J. F. Freedman

Residual Stress in Single-Crystal Nickel Films*

Abstract: Experiments are described which directly indicate the source, magnitude and direction of the residual stress on single-crystal nickel films evaporated on rock salt. A theoretical analysis using the five-constant magnetostriction equation shows how this stress affects the magnetic properties. X-ray diffraction studies indicate that the as-evaporated films exist in a highly strained condition, resulting in a tetragonal distortion of the cubic symmetry. The parameters of the unit cell are $a = b = 3.500 \pm 0.004$ A in the plane of the film and $c = 3.546 \pm 0.002$ A normal to the film. The stress causing this strain is elastic in character. When the film is floated off the rock salt, the parameter decreases to bulk unstrained Ni, $a_0 = 3.524$ A.

The calculation using bulk elasticity data yields a planar compressive stress $\sigma = 1.0 \times 10^{10}$ dyne/cm^2, from differential thermal expansion between nickel film and NaCl substrate. This externally applied stress system influences the magnetic state of the film by contributing to the total energy of the system through a magnetoelastic interaction.

Introduction

It has been reported by Chikazumi[1] that the first-order crystal anisotropy constant, K_1 was anomalously large for single-crystal nickel films evaporated epitaxially on NaCl. The origin of the anomalous part of K_1 was assumed to be a large, isotropic tension in the film parallel to the surface, arising from the epitaxial misfit between substrate and film. Similarly, Kuriyama et al.[2] have reported anomalously low values for the saturation magnetization when calculated from ferromagnetic resonance measurements on single-crystal nickel films grown on rock salt. They also indicated a large, planar isotropic tensile stress would explain their data.

There are three major sources for the introduction of stresses and strains into an evaporated film: (1) a structural contribution arising from the epitaxial misfit (as indicated by Chikazumi), (2) stresses arising from any defect or impurity distribution arising during deposition or subsequent thermal annealing, and (3) stresses arising from the constraints imposed by the thermal contraction of the substrate. In the case of a metal film evaporated on NaCl at elevated temperatures and then cooled to room temperature, the last factor would be quite large, resulting in a residual compressive stress in the film and not a tensile stress, as indicated previously.

In the present work experimental evidence is presented which directly indicates the source, magnitude and direction of the residual stress on pure single-crystal nickel films evaporated on rock salt. Theoretical justification is also given for the effect of this stress on the measured magnetic properties.

Experimental procedure

Nickel films were prepared by deposition from a tungsten filament in a vacuum of 10^{-5} mm Hg onto the heated (100) cleavage surface of NaCl in a manner similar to that described by Chikazumi.[1] His work indicated that, after a preheating of one hour at 500°C, the critical temperature for single-crystal growth was 200°C. As a result all films studied in this investigation were grown at temperatures ranging from 200° to 500°C. No special care was taken to control the rate of evaporation (which varied from 10 to 30 A per second) because the results of the magnetic measurements appeared to be independent of rate within this range.

Magnetic anisotropy measurements were made on an automatic torque magnetometer[3] which is capable

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of detecting 0.002 dyne-cm. Anisotropy measurements in the plane of the film and perpendicular to the plane of the film were made with the film as evaporated on the rock salt. The samples were then floated off the rock salt in water, picked up on glass slides and the properties remeasured.

Structural measurements were made using standard electron diffraction and transmission techniques on specimens floated onto 200-mesh Cu grids. A Phillips EM-100 electron microscope operating at 100 kv was employed. X-ray diffraction measurements were obtained on specimens as evaporated, and after floating, employing a standard Norelco diffractometer unit. Because of the diffuseness of the peaks obtained, it was necessary to employ films greater than 500 A thickness for reliable x-ray analysis. Peeling of the film from the substrate restricted the upper limit of thickness to 1500 A. Since films greater than 1000 A could not be penetrated by the electron beam, a complete analysis could be performed only on films between 500 A and 1000 A thick. The x-ray and magnetic analyses indicated that there was no thickness dependence within this range. Consequently, thicknesses were only crudely controlled by evaporating to completion a weighted amount of evaporant, designed to yield a film 700 A thick.

Experimental results

Figure 1 shows the first-order crystal anisotropy constant, K_1, plotted as a function of substrate temperature during evaporation. Although there is considerable scatter of the data, the following conclusions can be drawn: (1) as previously noted, the critical evaporation temperature (i.e., the substrate temperature above which the film is a single crystal) is about 200°C; (2) the value for K_1 for the films on NaCl appears to be about twice as large as the bulk value, and apparently independent of temperature above 200°C; (3) this anomalous value of K_1 is reduced to the normal value when the film is removed from the rock salt substrate. The literature values of K_1 for the bulk material are shown by the banded straight line.

Figure 1 **Variation of the room temperature first-order crystal anisotropy constant, K_1, as a function of substrate temperature during evaporation.**
The circles represent the values for the individual films as evaporated at the indicated temperature on the rock salt; closed circles indicate values after the identical film has been floated off the rock salt and picked up on glass disks. The banded area represents the literature values for the bulk material.

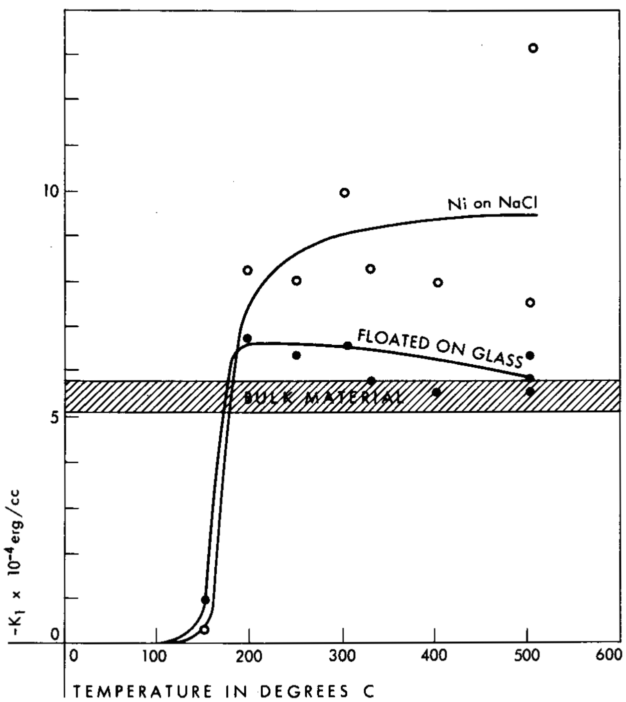

Figure 2 **Variation of the room temperature perpendicular anisotropy, $K_\perp$, as a function of substrate temperature during evaporation.**
The theoretical value of $K_\perp$ is $2\pi I_s^2 + K_1$. The circles represent the values for the individual films as evaporated at the indicated temperature on the rock salt; closed circles represent values after the identical film has been floated off the rock salt onto glass disks.

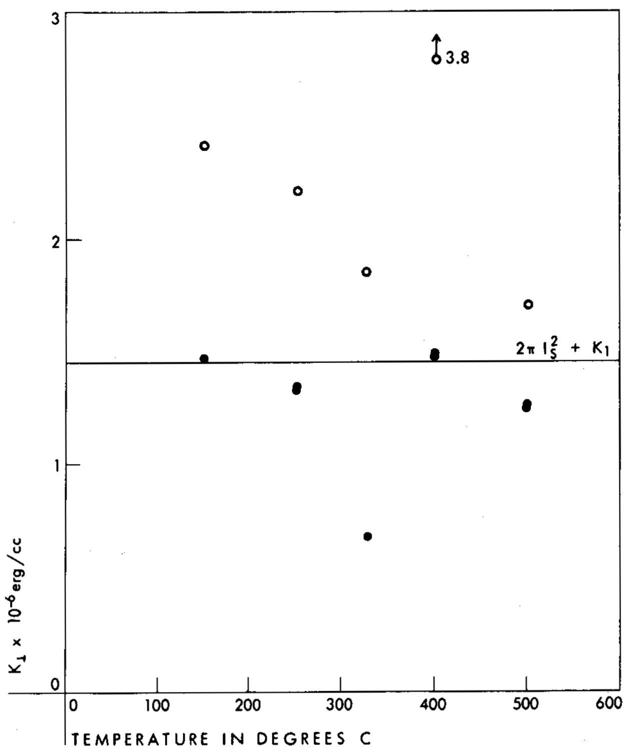

Figure 2 indicates the values of the perpendicular anisotropy versus the substrate temperature during evaporation. This is the anisotropy resulting from the rotation of the magnetization out of the plane of the film. The value of $K_\perp$ should be $2\pi I_s^2 + K_1$ when the origin is merely shape anisotropy and crystalline anisotropy. Again it is noted that the value of $K_\perp$ is anomalously large when the nickel is constrained to the NaCl and reduces to the theoretical value when floated off and picked up on glass.

Electron transmission diffraction patterns taken on these specimens indicated single-crystal orientation in the (100) plane, in accord with the magnetic results.

It is obvious that the factor contributing to the magnetic anisotropies disappears when the film is removed from the NaCl. It was also observed, however, that films exposed to air for a period of time experienced a decay of the anisotropies from the anomalous values to normal bulk values.

The photomicrographs in Fig. 3 show a change in surface morphology which accompanies the decay of the anisotropies. The dark striations in Fig. 3b were examined by oblique lighting techniques and by transmission optical microscopy, and these areas were found to have a "mountain" profile. This was interpreted as indication of the fact that the nickel films were under considerable elastic compressional stress, which is relieved when the film is physically removed from the constraining substrate, or when sufficient water vapor is absorbed during exposure to atmospheric conditions to accomplish the same result.

To detect this stress, x-ray traces were taken using CoKα radiation and a Geiger tube diffractometer. Since monochromatic radiation was used, the single crystal produced a particular reflection only when the specimen was oriented such that the normal to that set of planes was contained in the plane of the diffractometer circle and bisected the angle between the incident and diffracted beams. This condition for any set of planes was fulfilled by mounting the film on a goniometer which allowed the desired rotations. Since the films were thin enough to allow penetration of the x-ray beam through the nickel film and into the substrate, a corresponding set of the NaCl planes was used for purposes of alignment and hence supplied an internal standard for obtaining absolute values for the nickel spacings. A typical set of diffraction results obtained from a single-crystal film on NaCl grown at 375°C, and exhibiting the anomalous magnetic behavior, is tabulated in Table 1. Listed are the Bragg angle θ, (obtained from the position of the maximum of a diffraction peak), the corresponding peak half-width, and a_0, the lattice parameter calculated assuming a cubic unit cell. It was impossible to obtain any higher order nickel lines because of the loss of intensity at large angles of reflection.

There is excellent agreement for the cell constant obtained from various planes of the NaCl, implying a

Figure 3a **Macrostructure observation of a nickel film grown on NaCl at elevated temperatures.**
Magnification 100×. The dark spot is a hole placed in the film for purposes of area identification.

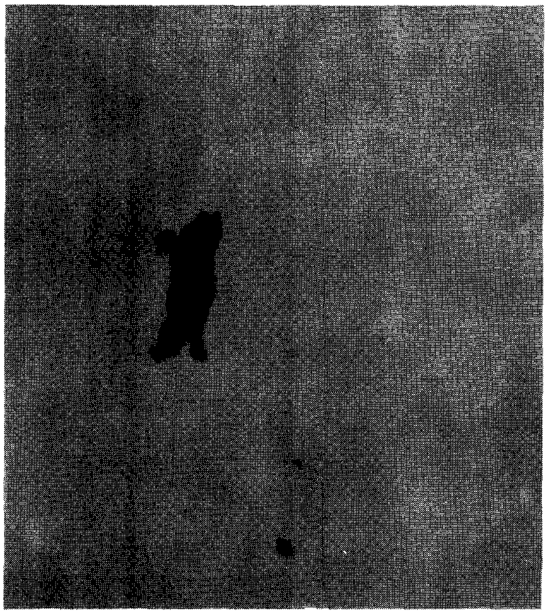

Figure 3b **Macrostructure observation of identical area of Fig. 3a, after exposure to moist atmosphere.**
Magnification 100×. The dark striations are actually wrinkles (or "mountains") on the surface.

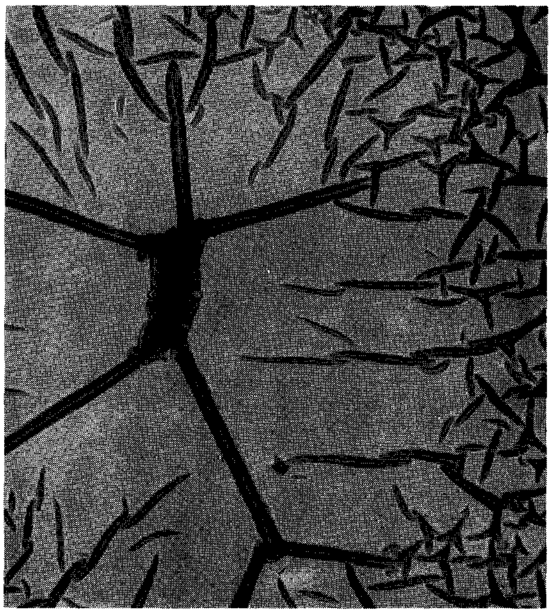

Table 1 **Typical set of diffraction results for nickel film grown on rock salt at 375°C.**

(*hkl* plane) Ni	NaCl	θ (degrees)	a_0(A)	half-width (degrees)
(002)		30.29	3.546	0.46
	(004)	39.56	5.641	0.31
(022)		45.92	3.521	0.64
	(333)	55.48	5.640	0.25
(222)		61.68	3.510	0.90
	(044)	63.76	5.641	0.17

a_0NBS Ni = 3.5238 A at 26°C
a_0NBS NACl = 5.6402 A at 26°C

precision of about 1 part in 10,000. However, the peaks were quite sharp and for all sets of planes there was complete resolution of the $K\alpha_1$ and $K\alpha_2$ reflection peaks. The error in the cell constant of the nickel lines is considerably larger because of particle size broadening of the diffraction peaks and because of internal strains. In particular, the broadening was so extensive (Table 1) that it was impossible to resolve the $K\alpha_1 K\alpha_2$ doublet for any set of nickel planes. The position of the peak was obtained by a linear extrapolation of the straight sides of the unsymmetrical experimental peak. Coupled with the smaller intensity of the nickel reflections, the estimated error for the nickel values ranges from 1 to 3 parts per 1000, increasing from low to high θ. The experimental deviations, from the (022) and (222) reflections, fall on the fringe of the estimated error; however, the deviation of the (002) reflection, which possesses the greatest reliability, falls well outside the estimated error.

A reasonable explanation of the observed parameter values can be obtained by assuming a tetragonal distortion of the unit cell such that $a = b = 3.500$ A $\pm$ 0.004 A, $c = 3.546$ A $\pm$ 0.002 A.

This tetragonal distortion is assumed to arise from an isotropic compressive stress in the plane of the film and a Poisson's extension normal to the film. Since this stress must be an elastic stress which is completely relieved when the film is removed from the constraining NaCl substrate, Ni specimens on NaCl were mounted on the diffractometer and a trace of the (002) planes taken, yielding values similar to those tabulated in Table 1. Without disturbing the film in its mounting, moist air was blown over the surface. An immediate rescan of the (200) line revealed an increase of θ to 30.50°, resulting in a corresponding decrease in cell parameter a_0 to 3.524, the normal bulk nickel parameter. [The (022) and (222) planes were not scanned in this manner since the possible observable change was on the fringe of experimental error.] Observations of the surface indicated a change similar to that depicted in Fig. 3; measurements of the magnetic anisotropies before and after the x-ray traces yielded changes in accord with Figs. 1 and 2. It is concluded, therefore, that an elastic compressional stress is acting in the film, and the presence of water or water vapor relieves the stress by destroying the film-substrate bonding. This could account for the considerable scatter in magnetic data of the as-evaporated films reported in Figs. 1 and 2. Since the measurements were made in air, the film tension could have been partially relieved due to adsorption of water vapor prior to the measurement.

Discussion of results

• *Magnitude of stress*

The stress, σ_i, present in the plane of the film can now be calculated from the following equations derived from classical elasticity theory for a cubic system:

$$\varepsilon_x = S_{11}\sigma_x + S_{12}\sigma_y \tag{1}$$

$$\varepsilon_y = S_{12}\sigma_x + S_{11}\sigma_y \tag{2}$$

$$\varepsilon_z = S_{12}\sigma_x + S_{12}\sigma_y\,, \tag{3}$$

where ε_j is the strain, σ_i the stress, and S_{ij} the elastic compliances.

It is to be noted that for a film supported on a substrate, the surface of the film is without constraint, resulting in $\sigma_z = 0$. The strain in the z direction, ε_z, is therefore the effective Poisson's contraction set up normal to both σ_x and σ_y.

Within the limits of experimental accuracy, the x-ray diffraction data shows that $\varepsilon_x = \varepsilon_y$. Solution of Eqs. (1) and (2) therefore yields $\sigma_x = \sigma_y$. As stated earlier, the most reliable data for the strain is obtained from the (002) planes rather than from the reflections occurring at higher θ. Defining the strain as

$$\varepsilon = \frac{a_{\text{strain}} - a_{\text{bulk}}}{a_{\text{bulk}}}\,,$$

one obtains $\varepsilon_z = 0.0063$ cm/cm directly from the experimental data.

Substituting the literature values for the compliances[4] into Eq. (3) [$S_{12} = -0.312 \times 10^{-12}$] one obtains for the stress $\sigma = -1.0 \times 10^{10}$ dynes/cm^2, a compressive stress in excess of the yield point for normal bulk materials. Such large elastic stresses are, however, in accord with other work in thin films.[5-11]

• *Origin of stress*

There are three primary sources of strain in single-crystal films:

1) Strains resulting from epitaxial growth: there is a 38% misfit, the rock salt having the larger parameter. One would anticipate that a large portion of this strain would be taken up in growth structure of the film.

2) Strains resulting from defects in the material resulting from the evaporation process, i.e., an intrinsic strain. Although there is no available information

on nickel films to estimate the magnitude of such effects, work by Hoffman *et al.*[12-14] indicates this intrinsic stress would be of tensile character.

3) Strains resulting from the difference in thermal expansion of the NaCl and the Ni, coupled with the constraints imposed by the bonding between the film and the substrate. The resulting strain would be given by

$$\varepsilon_T = \left(\frac{\delta l}{l}\right)_{\mathrm{Ni}} \Delta T - \left(\frac{\delta l}{l}\right)_{\mathrm{NaCl}} \Delta T \,, \qquad (4)$$

where

$$\left.\begin{aligned} \left(\frac{\delta l}{l}\right)_{\mathrm{Ni}} &= 13.7 \times 10^{-6} \\ \left(\frac{\delta l}{l}\right)_{\mathrm{NaCl}} &= 40.4 \times 10^{-6} \end{aligned}\right\} \text{the average coefficients of expansion,}^{15} \text{(per deg C)}$$

and ΔT = temperature of evaporation minus the temperature of measurement.

Of the three possible sources, the stresses resulting from the thermal expansion effects are of the right sign and magnitude. Moreover, since they depend on the substrate film bonding, the stress is relieved when this bond is broken. One notes that in order to explain the observed strain a temperature of 275°C is required, which is only slightly in excess of the critical temperature of epitaxy. If this strain corresponds to the elastic limit for the metal film, any further increase in stress resulting from an increase in the temperature interval of cooling (i.e., evaporating at a substrate temperature higher than 275°C) would be relieved by plastic deformation. As the calculations in the Appendix show, a change in interatomic distance over the entire volume is needed to effect a change in K_1 or in $K_\perp$. Since this is accomplished by a uniform elastic strain, but not by plastic deformation, one would not anticipate any temperature dependence of the anomalous parts of K_1 and $K_\perp$ above the temperature required to introduce sufficient stress to exceed the yield point of the metal film.

To prove that the observed strain did result from thermal expansion differences, specimens which had been relieved by water vapor, but still remained on the rock salt, were placed in a hot-stage microscope. The stage and specimen were heated slowly and the temperature at which the wrinkles disappeared, indicating a matching of film and substrate, was obtained. For a series of three different specimens the temperature obtained was 275° ± 25°C. (It was quite difficult to obtain an exact temperature since the disappearance was gradual.) On cooling from this temperature the wrinkles did reappear, indicating a reversible process.

Since the appearance and disappearance of the wrinkles is reversible with temperature, and since the point at which the wrinkles disappear, indicating the disappearance of the initial strain (i.e. a coherency of the film and substrate), occurs in the temperature range calculated, it is concluded the origin of the strain is the difference in thermal expansion of the film and substrate.

• *Effect of stress on magnetic properties*

The influence of the isotropic planar stress upon the magnetic state of the film may be obtained by considering its contribution U_σ to the total effective anisotropy energies. In this fashion, it can be shown (for a detailed calculation refer to Appendix A) that the contribution of a planar stress to the first-order crystal anisotropy constant is

$$K_{1\sigma} = \sigma(\tfrac{2}{3}h_4 - 2h_3) = (K_{1\,\mathrm{meas}} - K_{1\,\mathrm{bulk}}) \,,$$

where σ is the applied planar stress, $S = \alpha_i^2 \alpha_j^2$ where the α's represent the direction cosines between the magnetization and the crystallographic axis, and h_3, h_4 are the third and fourth constants in the five-constant magnetostriction equation.

For a typical sample $(K_{1\,\mathrm{meas}} - K_{1\,\mathrm{bulk}}) \approx -4 \times 10^4$ erg/cc. (Refer to Fig. 1). Employing the values of the magnetostrictive constants of Bozorth and Hamming[17] for nickel,

$$\begin{aligned} h_1 &= -68.8 \times 10^{-6} & h_4 &= -7.5 \times 10^{-6} \\ h_2 &= -36.5 \times 10^{-6} & h_5 &= 7.7 \times 10^{-6} \,, \\ h_3 &= -2.8 \times 10^{-6} \end{aligned}$$

one obtains a compressive stress of 7×10^{10} dynes/cm². This is nearly an order of magnitude larger than the value estimated employing the observed strain and the elastic moduli. However, it should be stressed that since the calculation involves *the difference of two large terms* (both of which are subject to considerable experimental error) the agreement in direction of the stress may even be considered fortuitous. The sensitivity of the calculated stress to the choice of magnetostrictive constants is shown by the fact that Chikazumi, employing earlier data in which h_3 was chosen to be zero, calculated a tensile stress from his magnetic results. Hence the most important aspect of this calculation is the fact that an isotropic planar stress will affect the first-order crystal anisotropy constant.

The contribution of a planar stress to the anisotropy energies involved in rotating the magnetization out of the plane of the film $(\Delta K_\perp)$ is shown to be

$$\Delta K_\perp = \sigma(h_1 - h_3 - \tfrac{8}{4}h_4) \,.$$

The detailed calculation appears in Appendix B.

Since the experimental value of $\Delta K_\perp$ (refer to Fig. 2) is approximately 1×10^6 erg/cc, one requires a residual planar stress of -2.2×10^{10} dyne/cm², in good agreement with the elasticity calculations.

Conclusions

1) Nickel films grown epitaxially on rock salt exist in a highly stressed condition. This stress is of the order of 10^{10} dynes/cm², is compressive in character,

and results from the difference in thermal expansion between the nickel film and the rock salt substrate, and the restrictive bonding of the substrate on the film.

2) This planar compressive stress distorts the cubic symmetry of nickel, resulting in a tetragonal unit cell, $a = b = 3.500 \pm 0.004$ A in the plane of the film; $c = 3.546 \pm 0.002$ A normal to the film.

3) The predominant stress is completely elastic in character and can be relieved by floating the film off the rock salt, or it can be relieved from the substrate merely by exposing it to water vapor.

4) This externally applied stress system influences the magnetic state of the film by contributing to the total energy of the system through a magnetoelastic interaction. Such an interaction of a compressive stress on nickel results in an anomalously high first-order crystalline anisotropy value, K_1, as well as an anomalously high value of the anisotropy perpendicular to the plane of the film.

Acknowledgments

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Appendix A: Calculation of the stress effect on the crystalline anisotropy

The intrinsic magnetic crystalline anisotropy energy (i.e., the energy of a crystal which has not been permitted to deform, either due to magnetostriction or an externally applied stress) may be expressed to the sixth degree in α_i, where α_i is the direction cosine between the magnetization and the crystallographic axes, as:

$$U_K = K_1{}^iS + K_2{}^iP, \qquad \text{A(i)}$$

where

$$S = \alpha_1{}^2\alpha_2{}^2 + \alpha_2{}^2\alpha_3{}^2 + \alpha_3{}^2\alpha_1{}^2$$

and

$$P = \alpha_1{}^2\alpha_2{}^2\alpha_3{}^2 .$$

Under normal experimental conditions no external stresses are applied, and magnetic measurements are made such that the crystal is allowed to deform freely due to magnetization. Following the terminology of Baltzer,[16] the effective anisotropy energy becomes

$$U_{\text{tot}} = U_K + U_\lambda + U_e \equiv U_K + U_\lambda{}^0 . \qquad \text{A(ii)}$$

U_K, as defined above, is the anisotropy energy for an undeformed crystal, U_λ is the magnetoelastic coupling energy, and U_e is the elastic energy.

Equation A(ii) can be rewritten in the same form as Eq. A(i), yielding

$$U_{\text{tot}} = K_1S + K_2P , \qquad \text{A(iii)}$$

where K_1 and K_2 are now the effective domain anisotropy constants and are the values one obtains from torque magnetometer measurements. (The changes are actually small relative to $K_1{}^i$ and $K_2{}^i$.)[16]

Consider what happens to K_1 when an external stress is applied. The resulting strain is determined by the elastic properties of the material and is independent of the magnetization. The only contribution to the total magnetic energy is through the magnetoelastic coupling energy, such that

$$U_{\text{tot}} = U_K + U_\lambda{}^0 + U_\sigma , \qquad \text{A(iv)}$$

where U_K is again the anisotropy energy for an undeformed crystal, $U_\lambda{}^0$ is the magnetoelastic coupling energy for zero applied stress defined earlier, and U_σ represents the total contribution to the magnetic anisotropy energy due to the applied stress. This energy is just the work done as the magnetization rotates in the presence of the applied stress, σ, and is

$$U_\sigma = -\sigma\int_0^{\lambda_s} d\lambda = -\sigma\lambda_s . \qquad \text{A(v)}$$

The negative sign arises so that the energy is a minimum when the magnetization lies in the easy direction. The saturation magnetostriction λ_s is given by

$$\begin{aligned}\lambda_s = {} & h_1(\alpha_1{}^2\beta_1{}^2 + \alpha_2{}^2\beta_2{}^2 + \alpha_3{}^2\beta_3{}^2 - \tfrac{1}{3}) \\ & + 2h_2(\alpha_1\alpha_2\beta_1\beta_2 + \alpha_2\alpha_3\beta_2\beta_3 + \alpha_1\alpha_3\beta_1\beta_3) \\ & + h_3(S - \tfrac{1}{3}) + h_4(\alpha_1{}^4\beta_1{}^2 + \alpha_2{}^4\beta_2{}^2 + \alpha_3{}^4\beta_3{}^2 \\ & + \tfrac{2}{3}S - \tfrac{1}{3}) + 2h_5(\alpha_1\alpha_2\alpha_3{}^2\beta_1\beta_2 \\ & + \alpha_2\alpha_3\alpha_1{}^2\beta_2\beta_3 + \alpha_1\alpha_3\alpha_2{}^2\beta_1\beta_3) . \qquad \text{A(vi)}\end{aligned}$$

S, P, and α have their previous meanings, and β is the direction cosine between the stress axis and the crystallographic axis.

Again in terms of Eq. A(i) we can express the total energy as

$$U_{\text{tot}} = (K_1 + K_{1\sigma})S + (K_2 + K_{2\sigma})P = K_1{}'S + K_2{}'P . \qquad \text{A(vii)}$$

$K_{1\sigma}$ and $K_{2\sigma}$ therefore represent the stress contribution to the anisotropy and are equal to the difference between the experimental values and the bulk values. The theoretical values are obtained by substituting in the appropriate direction cosines into Eqs. A(v) and A(vi) and collecting terms with the proper α_i dependence.

For an (001) oriented planar film, it is to be noted that $\alpha_3 = 0$, $\alpha_1{}^2 + \alpha_2{}^2 = 1$. If one represents the isotropic planar stress system by a biaxial stress, aligned parallel to the crystallographic axis such that $\beta_3 = \beta_2 = 0$, $\beta_1 = 1$ for $\sigma\langle 100\rangle$ and $\beta_3 = \beta_1 = 0$, $\beta_2 = 1$ for $\sigma\langle 010\rangle$, insertion of these direction cosines into the expression for λ_s yields

$$\begin{aligned}U_\sigma = {} & -\sigma[\tfrac{1}{3}h_1 + h_4(\alpha_1{}^4 + \alpha_2{}^4 + \tfrac{4}{3}\alpha_1{}^2\alpha_2{}^2 - \tfrac{2}{3}) \\ & + 2h_3(\alpha_1{}^2\alpha_2{}^2 - \tfrac{1}{3})] \qquad \text{A(viii)}\end{aligned}$$

when the effects are calculated independently and added.

Using the appropriate algebra yields

$$U_\sigma = +\sigma[(\tfrac{2}{3}h_4 - 2h_3)\alpha_1{}^2\alpha_2{}^2 - (\tfrac{1}{3}h_1 + \tfrac{1}{3}h_4 - \tfrac{2}{3}h_3)] \,. \qquad \text{A(ix)}$$

It is to be noted that the latter bracket in Eq. A(ix) is a constant and independent of θ; hence employment of the normal two-constant magnetostriction equation would not yield a stress contribution to the normal crystalline anisotropy constant.

Equating terms of Eq. A(ix) and Eq. A(vii) yields

$$K_{1\sigma} = +\sigma(\tfrac{2}{3}h_4 - 2h_3) \equiv K_{1\,\text{meas}} - K_{1\,\text{bulk}} \,. \qquad \text{A(x)}$$

Appendix B: Calculation of the stress effect on the perpendicular anisotropy

It has been shown by Chikazumi[1] that when the applied magnetic field H makes a small angle Ψ to the plane of the film, the measured torque is given by

$$L = -\frac{2K_\perp I_s H v}{2K_\perp + I_s H}\Psi \,, \qquad \text{B(i)}$$

where I_s is the saturation magnetization, v is the volume of the specimen and $K_\perp$ is the perpendicular anisotropy constant given by $2\pi I_s{}^2 + K_1$. Equation B(i) can be rewritten in the form

$$\frac{\Psi}{L}H = \frac{1}{I_s v} + \frac{H}{2K_\perp v} \,. \qquad \text{B(ii)}$$

$K_\perp$ is therefore obtained from the linear plot of $\Psi H/L$ versus H.

Consider now the effects of an isotropic planar stress. The total energy of a stressed film in a magnetic field H applied at a small angle Ψ to the plane of the film is given by

$$U_{\text{tot}} = U_D + U_K + U_\lambda{}^0 + U_\sigma - I_s v \cdot H \,, \qquad \text{B(iii)}$$

where U_D is the demagnetizing energy arising from the shape anisotropy and U_K, $U_\lambda{}^0$ and U_σ are defined as in Appendix A.

Equation B(iii) can be rewritten in the form

$$U_{\text{tot}} = K_\perp \sin^2\theta - \sigma\lambda_s - I_s v H \cos(\Psi - \theta) \,, \qquad \text{B(iv)}$$

where θ is the angle between the magnetization and the crystallographic axis and $K_\perp = K_D + K_1 \cos^2\theta = 2\pi I_s{}^2 + K_1$ for small θ.

If we again represent the planar stress by a binary stress system, the axis oriented in the same plane as the magnetization, we obtain:

$$\begin{aligned}\sigma\lambda_s &= [h_1(\cos^2\theta - \tfrac{1}{3}) + h_3(\cos^2\theta\sin^2\theta - \tfrac{1}{3}) \\ &\quad + \sigma h_4(\cos^4\theta + \tfrac{2}{3}\cos^2\theta\sin^2\theta)] \,. \qquad \text{B(v)}\end{aligned}$$

The energy is a minimum with respect to θ when

$$\begin{aligned}\frac{\partial E}{\partial \theta} = 0 &= 2K_\perp \sin\theta\cos\theta - MH\sin(\Psi - \theta) \\ &\quad - \sigma[(-h_1 2\cos\theta\sin\theta) + 2h_3(\sin\theta\cos^3\theta \\ &\quad - \cos\theta\sin^3\theta) + h_4(-4\cos^3\theta\sin\theta) \\ &\quad + \tfrac{4}{3}h_4(\sin\theta\cos^3\theta - \cos\theta\sin^3\theta)] \,. \qquad \text{B(vi)}\end{aligned}$$

Ignoring terms of the order of $\sin^2\theta$, since θ is small, this yields for the torque for a film of volume v:

$$L = \frac{2vK_\perp{}' I_s H}{2K_\perp{}' + I_s H}\Psi \,, \qquad \text{B(vii)}$$

where $K_\perp{}' = K_\perp + \sigma(h_1 - h_3 - \tfrac{8}{3}h_4)$.

The slope of a plot of $\Psi H/L$ versus H is now proportional to $K_\perp{}'$ and $\Delta K_\perp$; the anomalous part of the perpendicular anisotropy, is proportional to $\sigma(h_1 - h_3 - \tfrac{8}{3}h_4)$.

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