H. B. Bebb

A Polarimetric Method of Measuring Magneto-Optic Coefficients

Abstract: Formulas are established which allow polarimetric determination of the amplitudes and relative phases of the Kerr (and Faraday) coefficients for ferromagnetic metals. The results are valid only for oblique angles of incidence. Hence, the technique is most useful for measurement of thin films where the magnetization vector lies in the plane of the film.

Introduction

The magneto-optic Kerr effect is most commonly evaluated photometrically in terms of the electrical current change (signal) upon reversal of the magnetization in the ferromagnetic specimen and the inherent current fluctuations (noise).[1-5] However, when detailed properties of the Kerr effect are of interest, e.g., the evaluation of the reflectivity matrix defined by Eq. (1), photometric data is not convenient. In this paper, a new, completely polarimetric method is developed for determining the complex reflectivity matrix. The method depends on the use of oblique angles of incidence; thus it is probably most useful for thin films where the magnetization vector is in the plane of the film.

The optical properties of a magnetic material can be characterized by two reflection coefficients, one independent and one dependent on magnetic anistropy.[6-7] This assumption is consistent with the theory of wave propagation in a magnetic anisotropic medium.[8] The relation between the reflected and incident vector is:

$$\begin{pmatrix}\bar{A}_p\\ \bar{A}_s\end{pmatrix}=\begin{pmatrix}A_p{}^m\exp(i\varepsilon_p{}^m)\\ A_s{}^m\exp(i\varepsilon_s^m)\end{pmatrix}$$

$$=\begin{pmatrix}r_{pp}\exp(i\delta_{pp}) & r_{ps}\exp(i\delta_{ps}{}^m)\\ r_{sp}\exp(i\delta_{sp}{}^m) & r_{ss}\exp(i\delta_{ss})\end{pmatrix}\begin{pmatrix}E_p\\ E_s\end{pmatrix}, \qquad (1)$$

where $\begin{pmatrix}E_p\\ E_s\end{pmatrix}$ is the amplitude vector of the incident radiation in the usual coordinate system defined in terms of the propagation vector **k** and the plane of incidence (see Fig. 1). The diagonal elements of the matrix obey the Fresnel laws of reflection and shall henceforth be denoted by single subscripts. The off-diagonal terms account for the magneto-optic properties of the material. The superscript m denotes the two anti-parallel magnetization directions, $m = 1$ and $m = 2$. The Kerr coefficients r_{ps}, r_{sp} do not change if the magnetization is reversed only and not altered in magnitude, while the phase angles $\delta_{ps}{}^m$, $\delta_{sp}{}^m$ change by π.

Irradiance and polarimetric relations

The experimental technique relies on measurements made with the polarizer and analyzer nearly crossed. Equations for interpreting these measurements will be developed with reference to the *sp*-element of the matrix equation, (1). A second set of equations and measurements must be used to find the *ps*-element. (See last paragraph of this section.) The matrix equation can be written formally as $A = RE$, where R denotes the reflectivity matrix, A is the reflected amplitude vector and E is the incident amplitude vector $E\begin{pmatrix}B_1\cos\phi\\ B_2\sin\phi\end{pmatrix}$. The constants B_1, B_2 account for any partial polarization that might occur in the source after collimation into a parallel beam. The component A_t of the amplitude A in the transmission direction $\hat{t}=\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$ of the analyzer is found by taking the scalar product $\hat{t}\cdot A = A_t$. The polarizing elements can be represented by a vector only if they are assumed perfect.† However, the results obtained do not differ significantly if a more realistic matrix

† The case here should be contrasted to the one treated by P. H. Lissenberger,[8] appearing after the present paper was submitted for publication.

representation of the polarizers is used. The irradiance (flux/area) in the $\hat{t}$ direction is just the modulus $A_t^* A_t$ which, to second order in r_{sp}, the polarizer angle ϕ, and the analyzer angle $\theta = (\pi/2) - \alpha$, is

$$I(m)/I_0 = A_t^* A_t/I_0 = (R_p\alpha)^2 + (R_s\phi)^2 + R_{sp}{}^2 + 2R_pR_s\cos(\delta)\alpha\phi + 2R_{sp}R_p\cos(\Delta_p{}^m)\alpha + 2R_{sp}R_s\cos(\Delta_s{}^m)\phi\,. \tag{2}$$

Equation (2) is a simplified form valid for small angles ϕ, α, since in general, the Kerr coefficients are very small. The notation

$$R_p = B_1C_1r_p\,, \qquad R_s = B_2C_2r_s\,, \qquad R_{sp} = B_1C_2r_{sp}$$

$$\delta = \delta_p - \delta_s\,, \qquad \Delta_p{}^m = \delta_p - \delta_{sp}{}^m\,, \qquad \Delta_s{}^m = \delta_s - \delta_{sp}{}^m$$

is introduced for simplification. The constants C_1, C_2 account for any polarization effects in the optics between the analyzer and the photomultiplier.

The difference in irradiance ΔI for two reversed states of magnetization follows immediately from Eq. (2).

$$[I(m=2) - I(m=1)]/I_0 = \Delta I/I_0 = -4R_{sp}[R_p\cos(\Delta_p{}^{m=1})\alpha + R_s\cos(\Delta_s{}^{m=1})]\,. \tag{3}$$

The expression is valid for arbitrary settings ϕ, α (small angles). However, particular angles can be experimentally determined by a trial and error search for $\phi = \phi(m)$ and $\alpha = \alpha(m)$ yielding $I(m)$ a minimum. The extremal values of ϕ and α are found by taking derivatives of Eq. (2) and are given by

$$\phi(m) = -\frac{R_{sp}\sin\Delta_p{}^m}{R_s\sin\delta} \tag{4}$$

and

$$\alpha(m) = -\frac{R_{sp}\sin\Delta_s{}^m}{R_p\sin(-\delta)}\,. \tag{5}$$

A direct substitution of Eqs. (4), (5) into Eq. (2) reduces to $I(m=1) = 0$, implying that when the polarizer angle is $\phi(m=1)$, the light is linearly polarized and $\alpha(m=1)$ is the analyzing angle which gives extinction. For any other polarizing angle $\phi \neq \phi(m=1)$, the light is elliptically polarized and total extinction cannot be obtained. Note that upon reversal of the magnetization from $m=1$ to $m=2$, the light again becomes elliptically polarized. Hence, it is not possible to obtain extinction for $m=2$ by changing just the analyzer setting. An exception occurs when $\Delta_p{}^m = \delta_p - \delta_{sp}{}^m = 0, \pi$. In this case, the polarizing angle $\phi(m)$ which yields the reflected beam linearly polarized is zero for both states of magnetization.

The irradiance difference ΔI for the minimal values of ϕ and α defined by Eqs. (4), (5) is given by

$$\Delta I/I_0 = 4R_{sp}{}^2 = 4(B_1C_2r_{sp})^2\,. \tag{6}$$

For the sake of comparison, it is interesting to consider also the case when the incident polarization corresponds to the plane of incidence, i.e., $\phi = 0$. The corresponding expression is

$$\Delta I/I_0 = 4R_{sp}{}^2\cos^2(\Delta_p{}^{m=1})\,. \tag{7}$$

The difference between the physical situations leading to Eq. (6) and Eq. (7) is just the presence of a component oscillation, introduced when $\phi \neq 0$, which is capable of interference (coherent) with the Kerr oscillation. This possibility was recognized by Heinrich[2] but was not investigated because, it is presumed, of the obvious difficulties inherent in doing so when the light is normally incident on the surface of the specimen. The presence of interfering components permits polarimetric determination of the amplitude and phase of the Kerr coefficient.

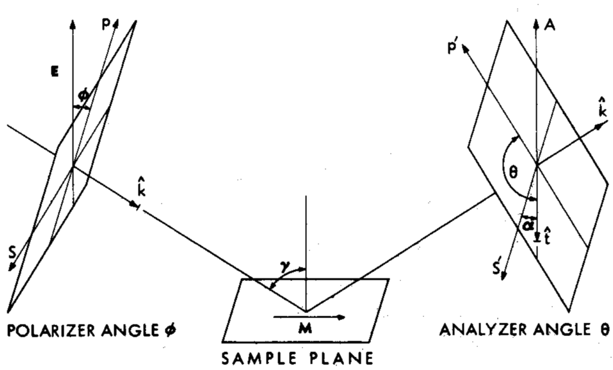
If the rotation $\Delta\psi$ of the major axis of the ellipse is measured, it is possible to determine the product of the amplitude and the cosine of the phase of the Kerr coefficient but not the amplitude and phase separately. A slight generalization of the results of Heinrich[2] gives the change in ψ upon reversal of the magnetic field as

$$\Delta\psi = \frac{2C_2r_{sp}}{C_1r_p}\cos(\Delta_p{}^{m=1})\,. \tag{8}$$

Equations (4), (8) are sufficient to polarimetrically determine the magnitude and relative phase of the Kerr component, r_{sp} and $\Delta_p{}^m = (\delta_p - \delta_{sp}{}^m)$ which are of interest in the case where the polarization angle ϕ is small so that $E_p \gg E_s$. Similar expressions involving r_{ps} and $(\delta_s - \delta_{ps}{}^m)$ are easily found by interchanging all subscripts s, p and 1, 2 and measuring the angles α and ϕ relative to the normal of the plane of incidence rather than to the plane.

Figure 1 **Resolution of vectors into p and s components at the polarizer and analyzer.**

$$E = E\begin{pmatrix}B_1\cos\phi\\ B_2\sin\phi\end{pmatrix},\ \hat{t} = \begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix},\ \text{and}\ \alpha = (\pi/2) - \theta$$



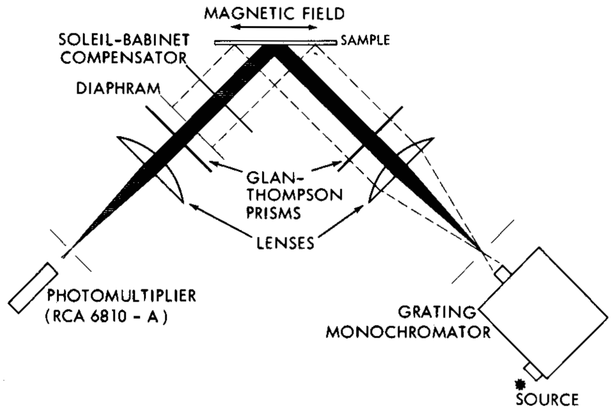


Figure 2 **Experimental arrangement.**

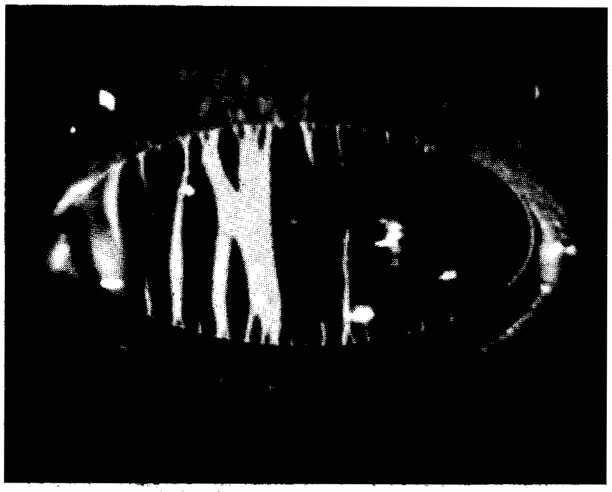


Figure 3 **The photograph illustrates the extinction possible for one magnetization state, showing that the reflected light is linearly polarized.**
The domains are magnetized in the opposite direction.

Experimental arrangement

The experimental sample used to illustrate the measurement technique was an iron film evaporated onto a glass substrate elevated to a temperature of 300°C at a pressure below 10^{-5} mm Hg in the presence of a uniform magnetic field of 15 gauss. The iron film was about 1000 A thick and possessed a uniaxial easy direction of magnetization with high remnant magnetization. The sample was mounted with the easy direction of magnetization parallel to the plane of incidence and the magnetic field used for switching. Glan-Thompson prisms mounted in verniers with an angular resolution of 0.005° and a Soleil-Babinet compensator arranged as shown in Fig. 2, were utilized for the measurements. The compensator was used only for determining the diagonal matrix elements A 1000 w high-pressure mercury-xenon short arc lamp driven by a well-regulated dc power supply yielded a short-term intensity fluctuation of $\pm 2\%$. The light was collimated through a grating monochromator and focused on a pin hole. The light emanating from the pin hole was collimated through the polarizer and reflected from the sample. The reflected beam passed through a circular aperture, the analyzer, and was focused on a diffusing frosted glass placed in front of the photomultiplier aperture. The photomultiplier tube (6810A) was magnetically shielded. A magnetic field of about 100 gauss was used to switch and orient the magnetization direction in the sample. The polarization of the source was measured to determine $B_1/B_2 = 1.12$. The readings of the photomultiplier were found to be independent of the polarization so $C_1/C_2 = 1$.

Measurement of r_{sp} and $\delta_p - \delta_{sp}{}^m$

The polarimetric method described above is illustrated by measurements on an evaporated iron film. From the polarimetric measurements of $\phi(m)$ and $\Delta\psi$, it is now possible to infer the values of the Kerr coefficient, r_{sp}, and its relative phase angle $\Delta_p{}^m = \delta_p - \delta_{sp}{}^m$ through the use of Eqs. (4), (8). It is first necessary to determine the normal reflectivity coefficients r_p, r_s and the phase angle $\delta = \delta_p - \delta_s$. Techniques used for measuring these parameters, plotted in Figs. 4 and 7, are discussed extensively elsewhere[10] and therefore, are not repeated here.

The experimental determination of the extremal values of ϕ and α is now described. The polarizer inclination $\phi(m)$ which yields the reflected beam linearly polarized is found by adjusting both the polarizer and the analyzer until a minimum is obtained. The specimen is then saturated in the opposite magnetization direction and the procedure repeated. The difference in the two settings $\phi(m = 2) - \phi(m = 1)$ is taken as $2\phi(m)$. This technique avoids the difficulty of finding directly the plane of incidence. When $\phi = \phi(m)$, it should in theory be possible to completely extinguish the reflected linearly polarized beam by crossing the analyzer with the plane of oscillation. That this cannot always be accomplished is due to imperfections in the sample and optical components. However, for a reasonably clean, flat sample, the remaining signal falls below the dark current in the photomultiplier (see Fig. 3).

The value $\Delta\psi$ measured with the analyzer is independent of the polarizer inclination as long as the latter remains within a few degrees of the plane of incidence. The measurement of the magnetically induced shift $\Delta\psi$ in the orientation of the ellipse, representing the polarization of the reflected beam, is made by noting the difference in the analyzer setting which gives a minimum irradiance for each of the two reversed magnetization states. Note that $\Delta\psi$ is not equal to $\alpha(m = 2) - \alpha(m = 1)$ except when the light is linearly polarized for $\phi(m = 2) = \phi(m = 1) = 0$.

Table 1 **Data for $2\phi(m)$ and $\Delta\psi$ versus angle of incidence for Hg green line ($\lambda = 546$ mμ).**

γ (deg)	$2\phi(m)$ (deg)	$\Delta\psi$ (deg)
45°	0.11°± 0.01°	0.13° ± 0.01°
50	0.12	0.15
55	0.13	0.14
60	0.15	0.15
65	0.14	0.13
70	0.12	0.06
75	0.08	0.00

Table 2 **Data for $2\phi(m)$ and $\Delta\psi$ for several lines of the Hg spectrum.**

λ (mμ)	$2\phi(m)$ (deg)	$\Delta\psi$ (deg)
365	0.12° ± 0.01°	0.10° ± 0.01°
404	0.11	0.10
435	0.09	0.12
546	0.11	0.13
577	0.11	0.14

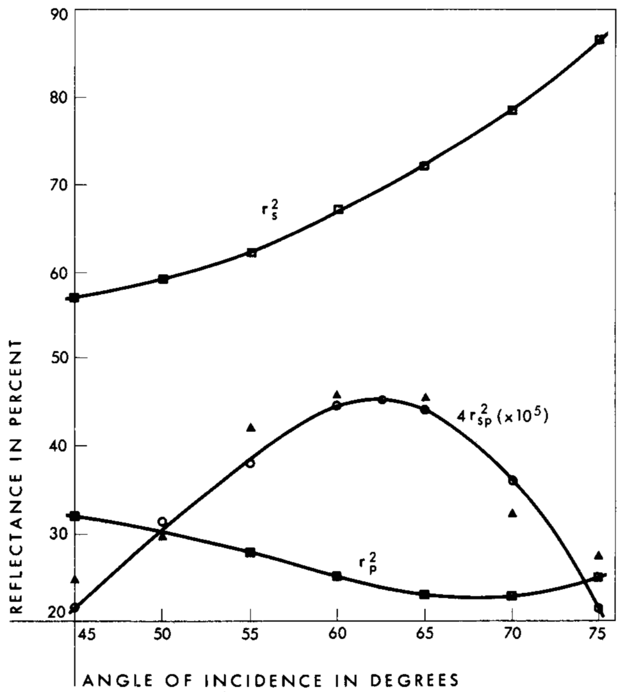
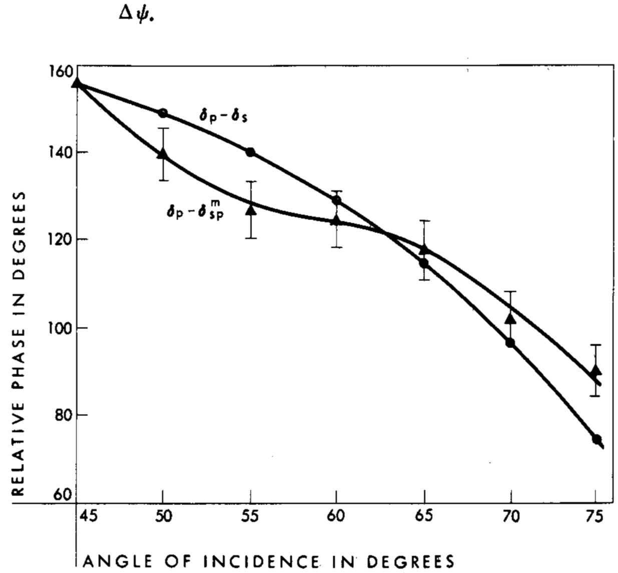


Figure 4 **The reflection coefficients of Fe as a function of incidence angle for Hg green line ($\lambda = 546$ mμ).**
The uncertainty in $4r_{sp}^2$ is 5%. ▲ represents data points measured photometrically. • represents polarimetrically inferred values of $4r_{sp}^2$.

The values of $2\phi(m)$ and $\Delta\psi$ for several angles of incidence and several wavelengths are given in Tables 1 and 2. The data recorded is averaged over 20 measurements.

The value of the Kerr coefficient r_{sp} and its relative phase angle $\delta_p - \delta_{sp}{}^m = \Delta_p{}^m$ is easily determined by substitution of the necessary data into Eqs. (4), (8). The amplitude of the Kerr component goes through a broad maximum for angles of incidence around 60° (see Fig. 4). It is interesting to note that the relative phase goes through 90° (Fig. 5) when the angle of incidence $\gamma = 74°$, implying that $\Delta\psi = (2R_{sp}/R_p)\cos(\delta_p - \delta_{sp}{}^m)$ goes to zero and then changes sign. Hence, the phase information affords a clear interpretation of the observed sign reversal in the Kerr rotation. The dispersion of r_{sp} and its relative phase for the case of a 45° angle of incidence appears in Figs. 6 and 7. It is seen that the phase difference $\delta_s - \delta_{sp}{}^m = \Delta_s{}^m$ between the s-component of the normal oscillation and the Kerr oscillation is near 10° for the 365 mμ Hg line but approaches 0° for the longer wavelengths where r_{sp} is increasing. The Kerr amplitude monotonically increases up to about 600 mμ but, because of the diminishing sensitivity of the photomultiplier, was not investigated for longer wavelengths.

Figure 5 **Relative phase of the Kerr and normal reflection coefficients of Fe for the Hg green line.**
The large uncertainty brackets occur because the phase is inferred from the expression $\tan(\delta_p - \delta_{sp}{}^m) = B_2r_s/B_1r_p \sin(\delta_p - \delta_s)2\phi(m)/\Delta\psi$.



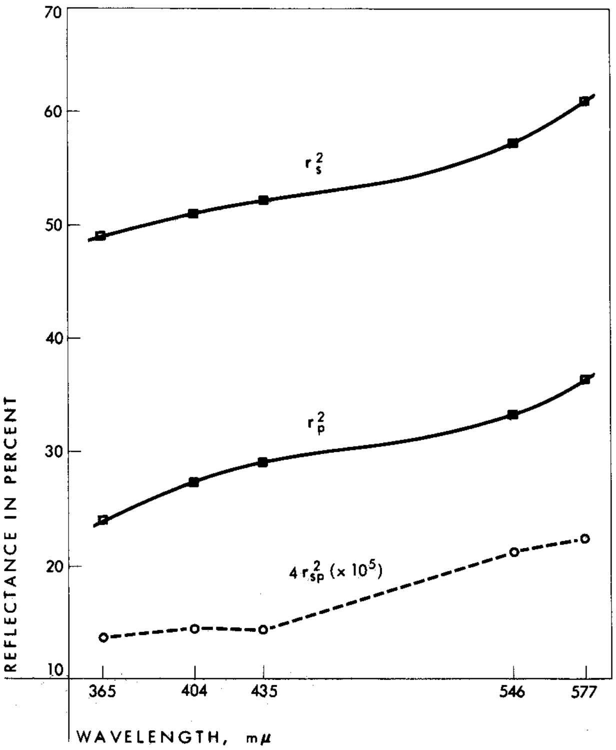


Figure 6 **Dispersion of the reflectance of Fe for an angle of incidence of 45°.**
The uncertainty in $4r_{sp}^2$ is 5%.

The polarimetrically measured values of r_{sp} can be compared with reality through Eq. (6) since $\Delta I/I_0 = 4R_{sp}^2$ is directly measurable by photometric methods. The agreement (see Fig. 4) is within experimental error, thereby validating for the case at hand the model assumed for the Kerr effect.

Measurements of several materials

The measurement technique was developed for evaluation of the magneto-optical properties of a variety of materials. The samples were fabricated by evaporation under approximately the same conditions as given above for the iron film.‡ The materials were selected with the hope of finding a large Kerr effect.§ The results from representative materials are given in Table 3 for reference. A comparison of the values for the two different iron samples illustrates the amount of sample to sample variation. It is believed that sample Fe(II) was oxidized more than sample Fe(I). The Fe = MgF_2 and FePt samples were made by simultaneously evaporating Fe and MgF_2 or Pt from two separate melts. The numerical subscript on $FePt_{0.42}$ denotes 42% platinum by weight.

‡ D. K. Seto developed many of the methods used in the evaporation process.
§ The compositions of the specimens were suggested by Dr. E. N. Adams on the basis of his theoretical analysis. (IBM Research Report 139 (1959)).

Table 3 **Kerr coefficients and phases of several materials for the Hg green line. Angle of incidence is 60°.**

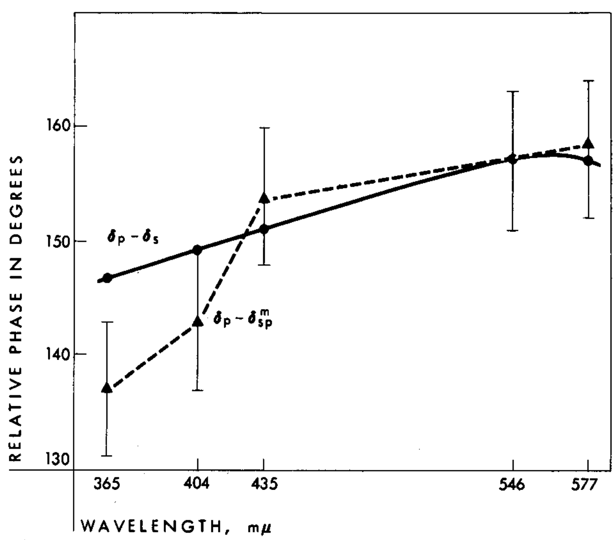
Sample	r_p^2	r_s^2	$4r_{sp}^2$	Δ_p^m
Fe(I)	0.44	0.50	1.7×10^{-6}	140°
Fe(II)	0.32	0.57	4.5×10^{-6}	157°
Fe + MgF_2*	0.27	0.51	4.0	90°
$FePt_{0.42}$	0.66	0.77	2.5	135°
$FePt_{0.08}$	0.53	0.77	6	126°

* For the Fe + MgF_2 sample, the rotation $\Delta\psi$ is zero since $\Delta_p{}^m + 90°$. However, the visible contrast between the reversed states of magnetization is excellent.

Conclusion

The polarimetric method for obtaining values of magneto-optical parameters is valid within the approximations made if r_{sp} is small compared to r_p and r_s. For example, if $\Delta\psi < 6°$ and $\phi = \phi(m = 1) < 6°$, the approximations are valid within 1%. If the Kerr effect is larger, then the anisotropic effects are resolvable by the more common polarimetric methods. The equations derived depend on $\delta = \delta_p - \delta_s \neq 0$ and therefore are not valid at normal incidence. However, they are valid for longitudinal, transverse, and polar magnetization states since Eq. (1) holds for all cases at oblique angles of incidence. Further, the method is valid for the Faraday effect at oblique angles of incidence where $r_p \to t_p$, $r_s \to t_s$, $r_{ps} \to t_{ps}$, $r_{sp} \to t_{sp}$, where t_p, etc., denotes the transmissivity.

Figure 7 **Dispersion of the relative phase of the Kerr and normal coefficients of Fe for an angle of incidence of 45°.**
See legend of Fig. 5.



The accuracy of the method is often limited by available equipment. The verniers on the Glan-Thompson prisms could be read to the nearest 0.01°. However, the Kerr effect of nickel, for example, is such that $2\phi \sim 0.05°$ so that the resolution is just barely adequate for materials of comparable anisotropy. If the Kerr effect of the specimen is small, extreme care must be taken in preparing the surface and maintaining it dust-free, since dust and imperfections decrease the signal-to-noise ratio, making it more difficult to accurately determine the polarization angles which yield a minimum. An error analysis shows that the possible error in $4r_{sp}^2$ is about 5% if the error in $\Delta\psi$ and 2ϕ is $\pm 0.01°$ for angles of the order of 0.1°.

From the analysis of the Kerr effect, it is shown that (1) the reflected beam can always be rendered linearly polarized (see Fig. 3) and (2) the change in irradiance ΔI is independent of the relative phase Δ_p^m only when $I(m)$ is rendered zero for one magnetization state.

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