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A Note on Hall Probe Resolution

Mapping magnetic fields by means of the Hall probe involves a study of the relationship between the Hall probe output and the actual magnetic field. This paper investigates this relationship, and describes a technique developed to obtain the corrected field from experimental measurements to within 1 to 2 per cent of the true magnetic field.

A Hall probe, rectangular in shape, is placed in a static magnetic field (Fig. 1). The sensitive area of the probe is defined as the intersecting area between two leads. The probe is moved along a straight line through the magnetic field, and the probe output can be represented by $G(x)$, where x is the coordinate of the probe position. The Hall probe resolution can be described as the relation between $G(x)$ and $g(x)$, where $g(x)$ represents the output function of a Hall probe approaching infinitesimal dimensions.

The following assumptions are made in the analysis:

1. The sensitive area of the probe is a square, each side $2h$ in length.
2. The Hall probe has constant sensitivity over its active area.
3. The Hall probe measures only that field component normal to its surface.
4. The normal field component is assumed to be independent of z (coordinate perpendicular to x in plane of probe) over the sensitive area of the probe.
5. $G(x)$ and $g(x)$ are continuous and differentiable for all values of x.
6. The output function $G(x)$ of an actual probe $(2h\times 2h)$ is a measure of the average field across the probe sensitive area, so that $G(x)$ and $g(x)$ are related by

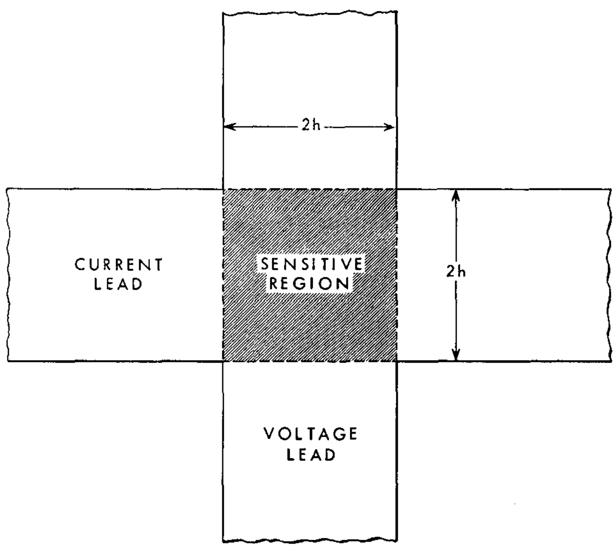
$$G(x)=\frac{1}{2h}\int_{-h}^{+h} g(x+a)\,da\,. \tag{1}$$

Equation (1) is integrable after substituting the Taylor expansion for $g(x+a)$ around any point, x. Then, substituting in the resultant equation the Fourier expansion of $G(x)$ and $g(x)$ in the interval $-mh\leq x\leq mh$, where m is a very large integer so that the interval $2mh$ adequately covers the range of x of interest,

$$\begin{aligned}G(x)&=A_0+\sum_{n=1}^{\infty}\left(A_n\cos\frac{n\pi x}{mh}+B_n\sin\frac{n\pi x}{mh}\right)\\&=a_0+\sum_{n=1}^{\infty}\frac{\sin(2n\pi h/2mh)}{2n\pi h/2mh}\\&\qquad\times\left(a_n\cos\frac{n\pi x}{mh}+b_n\sin\frac{n\pi x}{mh}\right).\end{aligned} \tag{2}$$

The a_n and b_n are the Fourier coefficients of the function $g(x)$ for an "ideal" Hall probe with an infinitesimally small sensitive area; they are related to the Fourier coeffi-

Figure 1 **Schematic of Hall probe sensitive region.**



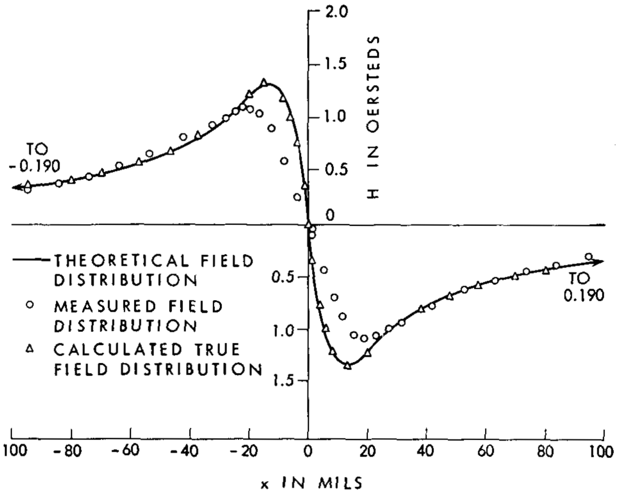


Figure 2 **Magnetic field intensity vs horizontal distance from wire.** *Hall probe size: 38×38 mils. Wire-to-probe distance: 13 mils. Wire current: 500 ma.*

cients A_n and B_n of the output function $G(x)$ of the actual probe ($2h \times 2h$) by:

$$\left.\begin{aligned} a_n &= \frac{2n\pi h/2mh}{\sin(2n\pi h/2mh)} A_n \\ b_n &= \frac{2n\pi h/2mh}{\sin(2n\pi h/2mh)} B_n \end{aligned}\right\} n=1, 2, 3 \ldots, \infty . \qquad (3)$$

From Eq. (2), if the true field, $g(x)$, contains harmonics whose wavelengths are 1, ½, ⅓, . . . times the dimension of the probe $2h$, corresponding to $n/m=s=1, 2, 3, \ldots, \infty$ these will not be present in the expansion for $G(x)$. Theoretically, to account for all these missing terms a second scan must be made with a probe having a sensitive area of $2h' \times 2h'$, where the ratio $h'/h > 1$ must be an irrational number.

$$\left.\begin{aligned} a_s &= \frac{s\pi(2h'/2h)}{\sin s\pi(h'/h)} A_s \\ b_s &= \frac{s\pi(2h'/2h)}{\sin s\pi(2h'/2h)} B_s \end{aligned}\right\} s=\frac{n}{m}=1, 2, 3 \ldots, \infty.$$

However, in practice, there are only a finite number of these missing terms that can be accounted for, limited by the accuracy of the probe dimensions (i.e., the irrational number has to be truncated in a practical case). For example, if $2h'/2h=1.11$, all the terms for s less than 100, or n less than 100 m, can be accounted for.

Also, the degree of accuracy in constructing the function $g(x)$ depends entirely on the accuracy of the measured output function of the probe, $G(x)$. Therefore, the accuracy of $g(x)$ so obtained is also limited by the noise level of the instrumentation system.

To demonstrate the applicability of the above analysis in reconstructing the true field, $g(x)$, from the measured field, $G(x)$, the field near a current-carrying wire has been mapped, using a probe with sensitive area of 0.038 × 0.038 in. (Fig. 1). The mapped field and the calculated true field (for $m=10$) are compared with the theoretical field in Fig. 2. Similar information for a 0.001 × 0.001 in. probe is shown in Fig. 3.

Neglecting terms of higher order in the integrated form of Eq. (1), it is possible to define the resolution of a probe at $x=x_0$ as follows:

$$\text{Resolution} = \frac{G(x_0)-g(x_0)}{2h} = \frac{g''(x_0)h}{12} = \frac{1}{12}\frac{h}{\Gamma(x_0)},$$

where $\Gamma(x_0)$ is the radius of curvature of the magnetic field at $x=x_0$.

For $|h/\Gamma(x)|<1$, it was found that the probe is small enough for accurate mapping of the magnetic field of the $1/x$ type (Fig. 3), while in the case of $|h/\Gamma(x)|>1$, it is necessary to apply the correction outlined to obtain the true field. When x is small (less than 13 mils), a slight discrepancy exists (about 4 to 6 per cent) between the calculated true field and the theoretical field. This is because A_{10}, A_{20}, A_{30}, . . ., ∞ are equal to zero in the expansion for $G(x)$. Using a different size probe for a second scan and applying the correction technique outlined previously, the agreement is within one to two per cent, which is within the experimental and computing errors.

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Figure 3 **Magnetic field intensity vs horizontal distance from wire.** *Hall probe size: 1×1 mil. Wire-to-probe distance: 4 mils. Wire current: 500 ma.*