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Ultrasonic Attenuation in Superconductors

Abstract: A brief review is given of the ultrasonic attenuation in metals arising from direct interaction of the elastic waves and conduction electrons, and the physical variables on which it depends. The drop in attenuation of longitudinal waves on entering the superconducting state is in good agreement with BCS theory, the various factors combining to make the relative attenuation, α_s/α_n, depend only on the energy gap; the measurements give evidence, however, for gap anisotropy or the presence of more than one gap. Recent measurements at Brown by Claiborne of shear wave attenuation in single-crystal Al, are in good agreement with a theory based on the Boltzmann and London equations. The steep drop at T_c is produced by shorting out of electromagnetic waves by supercurrents; the residual attenuation results from the effects of collision drag.

This paper has two purposes: to give a very brief review of ultrasonic measurements in superconductors, and to report on some as yet unpublished work that has been done recently at Brown.

It is well known, of course, that an elastic wave in a metal interacts with the conduction electrons and hence is attenuated. One of the principal parameters in determining the magnitude of this attenuation is the ratio of wavelength to electron mean free path. If q is the propagation constant ($q = 2\pi/\lambda$, where λ is the wavelength) and l is the mean free path, then there are two principal regimes depending upon the size of ql relative to unity. If $ql > 1$, the attenuation coefficient is proportional to the first power of the frequency, is independent of l, and depends only upon fundamental parameters of the metal. If $ql < 1$, the attenuation coefficient is reduced from the previous case approximately by the factor ql; hence it is proportional to the square of the frequency and to l.

If ql is sufficiently large (as it can be in pure metals at low temperatures) the ultrasonic attenuation by conduction electrons is sizeable and easily observed. In a metal which becomes a superconductor, this attenuation is radically altered by the appearance of superconductivity. The discussion here will be restricted to the ordinary ultrasonic region (up to, say, 200 Mc/sec) where the phonon energy is very small compared to the energy gap, and where the wavelength, although shorter than the mean free path, is longer than other possibly relevant distances such as the coherence length. Other papers in this conference have something to say about such other cases.[1]

The ultrasonic attenuation by electrons in the ordinary ultrasonic region decreases rapidly and apparently monotonically as the temperature is lowered below T_c. The temperature dependence of this decrease in attenuation for longitudinal waves is quite well accounted for by the BCS theory, the calculation being an extension of the usual quantum theory of the scattering of phonons by electrons. The BCS theory shows that three effects modify the scattering rate: the temperature-dependent energy gap, the increase in the density of states near the edge of the gap, and a coherence factor the form of which depends upon whether or not the spin is changed in the scattering. Now the structure of the BCS theory is such that the last two factors exactly cancel for the low-frequency ultrasonic region. Thus the decrease in attenuation reflects only the opening of the energy gap. The prediction is that the ratio of superconducting to normal state attenuation is given by the Fermi function of the energy gap; namely,

$$\alpha_s/\alpha_n = 2/(e^{\varepsilon/kT} + 1) = 2f(\varepsilon)\,, \tag{1}$$

where ε is the temperature-dependent BCS energy gap. Although the derivation of this equation suggests that it is valid only for $ql > 1$, Tsuneto[2] has shown that it also holds for $ql < 1$.

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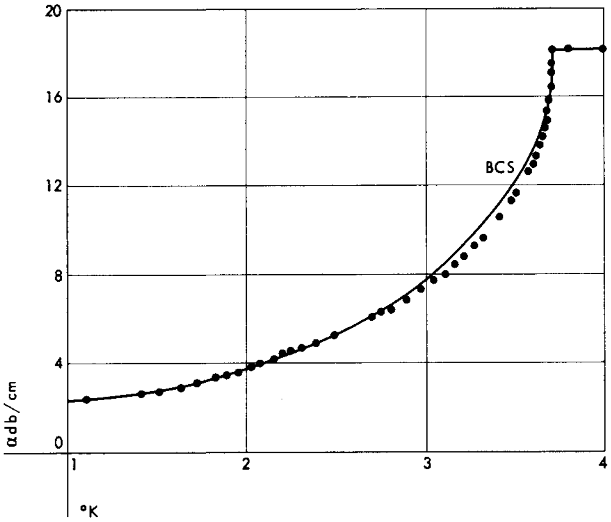

Figure 1 **Ultrasonic measurements in tin compared with the BCS prediction.**

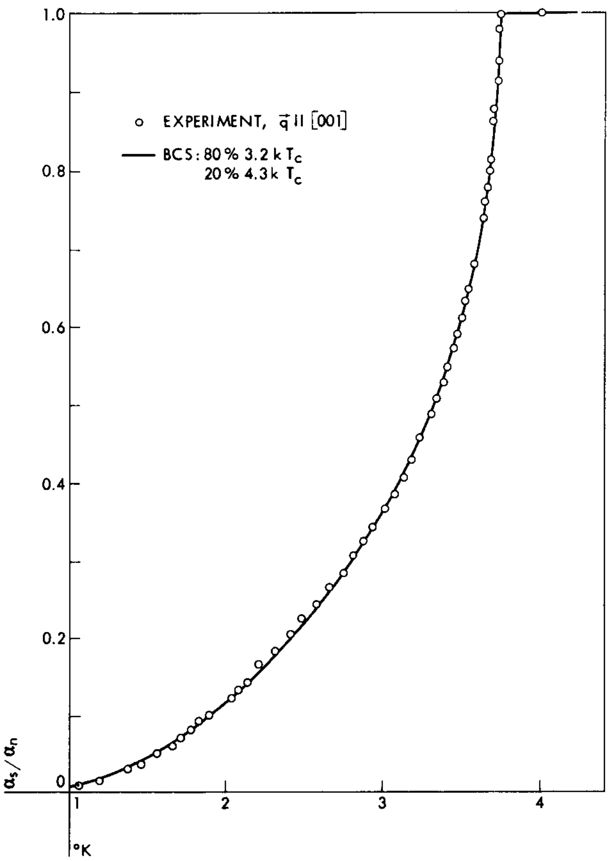

Figure 2 **Ultrasonic measurements in tin compared with a BCS prediction assuming two different energy gaps.**

Figure 1 shows how the above prediction typically compares with experimental results. There are two important ways in which such ultrasonic measurements give support to the BCS formulation. First of all, these results taken together with nuclear spin relaxation measurements verify that the spin dependence of an interaction is an important factor in determining scattering rates in a superconductor. In the nuclear spin case the rate goes up just below T_c, whereas it goes down with phonon scattering, results which are predicted by the BCS theory. The second point is that not just any energy gap theory would agree as well with the observed temperature dependence of the ultrasonic attenuation as does the BCS theory. One notes from Fig. 1 that the BCS theory does not predict quite as rapid a decrease below T_c as actually observed. A simple energy gap theory which did not have in it something like the coherence effect, would inevitably agree much worse with the measurements since the required density of states modification would lead to a greater scattering than that predicted by Eq. (1). Thus to obtain agreement one would be forced to quite a different temperature dependence of ε than is observed by other measurements.

The fact that the ultrasonic attenuation quite generally is found to decrease below T_c faster than predicted, could be due either to a small decrease in the lattice-electron interaction constant just below T_c, or to the presence of more than one gap. In the latter instance the largest energy gap value would be felt most near T_c and the smallest at low temperatures. Figure 2 shows how well one can fit tin data by adding the effects of two energy gaps. Such excellent agreement, of course, is not to be taken too seriously since any such fitting is not unique.

Ultrasonic measurements go further in suggesting either gap anisotropy or the presence of more than one gap, as some of us showed in tin a couple of years ago.[3] For $ql > 1$ the ultrasonic scattering picks out only those electrons which move in the direction of the wave with the sound velocity. Hence by propagating in different directions in a single crystal one might hope to select electrons from different parts of the Fermi surface. At the lowest temperatures the attenuation should fall exponentially, being determined by the limiting energy gap for the electrons which are selectively scattered. In tin we found that the apparent limiting energy gap indeed did depend upon the direction of propagation, varying from 3.2 to nearly 4.0 kT_c. The numbers we obtained in tin have since been verified by Mackintosh at Cambridge[4] and by Bezuglyi, Galkin, and Korolyuk in Russia.[5] Frankly I do not know how one can interpret these numbers in terms of the Fermi surface, or indeed if they have any real significance. However, they do suggest a directional dependence of the energy gap.

In the remaining time I would like to mention some very recent work done with Dr. Lewis Claiborne on shear wave effects in superconductors.[6] Some time ago

Morse and Bohm[7] found that the superconducting attenuation with shear waves shows a nearly discontinuous decrease near T_c followed by a gradual change similar in form to the longitudinal wave, i.e., one proportional to $f(\varepsilon)$. Claiborne has recently finished a study of the shear wave attenuation in aluminum crystals as a function of frequency, polarization and propagation directions, and temperature down to 0.28°K using adiabatic demagnetization. Special attention was given to the region very near T_c in order to examine the very rapid drop in detail. The ql range covered was from about 0.8 to 4.0. When the earth's magnetic field is cancelled, one finds that the very rapid decrease just below T_c has an observable width and the curve can be traced out point by point. Some results are shown in Fig. 3 as a function of frequency. Here one sees that the sharp drop in α_s/α_n just below T_c is significantly frequency dependent.

The total attenuation α_n was found by extrapolating the lower temperature results to $T = 0$. A typical temperature variation is shown in Fig. 4. (The results are not accurate enough for a good estimation of $\varepsilon(0)$.)

The frequency variation of α_s/α_n was also found to depend somewhat on the directions of propagation and polarization. Figure 5 shows another set of temperature variations near T_c.

Dr. Claiborne has made a theoretical calculation, approximately valid near T_c, which gives a reasonably tidy explanation of the results. He made a self-consistent, Boltzmann equation calculation similar to that made by Holstein[8] and others. Since the region of interest is very close to T_c, the superconducting behavior can be reasonably accounted for by use of the

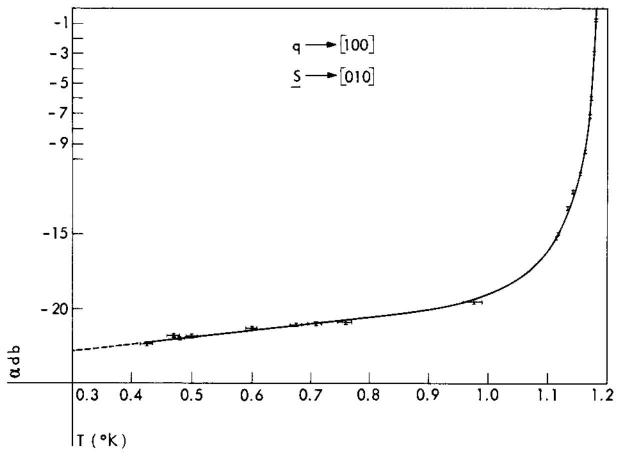

Figure 4 **α in *db* for shear waves in aluminum as a function of *T*.**

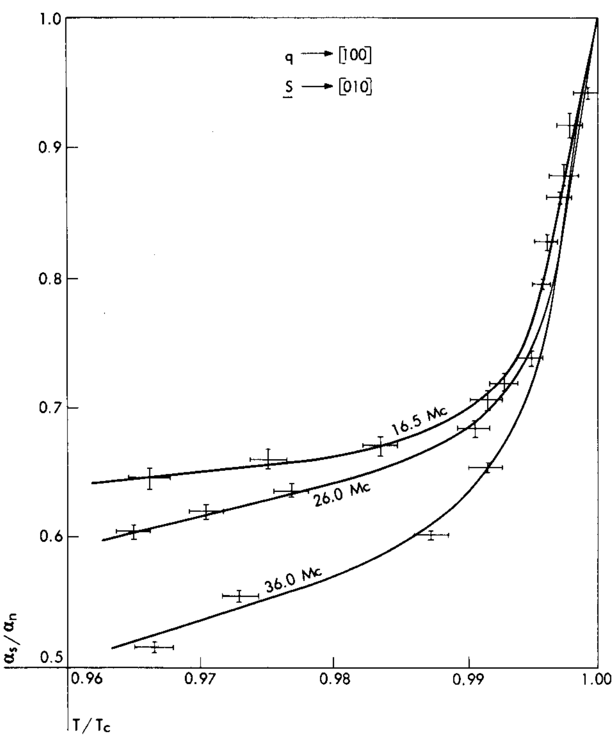

Figure 3 **α_s/α_n vs. T/T_c in aluminum near T_c.** *The direction of propagation is [100] and the polarization vector is along [010].*

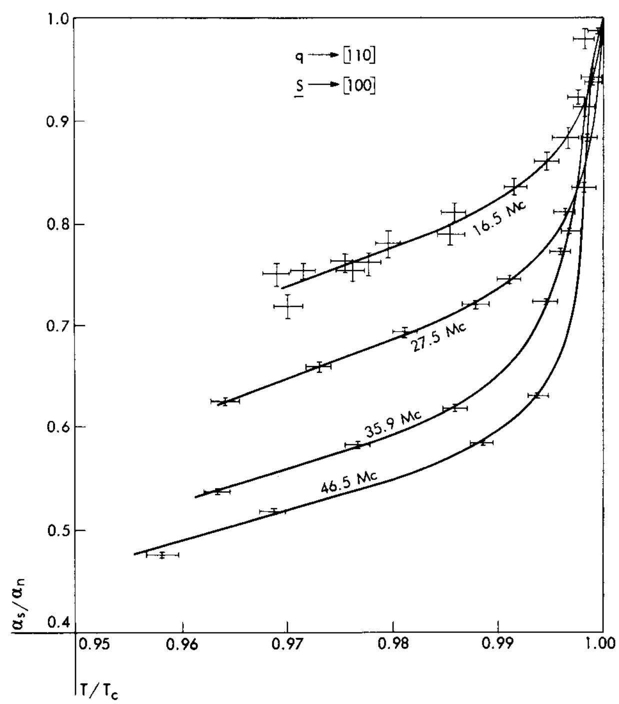

Figure 5 **α_s/α_n vs. T/T_c in aluminum near T_c.** *The direction of propagation is [110] and the polarization vector is along [100].*

London equation. Expressions for both longitudinal and shear wave attenuation were then obtained which are a function of the fractions of normal and superconducting electrons as well as ql and $\omega\tau$. He found, as expected, that α_s/α_n for longitudinal waves was equal to the fraction of normal electrons. Since in the BCS theory this should be $2f(\varepsilon)$, in his final results for the shear wave attenuation he replaced, by analogy, the normal electron fraction by $2f(\varepsilon)$. Without going into details here, it can be said that the calculations predict a behavior very much like that observed. Some typical results of the calculation are shown in Fig. 6. The rapid drop in α_s/α_n, which is due to screening by supercurrents, has a temperature width determined by $\omega\tau$. The residual attenuation is given by $g \cdot 2f(\varepsilon)$, where $g(ql)$ is a function which appears in the normal state shear wave calculation; it is plotted in Fig. 7. One sees that the shorting out should be negligible for very small ql and increase in significance as ql increases.

Now the residual term $g \cdot 2f(\varepsilon)$ results entirely from the inclusion of a collision drag term in the theory, i.e., the assumption that scattering produces a distribution which is in equilibrium with the local ion motions accompanying the wave. Tsuneto,[2] who made a more basic calculation for shear waves, did not find this

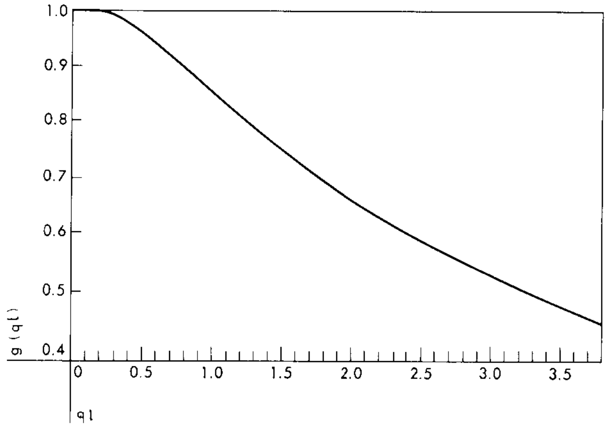

Figure 6 **A plot of g as a function of *ql*.**

$$g = \frac{3}{2ql^2}\left\{\frac{(ql)^2+1}{(ql)}\tan^{-1}(ql) - 1\right\}$$

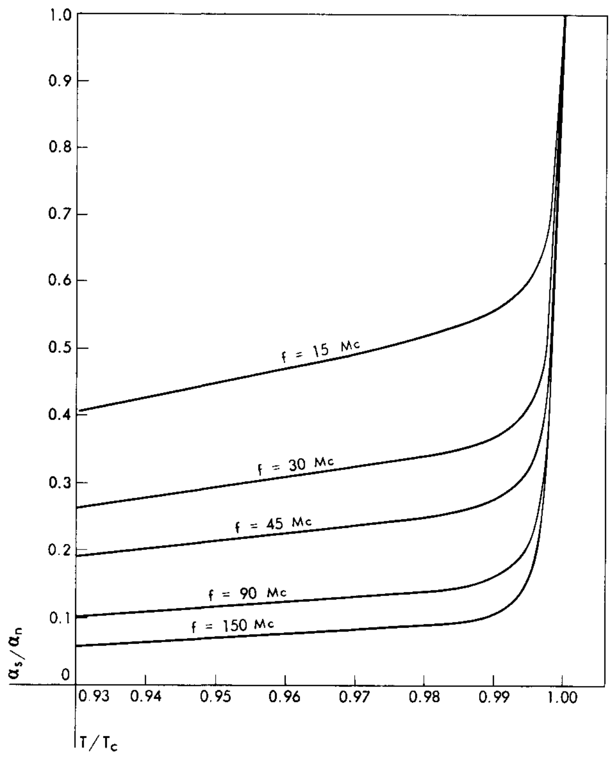

Figure 7 **Theoretical curves for shear waves, for fixed values of electron waves for fixed values of electron group velocity and relaxation time.** *($\tau = 0.3 \times 10^{-10}$ sec; $V_0 = 3.0 \times 10^8$ cm/sec).*

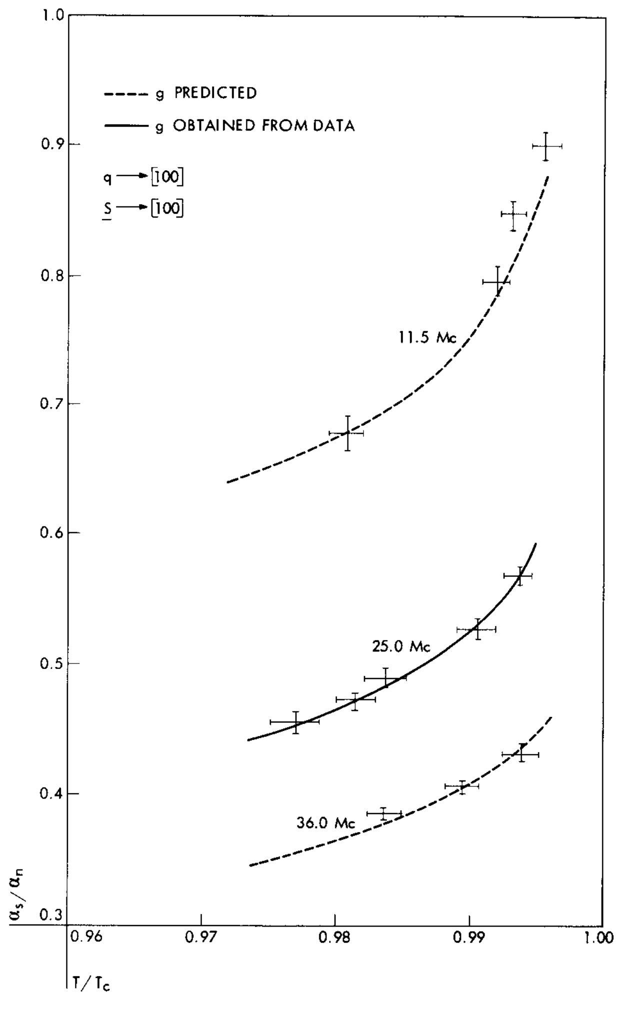

Figure 8 **A comparison between theory and experiment for the residual attenuation.** *The value of g is obtained by fitting the 25 Mc/sec data.*

residual term but found only the shorting-out effect, because he did not include the collision drag effect in calculating the attenuation even though he allowed for it in the distribution function.

Agreement between the calculations and the observations is quite gratifying and internally consistent. For example, if one estimates g by fitting the results at one frequency, then the curves for other frequencies are quite accurately reproduced. Such agreement is demonstrated in Fig. 8 for the data given in Fig. 3. The agreement, however, goes further than this. The normal-state attenuation, α_n, also depends upon g. As shown by Pippard[9] the shear wave attenuation is given by:

$$\alpha_n = K\frac{1-g}{g}, \tag{2}$$

where K is a constant independent of frequency. One finds that the g determined by fitting to the superconducting residual attenuation at one frequency also agrees with that which would be predicted for that frequency by the frequency dependence of α_n. Thus there is complete internal consistency by choice of a single g value (or l) for a given orientation of the propagation and polarization vectors. In going to another orientation one can also obtain another quite self-consistent fit to the data, but only by making a new choice of l. The differences in l's chosen for different orientations were about 20 percent and could be due either to actual variations in l or to other real-metal effects which were entirely ignored in the calculation.

In summary, the initial drop in the shear wave attenuation just below T_c seems to be due to a shorting out of the electromagnetic interaction between the lattice and the electrons, whereas the residual attenuation in the superconducting region seems to be explainable almost entirely by a collision drag interaction. Thus in aluminum it does not seem necessary to call upon real-metal effects to explain the residual attenuation observed with shear waves.

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Received June 15, 1961