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Nominal Clearance of the Foil Bearing

Symbols

h Clearance between foil and cylinder
h^* Nominal clearance
P Pressure in the lubricating film
R Radius of the cylinder
T Tension in the foil
U Velocity of the foil
x Dimensionless distance around the cylinder
μ Viscosity of the lubricant
ρ Radial coordinate of the foil
Θ Angular coordinate of the foil

Classic lubrication theory has generally been directed toward fluid films bounded by solid objects that were assumed to be infinitely rigid. This assumption, however, is contrary to the physical nature of things. Blok and van Rossum[1] introduced the concept that the rigid bearing could be replaced with a perfectly flexible band. This configuration has been termed the *foil bearing.* It represents the other extreme situation. Between these limits then are the cases of practical engineering interest. The hydrodynamic air lubrication of magnetic tape over magnetic heads, to prevent contact and wear, closely approximates the foil bearing. From the viewpoint of engineering design there is a great deal of interest in this configuration. The parameters of the foil bearing discussed in this Communication are shown in Fig. 1.

Three equations have been developed in the literature which represent the clearance as a function of the other parameters of the foil bearing. The first, Eq. (1) in Table 1, with its solution results from the most extensive set of simplifying assumptions that can be made. It was published by Blok and van Rossum. They also presented the second, Eq. (2), but without a solution. The third, Eq. (3), was published by Patel and Cameron[2], again without a solution.

The analysis presented here is similar to the analyses developed in the literature. The conventional approach has been directed towards incompressible fluids, infinitely wide and perfectly flexible foils, and negligible effects of fluid friction and foil inertia. The essential difference here is the order in which the analytic and simplifying steps are taken. In this analysis higher order terms are eliminated last. First, in Eq. (A), the pressure is written as a function of the radius vector and its derivatives with respect to the angle:

$$P = T[(\rho^2 + 2\rho'^2 - \rho\rho'')/(\rho^2 + \rho'^2)^{3/2}]. \tag{A}$$

Second, the derivative of this pressure function with respect to the angle is taken:

$$P' = T[(-\rho^3\rho' - 4\rho\rho'^3 - 3\rho'^3\rho'' + 3\rho\rho'\rho''^2 + 3\rho^2\rho'\rho'' - \rho^3\rho''' - \rho\rho'^2\rho''')/(\rho^2 + \rho'^2)^{5/2}]. \tag{B}$$

Third, the radius vector is replaced with its equivalent [Eq. (C)] in terms of the cylinder radius and clearance:

$$\rho = R + h. \tag{C}$$

Fourth, in the resulting expression for the rate of change of pressure, all powers greater than one of the clearance and of the first derivative of the clearance, are assumed infinitesimally small and consequently negligible. Similarly, the products of the clearance and its first derivative can be and are neglected. This results in a considerable simplification:

$$P' = -\frac{T}{R^2}h''' - \frac{T}{R^2}h' + 3\frac{T}{R^3}h'h'' + 3\frac{T}{R^4}h'h''^2 + 2\frac{T}{R^3}hh'''. \tag{D}$$

Fifth, the resulting expression for the rate of change of pressure is introduced into the first integral of the well-known Reynolds' Equation, which gives

$$P' = 6\mu UR\frac{h - h^*}{h^3}. \tag{E}$$

With the simplifying substitutions of Eqs. (F) and (G)

$$h = h^*H \tag{F}$$

$$\theta = \frac{h^*}{R}\left(\frac{T}{6\mu U}\right)^{1/3}x \tag{G}$$

the final form is obtained in Eq. (4), Table 1.

Table 1 **Clearance in terms of other parameters in the foil bearing.**

	Equation	$\frac{h^*}{R}\left(\frac{T}{6\mu U}\right)^{2/3}$
(1)	$H''' = 0$	0.426
(2)	$H''' = \frac{1-H}{H^3}$	0.642
(3)	$H''' + \left(\frac{h^*}{R}\right)^2\left(\frac{T}{6\mu U}\right)^{2/3} H' = \frac{1-H}{H^3}$	0.639
(4)	$H''' + \left(\frac{h^*}{R}\right)^2\left(\frac{T}{6\mu U}\right)^{2/3} H' - 3\frac{h^*}{R} H'H'' - 3\left(\frac{6\mu U}{T}\right)^{2/3} H'H''^2 - 2\frac{h^*}{R} HH''' = \frac{1-H}{H^3}$	0.650
(5)	$H''' + \left(\frac{h^*}{R}\right)^2\left(\frac{T}{6\mu U}\right)^{2/3} H' - 3\frac{h^*}{R} H'H'' - 3\left(\frac{6\mu U}{T}\right)^{2/3} H'H''^2 = \frac{1-H}{H^3}$	0.650
(6)	$H''' - 5\left(\frac{6\mu U}{T}\right)^{2/3} H'H''^2 = \frac{1-H}{H^3}$	0.650

Six equations and their numerical solutions are given in the table. As already pointed out, (1) and (2) are from Blok and van Rossum and (3) is from Patel and Cameron. Equation (4) is the one developed here. One of the terms in this equation $[2(h^*/R)HH''']$ was found to be insignificantly small and is omitted in (5). Furthermore, three of the remaining terms $[(h^*/R)^2(T/6\mu U)^{2/3}H'$, $3(h^*/R)H'H''$, $3(6\mu U/T)^{2/3}H'H''^2]$ are insignificantly small except where the clearance is large. However, in this region of the foil bearing, the quantity $(h^*/R)(T/6\mu U)^{2/3}$ is very nearly equal to the second derivative, H''. This approximation is included in (6).

From an examination of the numerical solutions in the table (right-hand column) it is apparent that the first is substantially smaller than the rest. This is directly attributable to the extensive simplifications incorporated in the equation. The answer to (2) is considerably greater than the answer to (1). Because fewer assumptions were made in the derivation of (2), it is reasonable to believe that its solution more nearly approximates the true solution to the foil bearing. Although (3) has the correct second term, its solution would seem to be further from the true value. This is so because there are more terms of the same magnitude that did not show up in the analysis. Equation (4) is believed to be more complete and to offer the most accurate solution because it not only has the first-order term but all the three second-order terms and one of the perhaps many third-order terms. The insignificance of the third-order term is demonstrated by the agreement between the solutions to (4) and (5). Furthermore, the insensitivity of the solutions to the exact form of the three second-order terms is demonstrated by the close agreement between the solutions of (4) and (6). The equations may be ordered for increasing accuracy and complexity as (1), (2), (5), and (4). Equation (6) in the table is particularly interesting and useful. Not only does it have a relatively accurate solution, but the solution may be quickly found by numerical methods because the unknown quantity, h^*/R, does not appear.

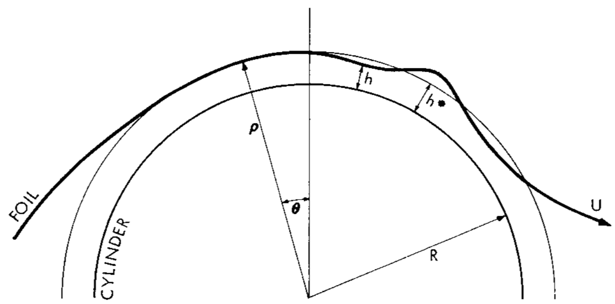

Figure 1 **The foil bearing**

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References

1. H. Blok and J. J. van Rossum, "Foil Bearing—New Departure in Hydrodynamic Lubrication," *Lubrication Engineering* **9**, 316 (1953).
2. B. J. Patel and A. Cameron, "The Foil Bearing," *Proceedings of the Conference on Lubrication and Wear*, London (Institution of Mechanical Engineers) October 1-3, 1957, p. 219.

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