

The Lightly Loaded Foil Bearing at Zero Angle of Wrap

Abstract: A method is developed for determining, to the first order, the deflection from a straight path of a perfectly flexible tape moving near a rigid cylinder. The case of a parabolic cylinder is considered.

Introduction

In most applications of fluid-film lubrication, the lubricant is contained between two rigid, or nearly rigid, surfaces. A notable exception is the foil bearing, in which the lubricant is entrained between a rigid surface and a membrane. Such a bearing is generated, for example, when a recording tape moves rapidly over a read-write head. For a more familiar example, we may cite the slipperiness and backlash of a roll of photographic film which is rapidly wound or unwound.

This paper treats the foil-bearing problem in two-dimensions, i.e., an infinitely wide membrane moving over a rigid cylinder. In the mathematical formulation, as set out in Section 1, the dynamical equations of the moving foil are coupled to the Reynolds equation for the pressure in the lubricant.

In Section 2, a first integral of the lubrication equation is considered. It is shown that there is a region where the pressure under the foil is subambient; in this region the foil is concave away from the cylinder.

The special case of incompressible lubricant and negligible drag is treated in Section 3. The dynamical equation of the foil is combined with the Reynolds equation, yielding a single, fourth-order, ordinary differential equation for the fluid film thickness.

In Section 4, we derive the approximate equations which apply when the bearing effect is small, causing the foil to deflect only slightly from a straight-line path. In Section 5, these equations are applied to the case of a foil moving near a parabolic cylinder, and the deflected profile is calculated explicitly.

1. The general problem of the inelastic foil bearing

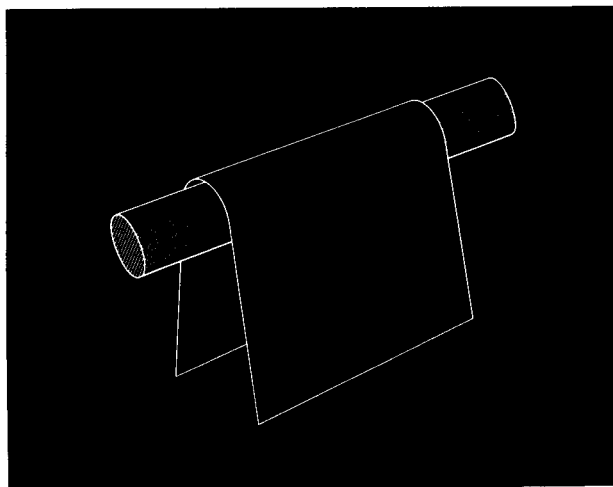
Previous authors^{1,2,3} have considered the problem of a flexible tape moving in proximity to a rigid cylinder, with a thin film of fluid entrained. The dynamical equations

governing the motion of the foil are coupled to the lubrication equation relating the fluid pressure to the film thickness.

The tape is customarily assumed to be a perfectly flexible, completely inextensible membrane. A further assumption, that the center line of the tape lies in a plane and that the tape itself is perpendicular to this plane, rules out cases in which the tape is twisted or bowed across its width. With these assumptions, the equations governing the motion of the tape are exactly those governing the motion of an inextensible string constrained to move in a plane.

Referring to Fig. 2, let T be the tension in the tape and let F denote the external force per unit area acting on the

Figure 1 The foil bearing.



tape. Denote by $\mathbf{X}$ the vector from the origin of reference to a point on the tape. Since the tape is in motion,

$$\mathbf{X} = \mathbf{X}(s, t), \quad (1.1)$$

where t is time and s is arc length along the tape measured from some reference point. The unit tangent vector α is given by

$$\alpha = \partial \mathbf{X} / \partial s. \quad (1.2)$$

The total force per unit width acting on a segment of length Δs is

$$-\alpha T + (\alpha + \Delta \alpha)(T + \Delta T) + \mathbf{F} \Delta s. \quad (1.3)$$

Passing to the limit of infinitesimal Δs and using (1.2), we obtain

$$[\partial / \partial s (T \partial \mathbf{X} / \partial s) + \mathbf{F}] ds. \quad (1.4)$$

If σ denotes the mass per unit area of the tape, Newton's second law then requires that

$$\partial / \partial s (T \partial \mathbf{X} / \partial s) + \mathbf{F} = \sigma \mathbf{A}, \quad (1.5)$$

where $\mathbf{A}$ denotes the acceleration of the infinitesimal segment of tape.

The normal and tangential components of (1.5) can be distinguished by using the relation

$$\partial^2 \mathbf{X} / \partial s^2 = -K \mathfrak{z}, \quad (1.6)$$

where $\mathfrak{z}$ is the unit normal, drawn to the right as we proceed in the direction of increasing s , and K is the curvature with the sign so chosen that, as we move in the direction of increasing s , K is positive when the tape is concave to the left. With (1.2) and (1.6), (1.5) becomes

$$\alpha (\partial T / \partial s) - \mathfrak{z} K T + \mathbf{F} = \sigma \mathbf{A}. \quad (1.7)$$

This paper is restricted to the case of a tape in steady motion parallel to itself with speed U . Thus, the velocity $\mathbf{V}$ of a point on the tape is given by

$$\mathbf{V} = U \alpha, \quad (1.8)$$

and its acceleration $\mathbf{A}$ is given by

$$\mathbf{A} = (d\mathbf{V} / ds) \dot{s} = U^2 d\alpha / ds = -K U^2 \mathfrak{z}, \quad (1.9)$$

a purely centripetal acceleration.

We neglect all forces acting on the tape except those arising from the lubricating film. Thus,

$$\mathbf{F} = -\tau \alpha + (p_f - p_a) \mathfrak{z}, \quad (1.10)$$

where τ is the viscous drag on the tape, p_f is the pressure at any point in the lubricating film, and p_a is the ambient pressure. The minus signs in (1.10) are appropriate for a configuration similar to that illustrated in Fig. 3.

Substituting (1.9) and (1.10) into (1.7), yields the tangential and normal equations of motion for the tape:

$$dT / ds = \tau, \quad (1.11)$$

$$(p_f - p_a) = K(T - \sigma U^2). \quad (1.12)$$

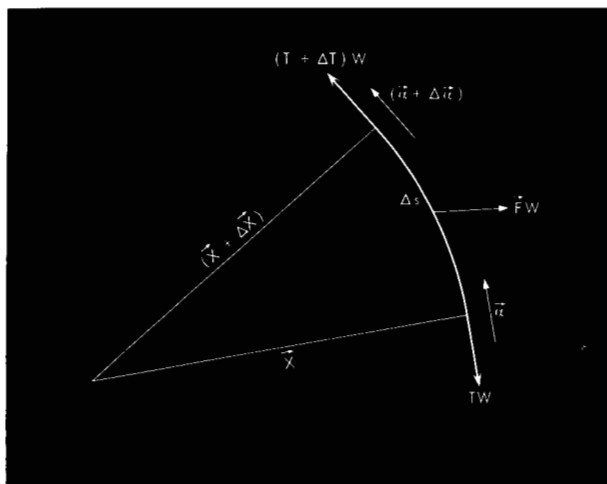


Figure 2 Forces on a segment.

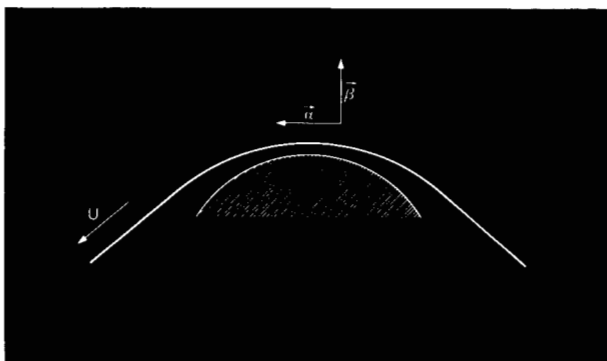


Figure 3 Tape configuration.

With the usual assumptions of steady-state lubrication theory, the pressure in the fluid film is governed by the Reynolds equation

$$(d/ds)(\rho h^3 dp_f / ds) = 6\mu U d(\rho h) / ds, \quad (1.13)$$

where ρ and μ denote, respectively, the density and viscosity of the lubricant, and h is the thickness of the fluid film.

In addition to Eqs. (1.11), (1.12), and (1.13), some assumption must be made about the relationship between p_f and ρ . In all of the references cited, it is assumed that the lubricant is incompressible, so that ρ is constant. In certain cases, it may be more realistic to assume that the lubricant is an isothermal gas, so that ρ is proportional to p_f .

2. A first integral of the lubrication equation

The Reynolds equation (1.13) can be integrated once, yielding

$$dp_f / ds = 6\mu U(\rho h - c) / \rho h^3, \quad (2.1)$$

where c is a constant of integration. Since

$$\lim_{s \rightarrow -\infty} h = \infty, \quad \lim_{s \rightarrow -\infty} \rho = \rho_a, \quad (2.2)$$

where ρ_a is the ambient density of the lubricant, (2.1) requires that for $|s|$ sufficiently large, $dp/ds > 0$. Because

$$\lim_{s \rightarrow -\infty} p_f = p_a, \quad (2.3)$$

the film pressure is subambient for s large and positive. Provided the tension T never falls below σU^2 , (1.12) then requires that the foil be concave away from the cylinder in the distant portion of the exit region.

This conclusion is based on the assumption that the Reynolds equation can be applied with negligible error, even where h is large. Strictly speaking, this is invalid, but where h is large, the pressure change is slight. The significant variation of pressure occurs where the Reynolds equation can validly be applied.

3. The incompressible foil bearing with negligible drag

The viscous drag on the tape is negligible compared with the lubricating pressure whenever μU is small compared with the minimum value of $T - \sigma U^2$. This is usually the case in practice.* Equation (1.11) then stipulates that the tape tension is constant. We shall henceforth assume that this is the case.

We assume further that the compressibility of the lubricant can be neglected. The fluid density and the ambient pressure then cease to be relevant quantities, so that (1.12) and (1.13) become, respectively,

$$p = K(T - \sigma U^2) \quad (3.1)$$

$$(d/ds)(h^3 dp/ds) = 6\mu U dh/ds, \quad (3.2)$$

where p is the gage pressure of the fluid, i.e.,

$$p = p_f - p_a. \quad (3.3)$$

The pressure can be eliminated from (3.1) and (3.2):

$$(T - \sigma U^2)(d/ds)(h^3 dK/ds) = 6\mu U(dh/ds). \quad (3.4)$$

First integrals of (3.2) and (3.4), respectively, are given by

$$dp/ds = 6\mu U(h - h^*)/h^3 \quad (3.5)$$

and

$$(T - \sigma U^2) dK/ds = 6\mu U(h - h^*)/h^3, \quad (3.6)$$

where h^* is a constant of integration.

4. Approximate solution for slight deflection

Let us now assume that the presence of the rigid cylinder causes the tape to deflect only slightly from a straight line. Thus, we assume that the angle of wrap is zero and that the pressure loading of the tape is small, in a sense best defined *a posteriori*. We specify that the undeflected tape

would move along the $y = 0$ axis of the x - y plane and that the actual path followed by the tape is the curve

$$y = \delta(x). \quad (4.1)$$

If the boundary of the rigid cylinder is the curve

$$y = -h_0(x) < 0, \quad (4.2)$$

the film thickness is given by

$$h(x) = h_0(x) + \delta(x). \quad (4.3)$$

We require that, for all x ,

$$|\delta(x)|/h_0(x) \ll 1 \quad (4.4)$$

and

$$|\delta'(x)| \ll 1. \quad (4.5)$$

If the arc length s is measured from the point $x = 0$, $y = \delta(0)$, then

$$s = \int_0^x \sqrt{1 + [\delta'(x)]^2} dx. \quad (4.6)$$

Since, by the requirement (4.4), $\delta'(x)$ is small compared with unity, s can be approximated by x . Furthermore,

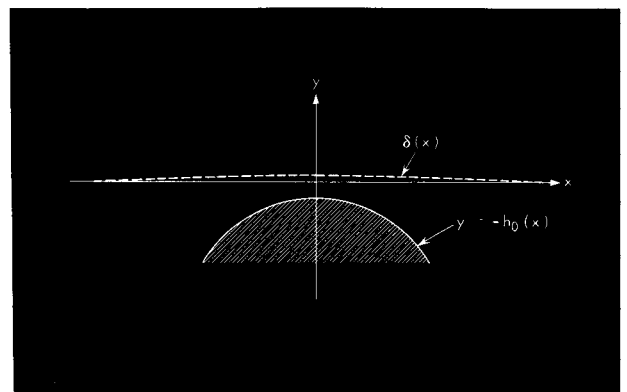
$$K \approx -\delta''(x), \quad (4.7)$$

so that the foil bearing equation (3.4) becomes

$$\begin{aligned} \frac{d}{dx} \left[(h_0 + \delta)^3 \frac{d^3 \delta}{dx^3} \right] \\ + \frac{6\mu U}{T - \sigma U^2} \left(\frac{dh_0}{dx} + \frac{d\delta}{dx} \right) = 0. \end{aligned} \quad (4.8)$$

We have not yet used the requirement (4.4); Eq. (4.8) has been established on the basis of (4.5) only. It is evident that (4.4) can hold only if the coefficient $6\mu U/(T - \sigma U^2)$ in (4.8) is at least of the same order of smallness as δ/h_0 . To appreciate the physical significance of this, consider a hypothetical experiment. Let the tape be constrained to move along $y = 0$. A bearing force will be generated be-

Figure 4 The slightly deflected foil.



* Conceivably $T - \sigma U^2$ could be negative. This would mean, however, that the tape is moving faster than its own sound speed, $\sqrt{T/\sigma}$.

tween the surfaces $y = 0$ and $y = -h_0(x)$. Now remove the constraints. The bearing force will tend to deflect the tape away from $y = 0$. This tendency will be resisted by the straightening effect of the tension in the tape. Thus, the deflection will be slight only if the tension is high.

Neglecting second-order terms in (4.8), we find that the deflection is determined by

$$(T - \sigma U^2)(d/dx)(h_0^3 d^3\delta/dx^3) + (6\mu U) dh_0/dx = 0, \quad (4.9)$$

subject to appropriate boundary conditions, which must include the vanishing of the tape curvature at each end of the lubricating film. Alternatively, we can determine a pressure field $p_0(x)$ by integrating the equation

$$(d/dx)(h_0^3 dp_0/dx) = 6\mu U dh_0/dx, \quad (4.10)$$

subject to the vanishing of p_0 at each end of the film, and then determine the deflection from

$$(d^2\delta/dx^2) + p_0/(T - \sigma U^2) = 0. \quad (4.11)$$

With $h_0(x)$ specified, neither of these approaches presents any conceptual difficulty.

5. An illustration: the deflection of a foil by a parabolic cylinder

Consider now the specific case in which

$$h_0(x) = G + x^2/2R, \quad (5.1)$$

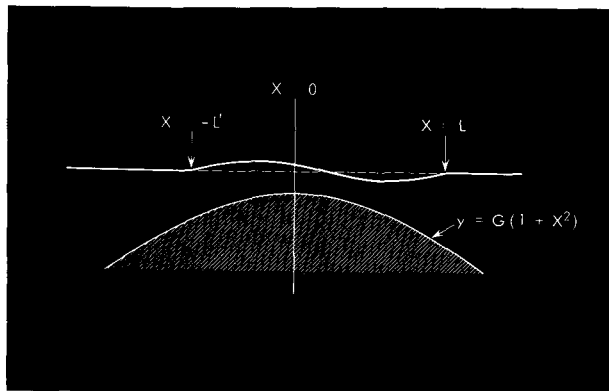
where G and R are constants. Integration of (4.10) then yields

$$p_0 = A + B \left[3 \arctan X + \frac{(5 + 3X^2)X}{(1 + X^2)^2} \right] - \frac{2\sqrt{2}\mu U}{G} \sqrt{\frac{R}{G}} \frac{X}{(1 + X^2)^2}, \quad (5.2)$$

where A and B are constants of integration and X is a "stretched" coordinate defined by

$$X = x/\sqrt{2RG}.$$

Figure 5 Location of the tape guides.



We assume for simplicity that the parabola (5.1) extends to $\pm\infty$ and, as discussed in Section 2, we ignore the inappropriateness of the lubrication equation where the film thickness is large.

We note in passing that the form for $h_0(x)$ specified by (5.1) provides a useful approximation for the case in which the undeflected path of the foil lies a distance G from a circular cylinder of radius R .

The pressure p_0 vanishes at $\pm\infty$ if we set $A = B = 0$ in (5.2). Thus

$$p_0 = -\frac{2\sqrt{2}\mu U}{G} \sqrt{\frac{R}{G}} \frac{X}{(1 + X^2)^2}. \quad (5.3)$$

Integration of (4.11) then yields

$$\delta = \frac{2\sqrt{2}\mu UR}{T - \sigma U^2} \sqrt{\frac{R}{G}} (a + bX - \arctan X), \quad (5.4)$$

where a and b are constants of integration.

Let us now assume that the tape is constrained by tape guides at $X = -L', L$, as illustrated in Fig. 5. Thus, the constants of integration in (5.4) are determined by setting

$$\delta(-L') = \delta(L) = 0, \quad (5.5)$$

so that

$$a = \frac{L' \arctan L - L \arctan L'}{L + L'}, \quad (5.6)$$

$$b = \frac{\arctan L + \arctan L'}{L + L'}, \quad (5.7)$$

and

$$\delta = \frac{2\sqrt{2}\mu UR}{T - \sigma U^2} \sqrt{\frac{R}{G}} \left[\frac{\arctan L}{L + L'} (X + L') + \frac{\arctan L'}{L + L'} (X - L) - \arctan X \right]. \quad (5.8)$$

Consider now a limiting case in which one of the tape guides is moved infinitely far from $X = 0$, e.g., let us assume that $L' \rightarrow -\infty$. With (5.6),

$$\lim_{L' \rightarrow -\infty} a = \arctan L \quad (5.9)$$

and, with (5.7),

$$\lim_{L' \rightarrow -\infty} b = 0. \quad (5.10)$$

Thus, when the only tape guide is at $X = L$,

$$\delta = \frac{2\sqrt{2}\mu UR}{T - \sigma U^2} \sqrt{\frac{R}{G}} (\arctan L' - \arctan X), \quad (5.11)$$

so that there is a finite deflection at minus infinity. Similarly, when the only tape guide is at $X = -L'$,

$$\delta = -\frac{2\sqrt{2}\mu UR}{T - \sigma U^2} \sqrt{\frac{R}{G}} \cdot (\arctan L' + \arctan X), \quad (5.12)$$

and there is a finite deflection at plus infinity.

If the guides are symmetrically placed at $x = \pm L$, then
 $a = 0$, (5.9)

$$b = L^{-1} \arctan L, \quad (5.10)$$

and

$$\delta = \frac{2\sqrt{2} \mu UR}{(T - \sigma U^2)L} \sqrt{\frac{R}{G}} \cdot (X \arctan L - L \arctan X). \quad (5.11)$$

As $L \rightarrow \infty$, $b \rightarrow 0$, so that

$$\begin{aligned} \delta &= -\frac{2\sqrt{2} \mu UR}{T - \sigma U^2} \sqrt{\frac{R}{G}} \arctan X \\ &= -\frac{2\sqrt{2} \mu UR}{T - \sigma U^2} \sqrt{\frac{R}{G}} \arctan \frac{x}{\sqrt{2RG}}. \end{aligned} \quad (5.12)$$

We have proceeded on the assumption that the method outlined in Section 4 leads to a valid approximation. By comparing (5.1) and (5.12), we see that the criterion (4.4) is satisfied if

$$\frac{2\sqrt{2} \mu U}{(T - \sigma U^2)} \left(\frac{R}{G}\right)^{3/2} \ll 1. \quad (5.13)$$

Differentiating (5.12), we see that (4.5) is satisfied if

$$\frac{2\mu U}{T - \sigma U^2} \frac{R}{G} \ll 1. \quad (5.14)$$

Since the lubrication approximation requires that G be small compared with R , the criterion (5.14) is satisfied whenever (5.13) is satisfied.

By way of example, if the tape moves at a speed of 100 in/sec through air (viscosity 2.7×10^{-9} lb sec/in²), and if $R/G = 1000$, then (5.13) is satisfied if $(T - \sigma U^2)$ greatly exceeds 2.4×10^{-2} lb/in.

References

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