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On the Influence of Free Path on the Meissner Effect

In an article bearing the same title which appeared in the previous issue of this Journal, von Hagenow and Koppe[1] published a calculation which was intended to prove that the electron mean free path has no influence on the free energy or the Meissner effect. However, besides being in contradiction with a consensus of theoretical and experimental evidence,[2,3] their calculation contains errors which we shall discuss below.

We start with their Eq. (6), which gives the second-order effect of the scattering centers on the free energy:

$$u^{(2)} = \frac{1}{2\Omega^2} \sum_{\substack{\mathbf{k},\mathbf{k}'\\ j,j'}} |v(\mathbf{k}-\mathbf{k}')|^2 \times \exp[i(\mathbf{k}-\mathbf{k}')\cdot(\mathbf{r}_j - \mathbf{r}_{j'})] L(\varepsilon, \varepsilon') . \tag{1}$$

Because $\mathbf{r}_j$ is a stochastic variable, nonvanishing contributions arise only from terms with $j = j'$. The sums over $\mathbf{k}$ and $\mathbf{k}'$ contribute a factor Ω^2, and therefore $u^{(2)}$ is correctly proportional to n_I (the number of impurity or scattering centers) and to a volume-independent energy. Next, we perform the following steps:

(a) Integrate over the angles of $\mathbf{k}$ and $\mathbf{k}'$.

(b) Use the formula of Ref. 1 for $L(\varepsilon, \varepsilon')$, simplify by using the symmetry of the summand under interchange of $\mathbf{k}$ and $\mathbf{k}'$, so that

$$L(\varepsilon, \varepsilon') = \frac{-2}{\varepsilon' - \varepsilon} \frac{\varepsilon}{E} \tanh \frac{E}{2kT} . \tag{2}$$

(c) Obviously, only the difference between normal and superconducting free energies is of interest. Denoting this by $\Delta u^{(2)}$, it is readily found to be

$$\Delta u^{(2)} \sim \frac{n_I}{{\varepsilon_F}^2} \int_{-\hbar\omega}^{+\hbar\omega} d\varepsilon \int_{-\hbar\omega}^{+\hbar\omega} d\varepsilon' \langle |v|^2 \rangle_{\theta,\theta'} \times \frac{1}{\varepsilon' - \varepsilon} \left[\tanh \frac{\varepsilon}{2kT} - \frac{\varepsilon}{E} \tanh \frac{E}{2kT} \right] . \tag{3}$$

This integral can be evaluated if $\langle v^2 \rangle$ = constant, and it follows that although $\Delta u^{(2)}(T_c) = 0$, that $\Delta u^{(2)}(T) \neq 0$ for $T < T_c$ in contradiction with the remark in Ref. 1.

Thus, the free path *does* influence thermodynamic properties of a superconductor. Another criticism of the method of calculation[1] is that the particular truncation of $\mathscr{H}$ which results in the BCS "reduced" Hamiltonian had not been intended to be valid in the presence of scattering,[2,3] and was not valid in the *approach* in Ref. 1 to the problems of dirty superconductors.

References

1. K. U. von Hagenow and H. Koppe, *IBM Journal* **6**, 12 (1962.)
2. J. Bardeen and J. R. Schrieffer, Chapter VI, Vol. III of *Progress in Low Temperature Physics*, C. J. Gorter, Editor, North-Holland Publishing Co., Amsterdam, 1961.
3. D. Mattis and J. Bardeen, *Phys. Rev.* **111**, 412 (1958).

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