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A Study of the Playback Process of a Magnetic Ring Head*

Abstract: The playback process of a magnetic ring head with finite permeability of head and tape are studied, using the theorem of reciprocity. In order to obtain an accurate result for the playback process, the field around a magnetic gap is studied by a Fourier method. The shift of the gap null for infinite permeability of the head as calculated by Westmijze is confirmed, and the new shift is found when the tape permeability is greater than one. A simplified gap-loss function is given for the case of finite parameters for tape and head.

Introduction

Several studies have been made of the playback process of magnetic recording. The first contribution was the determination of the approximate flux playback response of a magnetic head in contact with an infinitely thin recorded tape.[1,2] The approximate flux response was found to be of the form $\frac{\sin 2\pi/\lambda}{2\pi/\lambda} \cos (2\pi/\lambda)t$, where λ is the wavelength of the recorded signal on tape in the units of half-gap length. The output voltage from a head is the time derivative of the flux response, and therefore the voltage amplitude varies as $\sin 2\pi/\lambda$. Then Wallace[3] solved for the effects of finite head/tape separation[4] and finite thickness of the tape.

The next major step was taken by Westmijze[5] in 1953. He showed that the simple $\sin 2\pi/\lambda$ voltage response is only a rough estimate. The formula predicts a zero output at $\lambda=2/n$, where n is an integer, and he showed on the basis of an exact calculation that these null points are shifted toward longer wavelengths. He also determined the effect of head/tape separation and tape thickness by using a reciprocity theorem. However, Westmijze's result on gap loss can only be expressed in a complicated integral form. Hence the simple gap-loss formula is still widely used, especially for rough estimates. Furthermore, in all of the cases treated in the literature, the assumption has been made that the tape permeability is one and the head permeability is infinite. The case of finite permeability of head and tape still appears open.

It appears that the methods[5] based on the Schwartz-Christoffel transformation do not apply for solving the case with finite parameters.[6] In the present paper a Fourier method is used to obtain the field outside of a gap. Then, by using the reciprocity theorem, a simplified gap-loss function is derived, permitting the voltage response of a playback head with arbitrary tape permeability to be solved rigorously. It also enables us to obtain interesting qualitative results when the permeability of the head is finite. The problem considered by Westmijze is reconsidered here as a special case. A first-order series expansion is used to obtain numerical results, and excellent quantitative agreement with Westmijze's work is found. The reciprocity theorem is derived from Maxwell's equations[9] for the first time, and the derivation includes the case in which the head is loaded with a finite impedance. That case is of some interest for high-frequency recording since it is difficult to obtain high-input-impedance amplifiers in the megacycle range.

Semi-infinite head with infinite permeability

We shall first consider the case in which the permeability of the head material is infinite. The same case was solved by Westmijze[5] by using a different method. The physical configuration of the reproducing head surface can be represented by a semi-infinite gap as shown in Fig. 1. The two pole-pieces are closed at the back and a winding is placed on the back side of the pole-piece. When a pre-recorded tape passes the head surface, the winding gives an output voltage.

* The first part of the work reported here, the section on infinite permeability, was done under the supervision of Prof. E. T. Jaynes, while the author was a student at Stanford University.

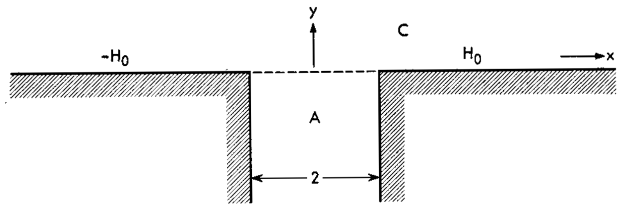


Figure 1

It is possible to calculate the output voltage from the winding by using a reciprocity theorem, as shown in Appendix I. The output voltage is:

$$V_{oc}=\int_v \mathbf{H}(x,y)\frac{\partial \mathbf{M}(x,y)}{\partial t}dv, \tag{1}$$

where $\mathbf{H}(x,y)$ is the magnetic field produced by a unit current passing through the head winding and $\mathbf{M}(x,y)$ is the magnetization of the tape. The integration is taken over all space.

The field near a gap

There are several methods for obtaining the field $\mathbf{H}(x,y)$ near a gap. Our approach is to solve Laplace's equation for the potential function in the region of interest and then obtain the field by differentiation.

When a current passes through the winding, the two pole pieces will be at a different magnetic potential. Since the head material is infinitely permeable, there can be no potential drop in the pole piece. Thus, the whole potential drop is across the gap. We denote *the potential on the pole pieces* by H_0 and $-H_0$, respectively. The precise boundary condition is indicated in Fig. 1.

We separate the region outside of the pole pieces into regions A and C (as shown in Fig. 1). The general solution to the Laplace equation in each region is:

$$\psi_A(x,y)=H_o x+\sum_{n=1}^{\infty}A_n \sin n\pi x e^{n\pi y} \qquad \begin{matrix}0<x<1\\ y<0\end{matrix} \tag{2}$$

where A_n are constants to be determined, and

$$\psi_C(x,y)=\int_0^{\infty}C(k)\sin kx e^{-ky}dk \qquad \begin{matrix}y>0\\ 0<x<\infty\end{matrix} \tag{3}$$

Matching the potential at $y=0$:

$$\int_0^{\infty}C(k)\sin kxdk=\sum_{n=1}^{\infty}A_n\sin n\pi x+H_0x \quad 0<x<1 \tag{4}$$

$$=H_0 \qquad 1<x<\infty$$

If we make a Fourier sine transformation on both sides with σ as a parameter in the respective regions and use the identity

$$\int_0^{\infty}\sin\sigma x\sin kxdx=\frac{\pi}{2}\delta(\sigma-k), \tag{5}$$

where $\delta(\sigma-k)$ is a Dirac's delta function, we obtain

$$\frac{\pi}{2}C(\sigma)=H_0\frac{\sin\sigma}{\sigma^2}+\sum_{n=1}^{\infty}A_n(-1)^n\frac{n\pi\sin\sigma}{\sigma^2-(n\pi)^2}. \tag{6}$$

Matching the y derivatives at $y=0$, we have

$$-\sum_{n=1}^{\infty}A_n n\pi\sin n\pi x=\int_0^{\infty}C(k)k\sin kxdk \quad 0<x<1. \tag{7}$$

Next, we multiply by $\sin(m\pi x)$ and integrate to obtain

$$\frac{A_m}{2}=(-1)^{m+1}\int_0^{\infty}C(k)\frac{k\sin k}{k^2-(m\pi)^2}dk. \tag{8}$$

From (6) and (8) we can eliminate the coefficients $C(k)$ and obtain

$$\frac{A_m}{2}=\frac{2}{\pi}(-1)^{m+1}\sum_{n=1}^{\infty}A_n(-1)^n n\pi$$

$$\times\int_0^{\infty}\frac{k\sin^2 kdk}{[k^2-(m\pi)^2][k^2-(n\pi)^2]}+H_0\int_0^{\infty}\frac{\sin^2 kdk}{k[k^2-(m\pi)^2]}, \tag{9}$$

where

$$\int_0^{\infty}\frac{k\sin^2 kdk}{[k^2-(m\pi)^2][k^2-(n\pi)^2]}=\frac{1}{2m\pi}Si(2m\pi) \qquad m=n$$

$$=\frac{1}{2\pi^2(m^2-n^2)}$$

$$\times\left[Ln\frac{m}{n}-Ci(2m\pi)+Ci(2n\pi)\right] \quad n\neq m \tag{10}$$

and

$$\int_0^{\infty}\frac{\sin^2 kdk}{k[k^2-(m\pi)^2]}=\frac{-1}{m\pi}[\gamma+Ln(2m\pi)-Ci(2m\pi)], \tag{11}$$

where γ is the Euler constant,

$$Ci(2m\pi)=-\int_{2m\pi}^{\infty}\frac{\cos x}{x}dx$$

$$Si(2m\pi)=\int_0^{2m\pi}\frac{\sin x}{x}dx.$$

We denote the right-hand side of (10) and (11) as K_{nm} and L_m respectively. Thus, (9) becomes

$$\frac{A_m}{2}=\frac{2}{\pi}(-1)^{m+1}\sum_{n=1}^{\infty}A_n(-1)^n n\pi K_{nm}+H_0L_m. \tag{12}$$

Equation (12) is a set of algebraic equations with known coefficients K_{nm} and L_m. Thus, the numerical values of A's can be obtained. It was evaluated that $A_1=-0.082H_0$, $A_2=0.027H_0$, $A_3=-0.014H_0$.

The potential function in the upper half-plane is then

$$\psi_C(x,y)=\frac{2H_0}{\pi}\int_0^{\infty}\frac{\sin k\sin kx}{k^2}e^{-ky}dk+\sum_{n=1}^{\infty}A_n 2n(-1)^n$$

$$\times\int_0^{\infty}\frac{\sin k\sin kx}{k^2-(n\pi)^2}e^{-ky}dk. \tag{13}$$

The terms in the series are small and drop off rapidly because the integral becomes very small as n increases. The higher order terms therefore have very little influence on the potential in the upper half-plane. Good numerical values can be obtained just by considering a few terms. In the case of the magnetic recording process, the zero-order term is sometimes sufficient, while in the playback process one has to consider the first-order approximation so that the gap loss can be accurately calculated.

From the potential function in (13), one can obtain the x and y components of the magnetic field H_x, H_y as:

$$H_x=\frac{2H_0}{\pi}\int_0^{\infty}\frac{\sin k\cos kx}{k}e^{-ky}dk+\sum_{n=1}^{\infty}A_n 2n(-1)^n \times\int_0^{\infty}\frac{\sin k\cos kx k e^{-ky}}{k^2-(n\pi)^2}dk \tag{14}$$

and

$$H_y=-\frac{2H_0}{\pi}\int_0^{\infty}\frac{\sin k\sin kx}{k}e^{-ky}dk+\sum_{n=1}^{\infty}A_n 2n(-1)^{n+1} \times\int_0^{\infty}\frac{k\sin k\sin kx e^{-ky}}{k^2-(n\pi)^2}dk, \tag{15}$$

where the H_x is an even function of x while H_y is odd.

The playback process

Consider a sine wave recorded on the tape in the x-direction. As the tape moves over the head surface, the magnetization has the form

$$M_x=M\cos\frac{2\pi}{\lambda}(x-vt). \tag{16}$$

Thus, it can be written as:

$$M_x=M\left(\cos\frac{2\pi}{\lambda}x\cos\frac{2\pi}{\lambda}vt+\sin\frac{2\pi}{\lambda}x\sin\frac{2\pi}{\lambda}vt\right). \tag{17}$$

Since H_x is an even function, the sine term in Eq. (17) will not contribute to the integral over the x-axis. Thus, the only term that contributes is the cosine term, and from (1), (13) and (16) we obtain

$$V(\lambda)=M2\cos\frac{2\pi vt}{\lambda}\left(\frac{1-e^{-\frac{2\pi a}{\lambda}}}{\frac{2\pi}{\lambda}}\right)e^{-\frac{2\pi b}{\lambda}}\sin\frac{2\pi}{\lambda} \times\left[1+\sum_{n=1}^{\infty}\frac{(-1)^n}{H_0}\frac{A_n 4n\pi}{4-(n\lambda)^2}\right], \tag{18}$$

where a is the thickness of the tape and b is the separation between head and tape.

The first term on the right-hand side of (18) was sometimes used as an approximate formula for the playback process; it does give the right separation and thickness loss. However, it does not give the shift of the first gap null as calculated by Westmijze.[5] The various orders of approximation corresponding to the inclusion of the increasing values of n in the series in Eq. (18) are plotted in Fig. 2 for wavelengths greater than the first gap null. It is shown that the deviation of the second approximation from the first is very small and that the higher order terms have a negligible effect on the overall response. The first approximation to the playback response, including the $n=1$ term, is approximately

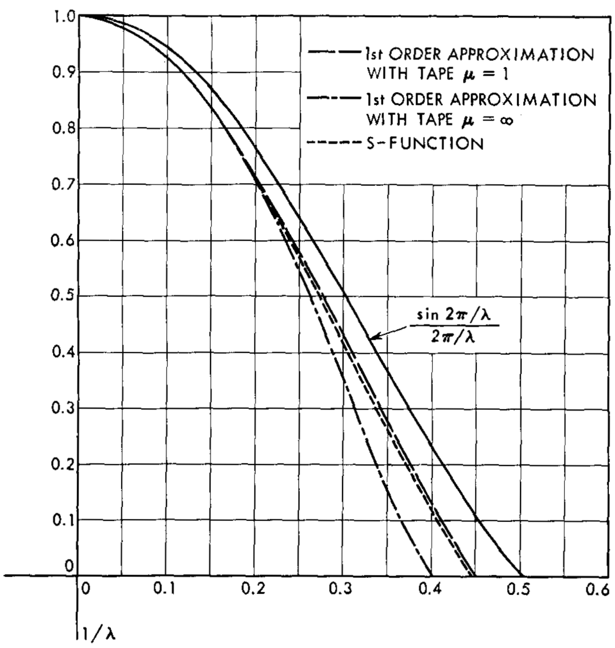
$$V(\lambda)=M\,2\cos\frac{2\pi vt}{\lambda}\left(\frac{1-e^{-\frac{2\pi a}{\lambda}}}{\frac{2\pi}{\lambda}}\right)\sin\frac{2\pi}{\lambda}\left[\frac{5-\lambda^2}{4-\lambda^2}\right]e^{-\frac{2\pi b}{\lambda}}. \tag{19}$$

Qualitatively, the shift of the null is due to the fringing field of the semi-infinite gap and the correction factor is simply $\left(\frac{5-\lambda^2}{4-\lambda^2}\right)$ for wavelengths greater than the gap length.

The net flux in the gap as shown in Eq. (17) has the amplitude of M cos $(2\pi/\lambda)x$. The field of the gap can be viewed as a weighting function in Eq. (1). The zero-order term gives a uniform weight to the flux in the gap; hence on integrating over the gap, the net flux is zero at $\lambda=2$. The first-order term reveals that the flux near the corner weighs about 16% more than that in the center; thus the null flux point comes at $\lambda\approx 2.24$.

In the above calculation, we have neglected the y component of the flux on tape. This is justified since the additional term that would have to be considered in Eq. (1) (i.e., $H_y M_y$) has the identical wavelength response as (19), and hence it is effectively included in the constant M.

Figure 2 **The gap loss of a ring head.**



Playback process with finite parameters of head and tape

The case in which the μ of the tape is other than one is also of interest. This can be calculated by the present method. However, the permeability of a tape is not a very meaningful concept, for with the present knowledge of fine-particle theory, it is extremely difficult, if not impossible, to specify some physically relevant parameter such as the μ of the tape. The analysis can only be viewed as an indicative calculation and is more meaningful when it is applied to a metallic medium.

The calculation with a finite permeability tape is similar to that of the previous section. The general solution of the potential function will be identical. Since the field has a jump when entering a medium with a different μ for a tape in contact with the head, Eq. (7) is changed to

$$\sum_{n=1}^{\infty} A_n n\pi \sin n\pi x = -\mu \int_0^{\infty} C(k) k \sin kx dk, \tag{20}$$

where μ is the permeability of the tape. Then performing the same operations as before, the values of the A's are obtained from

$$\frac{A_m}{2\mu} = \frac{2}{\pi}(-1)^{m+1} \sum_{n=1}^{\infty} A_n (-1)^n n\pi \times \int_0^{\infty} \frac{k \sin^2 kdk}{[k^2-(m\pi)^2][k^2-(n\pi)^2]} + H_0 \int_0^{\infty} \frac{\sin kdk}{k[k^2-(m\pi)^2]}. \tag{21}$$

Some values of these A's for different μ are tabulated as follows:

μ	A_1/H_0	A_2/H_0	A_3/H_0
1	−0.082	+0.027	−0.014
5	−0.128	+0.044	−0.017
10	−0.147	+0.049	−0.019
∞	−0.162	+0.053	−0.021

It is clear that the fringing effect is increased when the higher permeability tape is used. This checks well with physical intuition. The shift of the gap null due to the permeability of the tape can be calculated. Using the coefficient for infinite permeability tape, the correction factor for the first approximation to the gap loss now becomes

$$1+\frac{0.162}{4-\lambda^2}(4\pi) \cong \frac{6-\lambda^2}{4-\lambda^2}. \tag{22}$$

The null will be at $\lambda=2.44$; in other words, the shift will be approximately 22 per cent instead of 12 per cent.[12]

The above method can be applied to study the effect of the case in which the permeability of the head is finite. If the head permeability is finite, there will be a potential drop in the head; consequently there is also a field in the head pole piece. By the reciprocity theorem, such a field will also have some playback action.

The field in the pole piece is small, since the field has a jump of μ when it enters the gap. Thus, in applying the reciprocity theorem, one finds that the gap has a much more efficient playback process than that of the pole piece for the same length. For the short wavelengths on tape, over the surface of the pole piece there will be (roughly) equal amounts of flux in one direction as another, and the net flux over the head pole piece will tend to zero as the wavelength tends to zero. At longer wavelengths, the flux over the pole piece is the same direction (since the cosine part will play back as previously stated). *Hence, the integration in Eq. (1) is nonzero over the large volume above the total head pole piece, and the output voltage due to the pole piece can be comparable with that of a gap.* This would result in a higher output voltage at long wavelengths, and as the wavelength becomes shorter, the pole piece continuously loses its playback action.[13] This will result in a slower rise of output voltage than predicted by previous calculations.

If one neglects the playback process due to the pole piece (μ of head→∞), then the output voltage will rise 6 db/oct. However, in practice, one finds that it ranges from 3½ db to 5 db/oct. The ideal 6 db/oct. is never obtained. It is believed that this is due to the finite μ of the head. Previous experiments show that as the μ of the head improves, the rise becomes steeper; also, the rise is strongly dependent on the wrap-angle of the tape. These results qualitatively support the above suggestion.

Conclusion

From the above analysis, we have found that for all practical purposes, the gap-loss function can be written as

$$G_1 = \sin\frac{2\pi}{\lambda}\left(\frac{5-\lambda^2}{4-\lambda^2}\right) \tag{23}$$

in the case of infinite permeability.

When the μ of the recorded medium is high, there is no effect on separation and thickness loss of the medium, but the gap loss is changed to:

$$G_2 = \sin\frac{2\pi}{\lambda}\left(\frac{6-\lambda^2}{4-\lambda^2}\right). \tag{24}$$

To a first approximation the shift is independent of separation loss and tape thickness; since the calculation is exact for an infinite medium. If in each layer of the infinite medium the shift is the same, then the shift must not depend on the tape thickness and the separation.

When the μ of the head material is finite, a low-frequency boost is found to exist. This causes a discrepancy with the 6 db/oct. rise at long wavelengths predicted by previous calculations. No detailed calculation is presented since it strongly depends on the pole piece configuration and the actual value of the μ of the head.

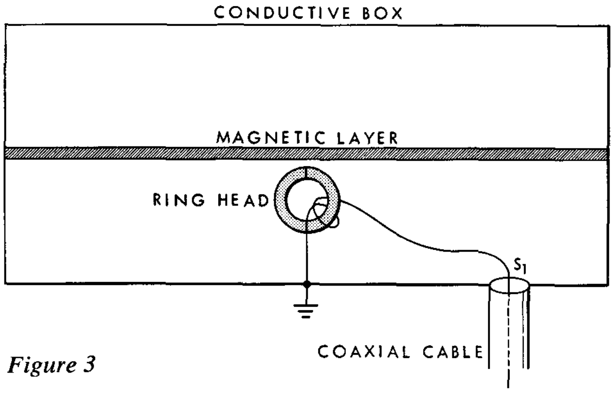


Figure 3

All calculations were based on a sine-wave response and therefore can be applied to any wave shape by resolving the waveform into its Fourier components.[14]

Appendix I

We put the ring head in a large conducting box with the output lead in the form of a coaxial line, with unit radius, as in Fig. 3.

Then let

$[\mathbf{E}_1, \mathbf{H}_1]$ be the electrical and magnetic field due to unit current in the winding

$[\mathbf{E}_2, \mathbf{H}_2]$ be the electrical and magnetic field due to a set of magnetic dipoles passing by the head gap.

Then, we use the divergence theorem, and write

$$\int_v \nabla\cdot(\mathbf{E}_1\times\mathbf{H}_2-\mathbf{E}_2\times\mathbf{H}_1)\,dv=\int_s(\mathbf{E}_1\times\mathbf{H}_2-\mathbf{E}_2\times\mathbf{H}_1)\cdot dS,$$

where v is the volume in the box and S is the surface over the volume. On the right-hand side the surface integral vanishes everywhere on the box, since there can be no tangential component of $\mathbf{E}$ field on the conductive boundary. And if the terminal is open, the value of $\mathbf{H}_1$ is zero on the surface of the coaxial, for there is no current in the coaxial due to the moving dipole, and the integral is reduced to:

$$-\int_{s_1}(\mathbf{E}_2\times\mathbf{H}_1)\cdot dS=\frac{1}{2\pi}\int_{s_1}(\mathbf{I}_\phi\times\mathbf{E}_2)\cdot dS=-V_{0C},$$

where S is the area shown in Fig. 3.

The left-hand side can be expanded as:

$$\int_v \nabla\cdot(\mathbf{E}_1\times\mathbf{H}_2-\mathbf{E}_2\times\mathbf{H}_1)\,dv$$

$$=\int(\mathbf{H}_2\cdot\nabla\times\mathbf{E}_1-\mathbf{E}_1\cdot\nabla\times\mathbf{H}_2-\mathbf{H}_1\cdot\nabla\times\mathbf{E}_2+\mathbf{E}_2\cdot\nabla\times\mathbf{H}_1)\,dv$$

then, using the Maxwell equations,

$$\nabla\times\mathbf{E}=-\partial B/\partial t \qquad \nabla\times\mathbf{H}=J+\varepsilon\partial\mathbf{E}/\partial t.$$

One finds in the box where J is zero, the only contributory term is the 3rd term. Since $\partial\mathbf{E}_1/\partial t=0$, $\partial\mathbf{H}/\partial t=0$ and $\mathbf{E}_1$ only exists in the coaxial where $\mathbf{H}_2$ is zero for an open circuit. Hence, the left-hand side becomes

$$-\int_v \mathbf{H}_1\frac{\partial B}{\partial t}dv.$$

This derivation also gives the result for a high-frequency reproducing head where the output impedance cannot be neglected. Then another term in the right-hand side cannot be ignored and thus the theorem shall be:

$$\int\left(\mathbf{H}_1\cdot\frac{\partial B}{\partial t}+\varepsilon\mathbf{E}_1\frac{\partial\mathbf{E}_2}{\partial t}\right)dv=V_{0C}-V_1I_2,$$

where V_1 is the applied voltage for unit current and I_2 is the current due to the moving dipole.

The restriction of the theorem is that the potential function of the head is not drastically changed by the tape and the remanent magnetization of the tape is high. If these conditions are not met, then one must calculate a new $\mathbf{H}_1$ and also one must integrate through all space instead of only the region where there is oxide magnetic material.

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