - n_0}\frac{N_V - p}{N_V - p_0}n_i^2 e^{h\nu/kT}\right)\left(\frac{N}{N_0 + 1}\right)\right] \Big/ \left[v_t\sigma_p(p + p_1)\frac{N_V - p}{N_V - p_0} + v_t\sigma_n\left(n\frac{N+1}{N_0+1} + n_1\frac{N}{N_0}\frac{N_C - n}{N_C - n_0}\right)\right]\right\}, \quad (3)$$

and the net gain becomes

$$g = \frac{\lambda^2 j\alpha}{8\pi n'^2 qd\Delta\nu} \times \frac{\left(pn - \dfrac{N_C - n}{N_C - n_0}\dfrac{N_V - p}{N_V - p_0}n_i^2 e^{h\nu/kT}\right)}{\left[pn(N+1) - N\dfrac{N_C - n}{N_C - n_0}n_i^2 e^{h\nu/kT}\right]}. \quad (4)$$

The gain at infinite current approaches an asymptotic value since N and j are linearly related[3] and is

$$g_\infty = \frac{\lambda^2 j\alpha}{8\pi n'^2 qd\Delta\nu N} \quad (5)$$

$$= \frac{(1 - R)(1 - \cos\theta)}{\sqrt{3}\, d}$$

if the photons are reflected at the boundary of the active region, and

$$= \frac{1}{\sqrt{3}\, d}$$

if the photons are not reflected but are immediately lost.

The saturated gain comes about because the photon flux prevents an arbitrarily large inversion of levels. The saturated gain is larger if there are no reflections at the boundary of the crystal since in this case a greater degree of inversion is possible.

Using the definition of g_∞ given in (5), Eq. (4) can be rearranged to yield

$$g = \frac{g_\infty}{1 + \left[(pn)\Big/\left(pn - \dfrac{N_C - n}{N_C - n_0}\dfrac{N_V - p}{N_V - p_0}n_i^2 e^{h\nu/kT}\right)\right]} \times \left[\frac{q(1-R)(1-\cos\theta)8\pi n'^2\Delta\nu}{\sqrt{3}\, j\alpha\lambda^2}\right]. \quad (6)$$

The quantity g is a monotonically increasing function of current. The threshold occurs when $g = \alpha_{\text{diff}} + T/l$. This equation is solved for current, and simplified by using Eq. (5) for the saturated gain. The threshold relation becomes

$$j_{\text{th}} = \left\{(pn)\Big/\left[pn - \frac{N_C - n}{N_C - n_0}\frac{N_V - p}{N_V - p_0}n_i^2 e^{h\nu/kT}\right] \times [1 - (T/l + \alpha_{\text{diff}})/g_\infty]\right\} \times \left\{\frac{8\pi n'^2 qd\Delta\nu}{\lambda^2\alpha}(T/l + \alpha_{\text{diff}})\right\}. \quad (7)$$

It should be noted that Eq. (7) is just Lasher's Eq. (6) for threshold current but modified by the factors involving the pn product and asymptotic gain. Actually, Eq. (7) is still an implicit relation for threshold current density since the current and the pn product are related. It does not appear possible to obtain an explicit form for the threshold current density but a great deal can be learned from the implicit relation. Firstly, the saturated gain at infinite current must exceed the losses to obtain a laser. The ratio of $\alpha_{\text{diff}}/g_\infty$ decreases as the active width increases. Thus a reasonably high mobility semiconductor is required. Also, a population inversion is necessary. If F_n and F_p are the quasi-fermi levels[2] for electrons and holes respectively, then the condition

$$pn - \frac{N_V - p}{N_V - p_0}\frac{N_C - n}{N_C - n_0}n_i^2 e^{h\nu/kT} > 0 \quad (8)$$

reduces to

$$F_n - F_p > h\nu. \quad (9)$$

For band-to-band recombination, the gain relation is identical to the relation for recombination through an impurity, even though the actual recombination rate appears to be quite different. Also, if the saturated gain is well in excess of the losses for the least lossy mode and if a population inversion is reached at a current well below the threshold, the threshold relation reduces to (1) with $\alpha_0 = 0$. If the current reaches the value calculated from (1) with $\alpha_0 = 0$ well before a population inversion is obtained then the most important criterion is that (9) be satisfied.

As applied specifically to a GaAs pn junction laser, the requirement that $pn > n_i^2 e^{h\nu/kT}$, which assumes nondegenerate statistics, can be written as

$$pn > N_C N_V \exp\{(h\nu - \mathcal{E}_g)/kT\}. \quad (10)$$

We do not know the magnitude of $\mathcal{E}_g - h\nu$ but it is certainly no more than 50 mv and is probably much less. We start our calculation with the formula for current in a diode:

$$j = \frac{qd}{\alpha\tau_n}(n - n_0), \tag{11}$$

where τ_n is the radiative electron recombination lifetime. If p and n are approximately equal at the threshold we have

$$n \simeq \sqrt{N_C N_C} \exp\{(h\nu - \mathcal{E}_g)/2kT\}$$

$$\simeq 2 \times 10^{18}/\mathrm{cm}^3 \left(\frac{T}{300}\right)^{3/2} \exp\{(h\nu - \mathcal{E}_g)/2kT\}. \tag{12}$$

We neglect the hole current injected into n- material since the lasing is thought to occur in the p-material.

The value of τ_n to be substituted into Eq. (11) should be the lifetime for radiative recombination. A value of 10^{-8} for τ_n seems reasonable and may be approximately justified as follows. In an ordinary GaAs diode operating at $T = 300°$K, $\tau_n = 10^{-9}$ sec is typical. Since a relatively small amount of light is observed from these diodes, we conclude that the 10^{-9} sec lifetime is due to nonradiative recombination processes, and that the lifetime for radiative processes must be 10^{-8} sec or greater. We further suppose that cooling the crystal increases the nonradiative lifetime but does not affect the radiative lifetime. Hence at liquid nitrogen or liquid helium temperatures, the radiative lifetime is the short one, and this determines the current.

With the aid of these approximations, Eq. (11) may be evaluated as follows:

$$J = \frac{1}{\alpha} 3.2 \times 10^4 \left(\frac{T}{300}\right)^{3/2}$$

$$\times \exp\{(h\nu - \mathcal{E}_g)/2kT\} \text{ amp/cm}^2. \tag{13}$$

This formula gives $j = 3.2 \times 10^4$ amp/cm^2 at $T = 300°$K; $j = 4.2 \times 10^3$ amp/cm^2 at $T = 77°$K; and $j = 48$ amp/cm^2 at $T = 4°$K if $\mathcal{E}_g - h\nu \ll kT$ and $\alpha = 1$. These values compare favorably with threshold current densities. Since the value of d has been taken from a laser which operates at liquid nitrogen temperatures, we expect the starting current to be most nearly correct there. Also, at liquid helium temperature, population inversion is reached well before the threshold so that formula (1) with $\alpha_0 = 0$ gives the correct value. At room temperature, the quantum efficiency α is considerably less than unity so the threshold is increased. The results of calculations and observations are listed below:

Calculation	*Observed*		*Conditions*
$3.2 \times 10^4 (\alpha = 1, h\nu = \mathcal{E}_g)$	10^5	(Ref. 4)	$T = 300°$K
$4.2 \times 10^3 (\alpha = 1, h\nu = \mathcal{E}_g)$	8×10^3	(Ref. 5)	$T = 77°$K
830 (as calculated by Lasher[1])	700	(Ref. 6)	$T = 4.2°$K
	80	(Ref. 7)	$T = 2°$K

The agreement is somewhat fortuitous since at 300°K the quantum efficiency is certainly less than unity. This discrepancy may be offset by having $\mathcal{E}_g - h\nu > 0$. For the helium temperature calculation, too little is known about capture cross sections to be confident of the lifetime values to use, and hence the width of the active region, d, is open to considerable question.

References

1. G. J. Lasher, *IBM Journal* **7**, 58 (1963).
2. M. Bernard and G. Duraffourg, *Physica Status Solidi* **1**, 699 (1961).
3. W. P. Dumke, *IBM Journal* **7**, 66 (1963).
4. G. Burns and M. I. Nathan, *IBM Journal* **7**, 72 (1963).
5. R. A. Laff, W. P. Dumke, F. H. Dill, Jr., and G. Burns, *IBM Journal* **7**, 63 (1963).
6. T. M. Quist, R. H. Rediker, R. J. Keyes, W. E. Krag, B. Lax, A. L. McWhorter, and H. J. Zeigler, *Appl. Phys. Letters* **1**, 91, (1962).
7. W. E. Howard, F. F. Fang, F. H. Dill and M. I. Nathan, *IBM Journal* **7**, 74 (1963).

Received January 31, 1963