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A Theoretical Solution for the Magnetic Field in the Vicinity of a Recording Head Air Gap

The problem of determining the characteristics of a magnetic field in the vicinity of a recording head air gap can be idealized to represent a boundary-value problem, as shown in Fig. 1. Here, the pole pieces are at potentials $+\pi$ and $-\pi$, respectively. A solution already exists for those cases where $\theta = 0$ and $\theta = \pi/2$.

This paper will show the development of a theoretical solution for the case where θ equals an arbitrary angle, so that the effect of the variation of θ on the field at normal recording distance can be calculated. The field intensity E at any point can be shown $= 1/(1 + e^{\psi})^{\alpha}$, where $\alpha = 1 - (\theta/\pi)$. This contradicts Booth's results[1] which state that E is monotonic with respect to θ. The numerical values of the field at this distance have been calculated for $\theta = 7/18\pi$ and compared with the fields for $\theta = 0$ and $\theta = \pi/2$. It seems that a head with $\theta = 7/18\pi$ is optimum for field intensity at the normal recording distance.

Boundary value problem

The potential field problem, as shown in Fig. 1, is described by the Laplace equation

$$\nabla_{u,v}{}^2\phi = \nabla_{u,v}{}^2\psi = 0\,, \qquad (Q = \phi + i\psi) \qquad (1)$$

where ϕ is the potential function, ψ is the flux function and $\nabla_{u,v}{}^2$ is the Laplace operator referring to the axes of u and v. The permeability μ of the magnetic circuit is assumed to be infinite compared with the permeability of air, which is 1. The boundary conditions are defined by having one pole piece at potential $+\pi$ and the other at potential $-\pi$.

Due to the symmetry of the potential field, it is only necessary to solve for the left half of the W plane ($W = u + iv$) shown in Fig. 1.

Following a procedure very similar to that used by Booth,[1] the region external to the pole piece (shaded area) and to the left of the zero potential line is mapped on the upper half of the z plane ($z = x + iy$) as shown in Fig. 2; the points W_1, W_2, W_3, W_4 are mapped into the corresponding points $z_1 = -\infty, z_2 = -1, z_3 = 0, z_4 = +\infty$, respectively. The mapping function, according to the Schwarz-Christoffel transformation, is

$$W = A\int[(z+1)^{\alpha}/z]dz + B\,, \qquad (2)$$

where $\alpha = 1 - (\theta/\pi)$.

The function $(z + 1)^{\alpha}$ in Eq. (2) may be replaced by its binomial expansion, valid for $|z| \leq 1$. From the resulting equation and the conditions,

$W = -h$, when $z = -1$

W jumps by h, when $z = 0$,

Figure 1 **Schematic of left half of the W plane.**

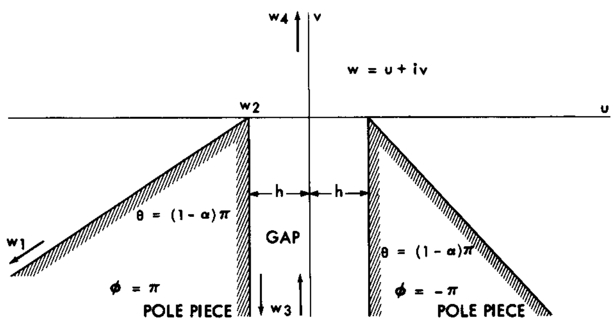


Figure 2 **Upper half of the z plane.**

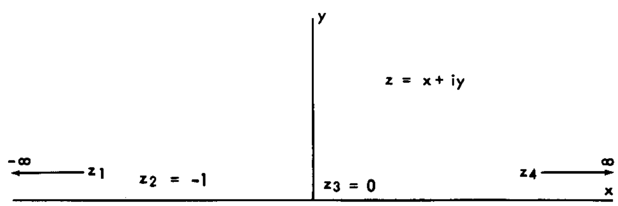
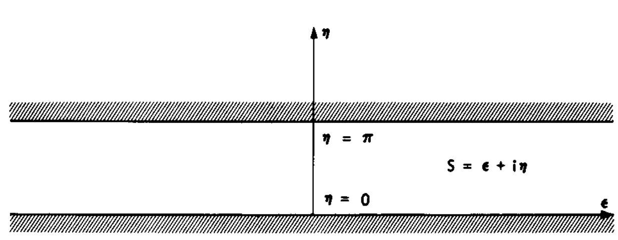


Figure 3 **Region between the two boundaries $\eta = 0$ and $\eta = \pi$.**



the two constants A and B can be determined to be

$$\left.\begin{aligned} A &= ih/\pi \\ B &= -(ih/\pi)\sum_{n=1}^{\infty}(1/n)\binom{\alpha}{n}(-1)^n \end{aligned}\right\} . \tag{3}$$

Hence,

$$W = u + iv = (ih/\pi)\int[(z+1)^{\alpha}/z]dz$$
$$-(ih/\pi)\sum_{n=1}^{\infty}(1/n)\binom{\alpha}{n}(-1)^n . \tag{4}$$

The next step is to map the upper half z-plane into the region between the two boundaries $\eta = 0$ and and $\eta = \pi$ (Fig. 3). It can be shown[1] that the mapping function is:

$$S = A'\int(dz/z) + B' . \tag{5}$$

With the conditions $x = 0$, when $z = 1$ and $x = \pi i$, when $z = -1$, it follows that

$$S = \varepsilon + i\eta = \log z \tag{6}$$

or

$$z = x + iy = e^S = e^{\varepsilon}(\cos\eta + i\sin\eta) . \tag{7}$$

The solution for the parallel plate potential problem in the S-plane is obviously

$$\left.\begin{aligned} \phi &= \eta \\ \psi &= \varepsilon \end{aligned}\right\} . \tag{8}$$

Therefore, x and y are related to ϕ and ψ in the following manner

$$\left.\begin{aligned} x &= e^{\psi}\cos\phi \\ y &= e^{\psi}\sin\phi \end{aligned}\right\} . \tag{9}$$

The dependence of u and v on ϕ and ψ can be obtained by expanding $(z+1)^{\alpha}$, for $1/2 \leqq \alpha < 1$, in a binomial series for each of the following conditions: $|z| \leqq 1$; $|z| > 1$ with $|x| > |y|$; $|z| > 1$ with $|x| < |y|$; and $|z| > 1$ with $|x| = |y|$, and substituting this series into Eq. (4).

The resultant equations and that for $\alpha = 1$ represent a solution which permits calculation of the field near the gap of a magnetic head with an arbitrary pole piece angle θ. These solutions reduce to those of Booth for $\theta = 0$ and $\theta = \pi/2$.

Using Eqs. (11) through (18), the equipotential lines $\phi = 0, \pi/4, \pi/2, (3/2)\pi, \pi$ and the flux lines ($\psi = -10, -9, -8, \cdots 0, 1, 2, 3, 4$) were calculated for an angle $\theta = 7/18\pi$, ($\alpha = 11/18$), and plotted as shown in Fig. 4.

The variation of intensity ($E = |dQ/dW|$) along the axis of symmetry (v axis) for the present head with $\alpha = 11/18$ and $h = \pi$ is plotted as shown in Fig. 5 together with Booth's results for the parallel plate case ($\alpha = 1$), and the rectangular gap case ($\alpha = 1/2$).

The 7/18 angle head yields an intensity about 16.4 per cent higher than that for a rectangular gap head

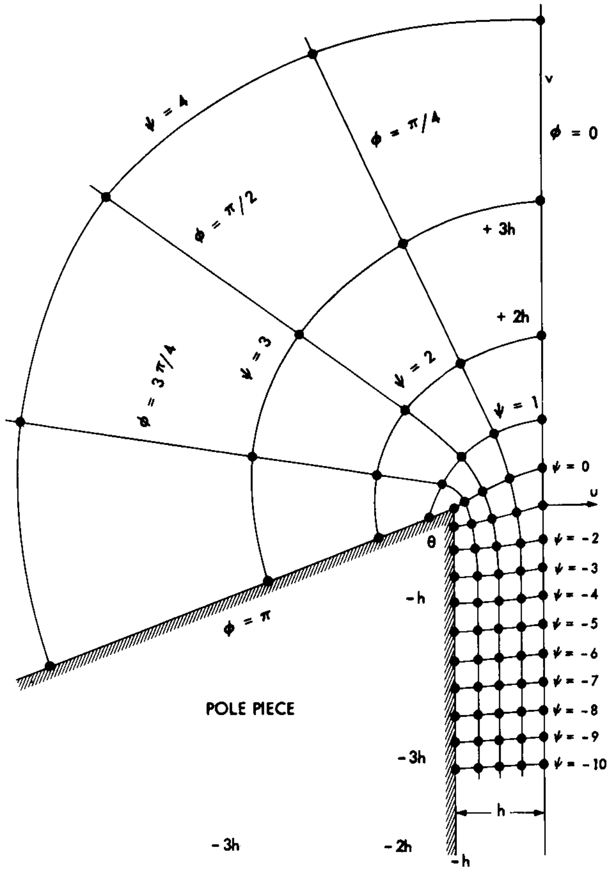
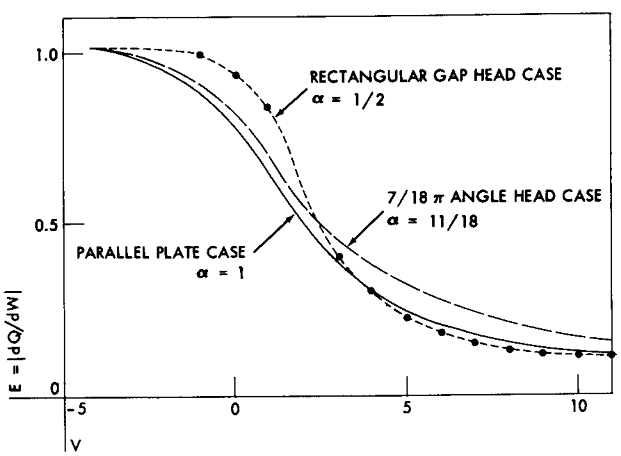


Figure 4 **Plot of equipotential and flux lines for an angle θ.**

Figure 5 **Plot of variation of intensity along the axis of symmetry for the present head, together with Booth's results for the parallel plate case and the rectangular gap case.**



(based on the intensity of the rectangular head), at a distance of one-half of the gap width in front of the gap, and 53.1 per cent higher at a distance of one gap width. These two distances are the limits of the normal separation between the recording medium and the recording head. It should be noted also that at distances less than 0.43 gap width, as can be realized in contact recording, the rectangular head will yield a higher intensity than that from any head whose angle θ is less than $\pi/2$.

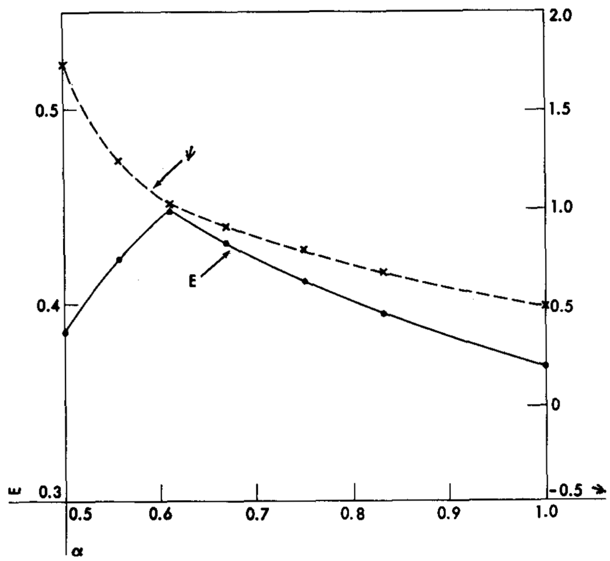


Figure 6 **Plot of variation of intensity and ψ with α.**

The expression for field intensity can be derived from Eq. (4), resulting in:

$$E = 1/(1 + e^{\psi})^{\alpha} . \tag{10}$$

When α decreases, the intensity E at a particular point tends to increase; however, at the same point, the value of ψ increases which tends to offset the increasing effect due to decreasing α. Figure 6 shows the variation of the intensity E with α at a point on the v-axis and at a distance of one-half the gap width in front of the gap. It clearly shows that when Booth concluded that the intensity is monotonic in α he had only calculated the two cases $\alpha = 1/2$ and $\alpha = 1$ and that the values of α and ψ for these two cases yield almost identical values of E. The above relationship is also responsible for the intensity curve, for $\alpha = 1/2$ drops below that for $\alpha = 1$ in Fig. 5 at large distances in front of the gap.

Reference

1. A. D. Booth, *Brit. J. Applied Physics*, **3**, 307 (1952).

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