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The Kapitza Resistance of Metals in the Normal and Superconducting States*

Abstract: A calculation has been made of the transfer of heat across a metal-liquid helium interface. In the normal state an appreciable transfer of energy occurs directly from the conduction electrons to the phonons of the fluid. Three mechanisms have been considered: the phonon-electron interaction within the metal, the modulation of the position of the interface and the phonon-electron interaction of the electrons which have tunnelled into the helium. All appear to be important. In the superconducting state each of these becomes ineffective at temperatures appreciably below the transition temperature and, consequently, the heat flow is reduced. The effect is sensitive to band structure and to the anisotropy of the energy gap.

When a solid is heated while immersed in liquid helium, it is found that a temperature discontinuity will occur across the solid-liquid interface.[1] The thermal resistance which gives rise to this discontinuity is called the *Kapitza resistance*. A reasonable understanding of this effect can be got by considering the transfer of energy by phonons across the interface.[2,3] The density and the velocity of sound of helium are both very much less than those of any solid; consequently at the interface a severe acoustic mismatch occurs and most phonons are reflected. This severely restricts the flow of heat across the surface and gives rise to the observed temperature discontinuity.

However, it is found that a considerably greater flow of heat occurs than that which can be accounted for by this mechanism alone. In view of this, some other mechanisms have been considered and it was realized that an additional transfer of heat could occur in the case of a metal by the direct interaction of the conduction electrons with the fluid phonons. It was suggested[3] then that one should expect a difference in the Kapitza resistance between a metal in the normal and one in the superconducting state. For in the superconducting state the electrons are prevented from taking part in such processes. Such a difference has now been observed by Challis[4] in Pb and by Gittleman[5] for In and Sn. To clarify what was meant by the earlier suggestion, a detailed calculation of the effect has been done and has been published elsewhere.[6] We give here a simplified outline of the calculation to illustrate the main physical features.

We consider a simple Sommerfeld model of a metal illustrated in Fig. 1. The electrons are bound in the potential well because of the smoothed-out charge distribution of the positive ions. This well terminates in a reasonably smooth manner at the plane $z = 0$, corresponding to the helium-metal interface. The probability density of the electrons at the Fermi surface, $\chi\chi^*$, is shown in the middle of the Figure. It should be noted that beyond the surface, this density decays away exponentially into the fluid. In the lower part of the Figure is shown the square of the amplitude of a phonon which is incident from the fluid. If the angle of incidence of this phonon exceeds the critical angle θ_c given by $\sin\theta = \alpha$ (α = ratio of the acoustic velocity of fluid to solid) total reflection will occur and n_1 will be real. A surface disturbance then exists whose amplitude decays exponentially away from the surface into the solid. The ratio α is approximately 0.1 for most solids and, consequently, total reflection occurs for a large fraction of incident phonons.

Three mechanisms of electron-phonon interaction

It is possible for the conduction electrons to interact with the phonons, then, in one of three ways. Referring now to the upper part of Fig. 1, we see there are three regions of interest. Firstly, the tail of the phonon in

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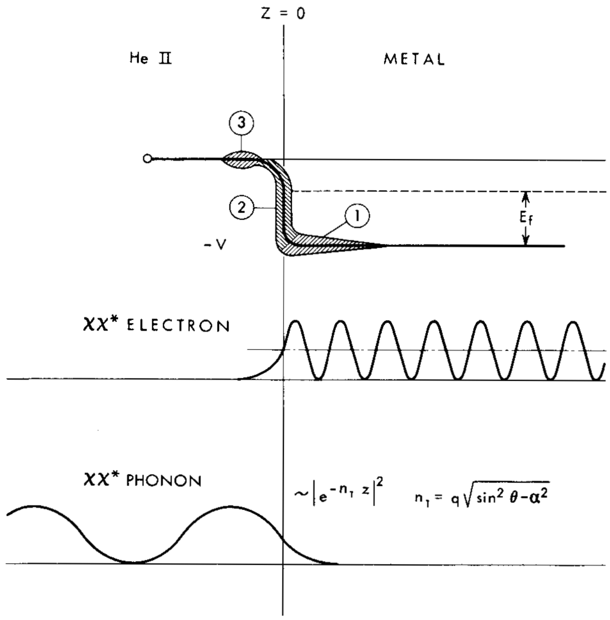


Figure 1 **A Sommerfeld model of a metal.** *The top diagram shows the electrons bound in the potential well, with Fermi energy E_f. The well terminates at the plane $Z = 0$, which is the helium-metal interface. The middle diagram shows the probability density of electrons at the Fermi surface. The bottom diagram shows the square of the amplitude of a phonon incident from the fluid.*

the solid can cause a compression or rarefaction of the charge density of the solid near the surface and thus can modulate the potential in which the electrons move. This is shown as Region 1. Secondly, the phonon impinging upon the solid surface can move it bodily back and forth and thereby modulate the potential as seen by an electron at a fixed point in space. This modulation is proportional to the gradient of the potential at the surface and to the amplitude of the phonon wave. It is shown as Region 2. Thirdly, in Region 3, the electron may tunnel into the fluid and interact there with the density modulation of the fluid. All these mechanisms provide a means of transferring energy between the electron and phonon systems.

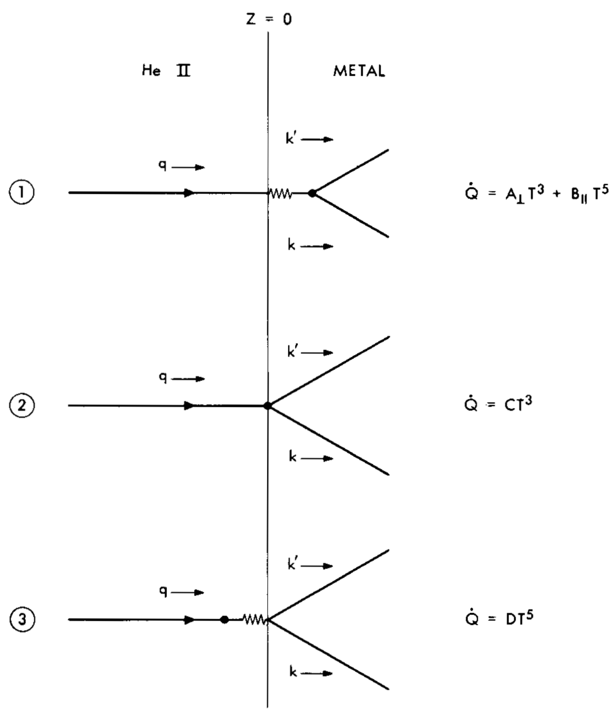
In Fig. 2 we show diagrammatically each of these processes together with the calculated heat flow across the surface. For Region 1 it is found that two groups of conduction electrons play a role. Those which are incident approximately normally can contribute to the heat flow and give a term proportional to T^3. Also, those electrons incident at very small angles add a term proportional to T^5. At helium temperatures this term turns out to be negligible compared to the first. In the second region one also obtains a term proportional to T^3. This differs from the similar term of Region 1 in that electrons from all parts of the Fermi surface contribute equally to the heat transfer here, whereas in Region 1 only that part of the Fermi surface corresponding to electrons of almost normal incidence contributed a significant part. The magnitude of this term appears to be comparable to that of the first term for the particular case considered, i.e., Pb—He II, but might not be so in other cases.

In Region 3, electrons may tunnel into the fluid and interact with phonons there. Bloch[7] has considered this process and finds a T^5 temperature dependence for it. Moreover, the magnitude of this term is almost a hundred times greater than that of the other terms and should be important. However, experimentally no such T^5 dependence has been seen. This is not unreasonable in view of the fact that an exceedingly thin dirt or oxide layer on the surface would greatly reduce the electron amplitude reaching the fluid and thus reduce considerably the heat flow.

Interaction in the superconducting state

In the superconducting state of the metal one can expect an interaction of the phonons with the electrons to occur in several ways. A quasi-particle may be excited out of the ground state, a thermally-excited quasi-particle may be scattered by the phonon, or the phonon may excite a collective mode of the superconducting state. In the first process, the phonon requires an energy of at least 3.5 kT_c, the energy needed

Figure 2 **The three processes for transfer of heat across a metal–liquid helium interface.** *The three diagrams illustrate the processes taking place in each of the regions marked in Fig. 1.*



to span the energy gap. The number of such phonons will decrease approximately exponentially with temperature well below T_c. Similarly, the second process will also become negligible well below T_c because the number of quasi-particles thermally excited also will decrease exponentially with T. The third process appears to be unlikely because the matrix elements for the excitation of a collective mode would be exceedingly small and also the number of such modes is comparatively small. Hence, at temperatures well below T_c all these processes become inoperative, and the Kapitza resistance is increased. For temperatures not quite so low it should be possible to determine the factor in the exponent and hence deduce the magnitude of the energy gap Δ. The fact that the electrons involved can come from a rather narrow region of the Fermi surface (approximately normal incidence) indicates that the angular dependence of Δ may be determined in this way as well as the band structure of the metal in the normal state.

References

1. P. L. Kapitza, *J. Exptl. Theoret. Phys.* (*U.S.S.R.*) **11,** 1 (1941).
2. I. M. Khalatnikov, *J. Exptl. Theoret. Phys.* (*U.S.S.R.*) **22,** 687 (1952).
3. W. A. Little, *Can. J. Phys.* **37,** 334 (1959).
4. L. J. Challis, *Proceedings of VIIth International Conference on Low Temperatures*, Toronto, p. 466 (1961).
5. J. I. Gittleman, *Bull. Am. Phys. Soc.* **6,** 3, 268 (1961).
6. W. A. Little, *Phys. Rev.* **123**, 435 (1961).
7. F. Bloch, private communication.

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