E. Hopner

Phase Reversal Data Transmission System for Switched and Private Telephone Line Applications

Abstract: A phase reversal data transmission system is described, capable of operating at 2000 bauds over private telephone lines and at 1200 bauds over switched networks. The advantages and limitations of the system are discussed from a theoretical and practical standpoint. The clocking problem in data transmission is considered and several approaches are indicated. A summary is given of extensive field tests over switched and private lines in Europe and on private (SAGE) lines in the U. S.

Introduction

An experimental modulation-demodulation scheme for data transmission was developed some time ago by the IBM Advanced Systems Development Laboratories in San Jose[1] for the purpose of exploring the practical value of phase modulation and synchronous detection systems for data transmission over telephone networks. This early work proved the feasibility of a basically simple phase modulation and synchronous detection system which performed as reliably under white noise conditions as was expected from theoretical considerations. The system also performed reliably under the impulse noise conditions characteristic of telephone line operations.

This paper presents a continuation of the above work and summarizes laboratory and field experiments performed in European countries on switched and private lines and on private lines (SAGE) in the United States over the last two years. The phase modulation subset used in the field tests, which obviates some of the shortcomings of the original subset while being markedly simpler in design, is described.

Pilot system

The phase modulation system described in Ref. 1 utilized phase reversal modulation, with the addition of a pilot frequency chosen so as not to interfere with the signal. At the receiver shown in Fig. 1, the subcarrier necessary for synchronous detection was recovered through modulation of the delayed input data by the output information. The timing was obtained by heterodyning the pilot frequency with the recovered subcarrier.

This exploratory system had some inherent limitations. It required a phase adjustment of the timing for different lines. The adjustment could have been made automatically, but the time delay involved, although acceptable on private lines, would have been undesirable in switched network applications. Also, because the pilot frequency used was transmitted at a null of the signal power spectrum, the system was limited to a speed of approximately 1,600 bauds. At that transmission speed, and with a 2,400-cps carrier, the power spectrum null occurs at 800 cps. At 2,000 bauds (with a 2,000-cps carrier) there is no null of the power spectrum in the telephone line pass band.

An advantage of the pilotclocking system, on the other hand, was that the method used for derivation of clocking information does not impose any code or information flow restrictions.

Experimental phase modulation system[2]

The basic modulation and detection system to be described here is actually a simplification of the pilot system, and consists of a total of 25 transistors (for transmitter, receiver and clock). In the pilot system, the transmitted information and the clocking were interlocked, whereas, in the simplified version, the information is derived independently of the clock. We will discuss the information transmission method first; a discussion of the detection method, and the derivation of the clock from the received data, will follow.

- *Transmitter*

The transmitter consists of a free-running multivibrator which can be synchronized with the incoming data signals if so desired. The information carrier frequency is derived from the multivibrator output by filtering and is fed into a phase modulator which reverses the phase of the information carrier, depending on the information to be transmitted. In order to avoid the ambiguity inherent in phase modulation systems, the information is carried in the phase transitions of the information carrier. (A diagram of the basic transmitter is shown in Fig. 2.)

If bit-synchronous transmission (rigid synchronism

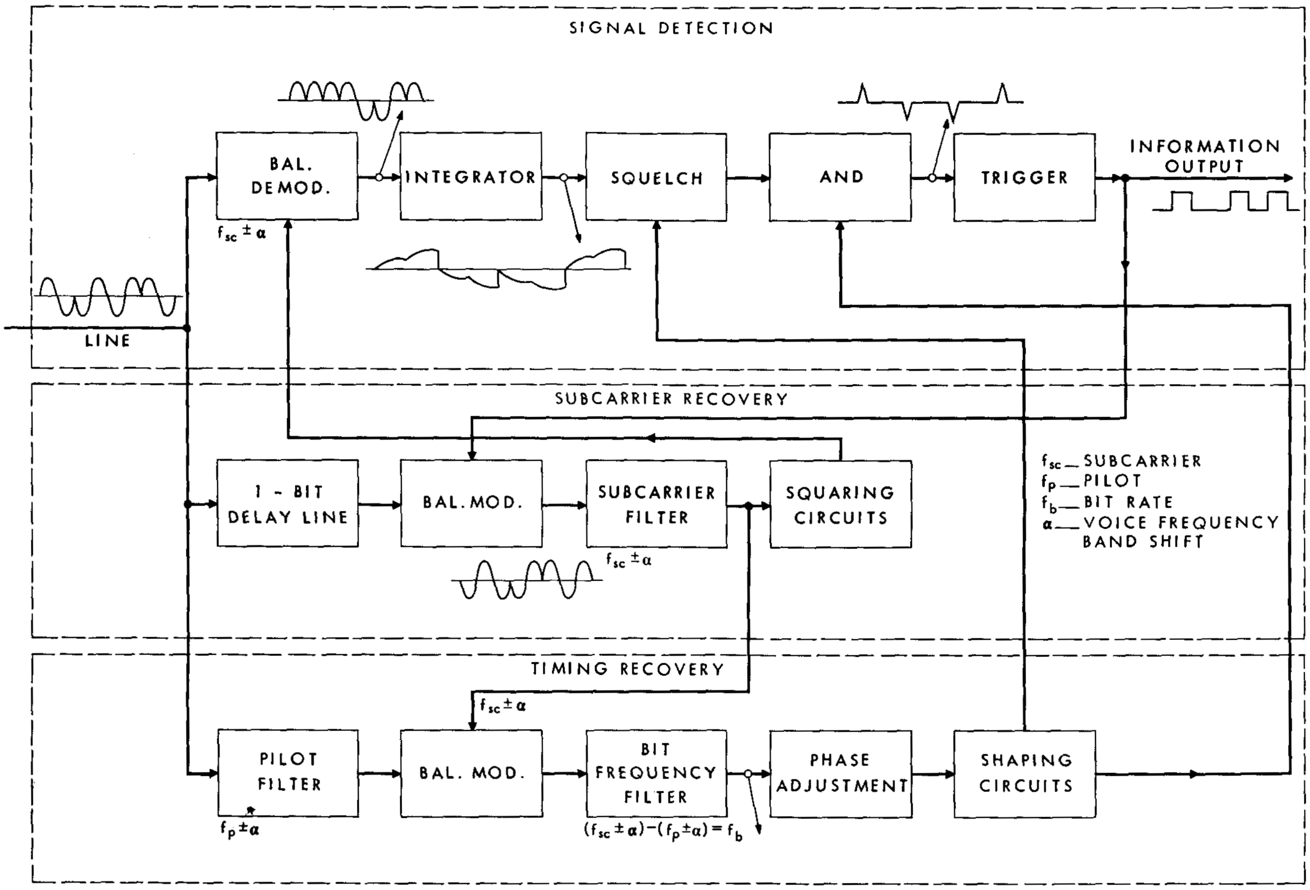


Figure 1 **Pilot system receiver.**

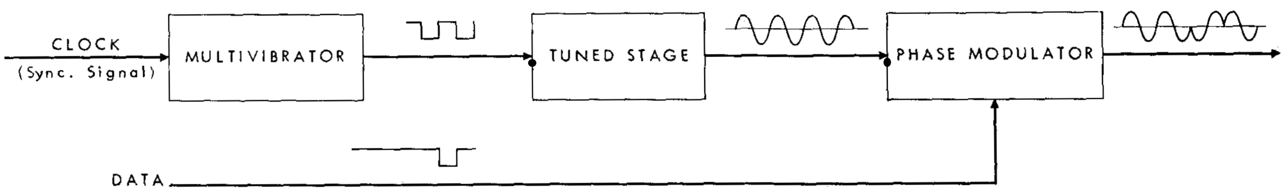


Figure 2 **Diagram of basic transmitter (coding logic not shown).**

between the information and the information carrier) is desired, the transmission speeds can be chosen so that they are submultiples of the carrier frequency. No changes in the transmitter are required to switch from one speed to another. The same modulator can also be used if bit-synchronous transmission is not required. In this case, the modulator will allow start-stop operation (see section on Clocking Problems).

Receiver

The receiver consists of the basic feedback loop of the pilot system, without the clock. (The clock was an inherent part of the pilot system because, with the straight integration and squelching method used, it was required for information recovery. After the synchronous detection process, the available energy was integrated on a bit-by-bit basis. The bit polarity was decided at the end of the bit time, after which the energy contained in the integrator was squelched under the control of the clock.) The energy available after the synchronous detection process is integrated by a low-pass filter as is shown in Fig. 3. The low-pass filter output is then transformed into square waves by wave-shaping circuits. As in the pilot system, the squared output signal is used to modulate the incoming signal on a bit-by-bit basis in a process which is the inverse of the modulation process at the transmitter. In other words, the phase of the incoming signal is reversed, bit by bit, as a function of the received information and the necessary detection carrier is thereby reconstructed. The signal coming from the modulator is then freed from noise and distortion to a great extent by narrow-band filtering so that it is essentially a noise-free detection carrier. The delay line in Fig. 3 is used to compensate for the delay introduced in the feedback loop by the low-pass filter (the delay line of the pilot system had to be of exact bit duration). The low-pass filter has to be optimized for every speed range in order to obtain minimum interbit interference and sensitivity to noise.

• *Clock*

The clock consists of a free-running multivibrator which is synchronized by the incoming data at each transition of the squared output of the demodulator (see Fig. 4). The free-running multivibrator frequency is adjusted at the bit rate and is exceptionally stable in spite of the simplicity of the clock. The multivibrator is synchronized by clamping its output at each data transition for ½ bit time. As soon as the clamp is released, the multivibrator changes state and this transition will provide a sampling point always occurring ½ bit after a data transition. To generate the clamping pulses, two single-transistor single-shot multivibrators are used. The single-transistor type is considered more reliable and less noise-sensitive than the two-transistor type, since it is not astable in nature.

The clock signal thus obtained is passed through a tuned circuit with a dual purpose. The flywheel effect of the tuned circuit very effectively reduces the jitter of the clock, which is introduced by the jitter in the data signal. The data signal jitter is, in turn, caused by noise, frequency spectrum shift of some telephone lines, interbit interference resulting from band-limited operation, etc. Further, without the tuned circuit, a single error occurring at the wrong time could cause the clock to lose or gain one clock pulse and thus upset the character synchronism of the received message. The flywheel effect permits such a thing to happen only if several bits in error occur at the wrong time.

The clock frequency is independent of possible frequency spectrum shifts, and is related only to the timing of the received data signal. The Q of the clock was chosen so that it required approximately one character-time (8 bits) to achieve synchronism with the received data signal.

Figure 3 **Diagram of basic receiver (coding logic not shown).**

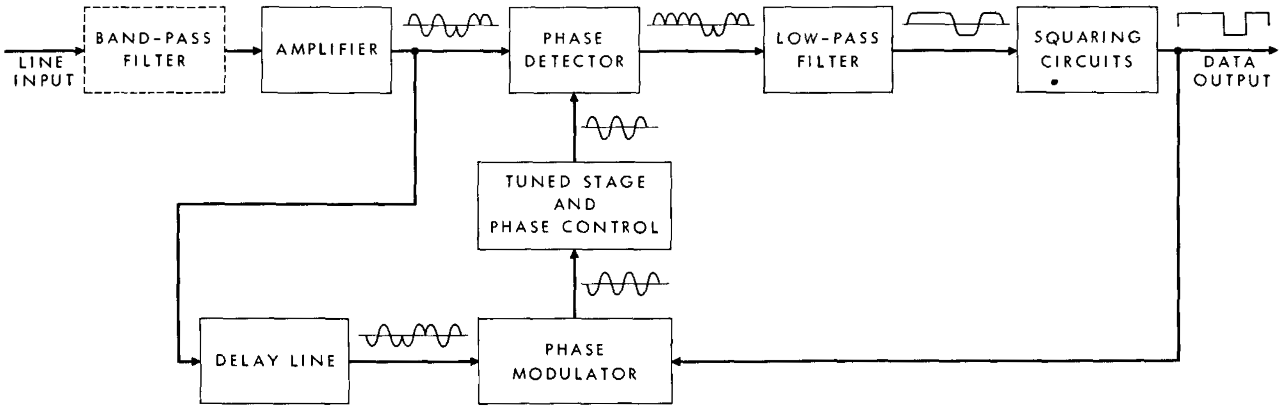
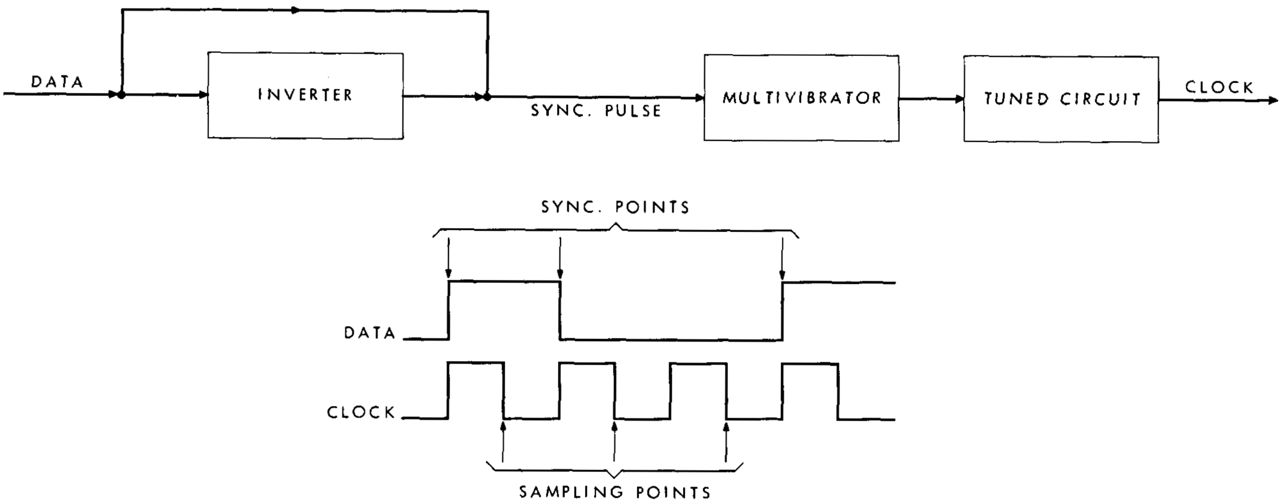


Figure 4 **Clock block diagram and timing.**



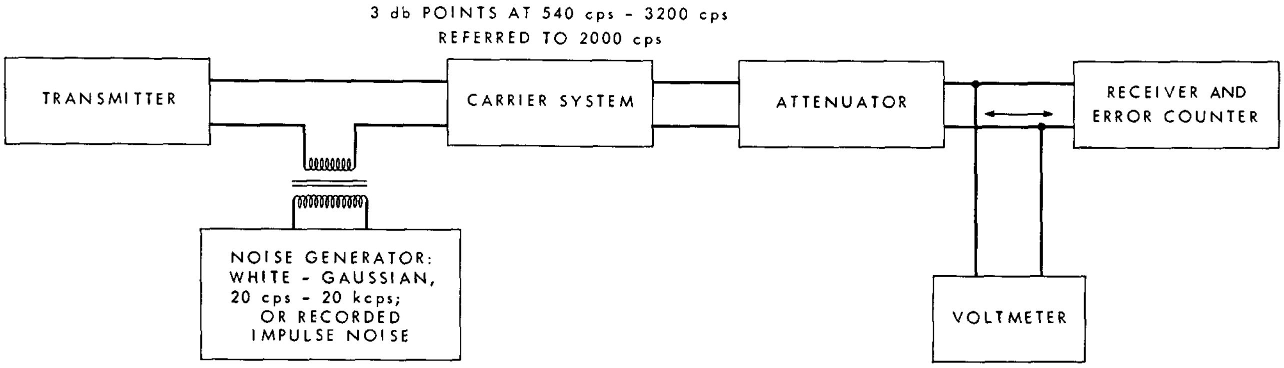


Figure 5 **Experimental configuration for white noise measurements.**

Test results

• *Dynamic range*

The dynamic range of a data transmission system is an important measure of its reliability. The experimental system, which does not have automatic gain control, can tolerate the sudden amplitude changes which occur in the telephone network without errors. A dynamic range of approximately 60 db was measured under laboratory conditions. Under band-limited field conditions this range is reduced to about 40 db.

• *White noise tests*

A good measure of the performance of a data transmission system is its error rate in the presence of white noise under laboratory conditions. This error rate is theoretically predicted in the Appendix as a function of speed, signal-to-noise ratio, and different types of post-detection filters. The theoretical predictions were substantiated by laboratory tests, the results of which can also be found in the Appendix.

For reasons explained in the previous paragraphs, information recovery is performed by a low-pass filter, which theoretically should result in a slightly more noise-sensitive system than if perfect integration and squelching techniques were used (as in the pilot system). The squelching techniques do not cause interbit interference, whereas the low-pass filter does.

A comparison between the test results obtained with the pilot system[3] and with a low-pass post-detection filter optimized for minimum interbit interference at 1200 bauds and 2000 bauds (see Figs. 5 and 6) shows only a small difference in the error performance between the two systems. The small advantage of the straight integration system was offset by the loading effect of the pilot signal.

• *Impulse noise tests*

Extensive testing and statistical evaluation of the impulse noise occurring on private (SAGE) lines in the United States and both switched and private lines in Europe have shown a clear distribution of the number of impulse noise spikes, depending on the noise threshold chosen and the type of line (switched network, private line, etc.). An example of this distribution, resulting from compilation of a ½-hour recording of impulse noise in a particularly noisy line, can be found in Fig. 7.[1] This distribution has been corroborated by statistical data and analysis reported by Pierre Mertz.[4]

The number of impulse noise spikes rapidly increases as their amplitude decreases. From the distribution graph of Fig. 7 it can be seen that the number of impulse noise spikes increases by a factor of 4 per 3 db change in amplitude. An improvement of 3 db in the impulse noise performance of a system would therefore result in a 4-to-1 improvement in error rates.

White noise tests are a good indication of the susceptibility of a system to noise, but since impulse noise and random noise are so radically different, comparative tests with impulse noise were also performed using the pilot and pilotless systems. Under laboratory noise test conditions essentially no difference in the performance of the two systems was found (see Ref. 3 and Fig. 6).

• *Tests over switched networks and private lines*

Basically, the same telephone network is used for both the switched network and private line applications. The difference is that, in a switched network connection, one has no control over the line quality. In order to establish an efficient data transmission service, the equipment must be adapted to the most probable line characteristics. But, to guarantee service under any conditions, the equipment must be adapted to the worst link in the chain. Therefore, two speeds seem to be indicated for switched network connections.

A switched network connection is established by automatic means, and therefore the connection goes through automatic exchanges, which are the main source of impulse noise—one of the most important factors affecting the reliability of data transmission. A private-line link is usually planned well ahead of time so that one has control over many important properties of the link. In addition, a private line usually does not go through the mechanics of an automatic exchange and is therefore, to

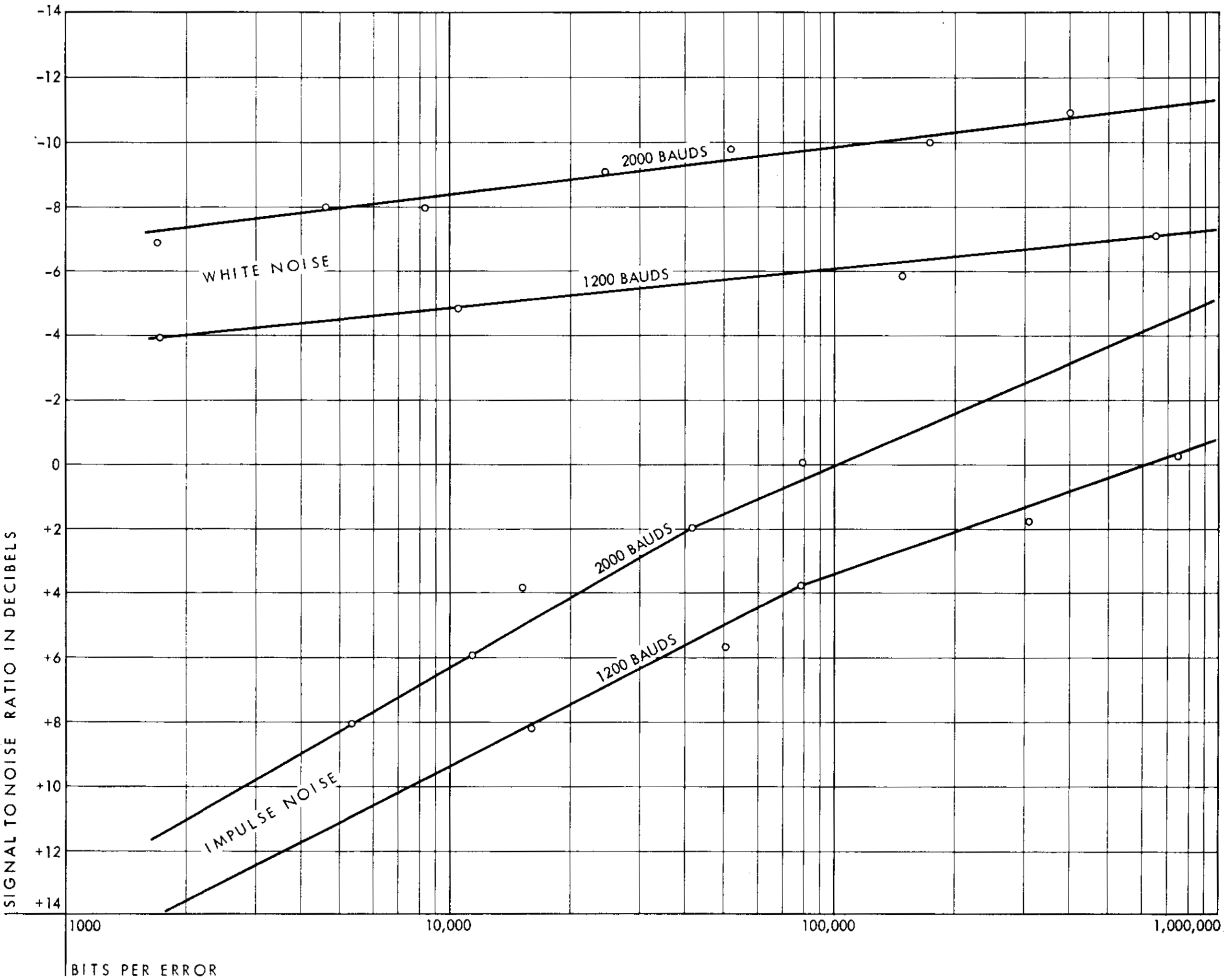


Figure 6 **Error rates of experimental data set as function of impulse and white noise levels.**

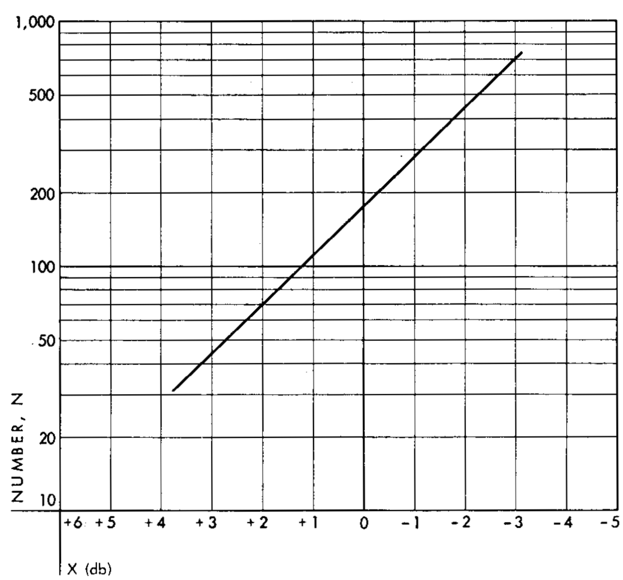
begin with, several orders of magnitude quieter than a switched line.

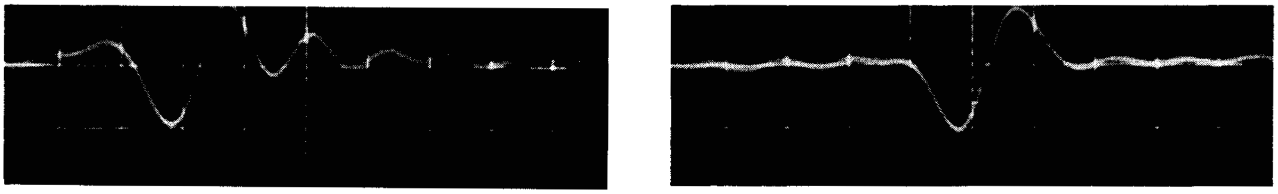
In the following paragraph, the experience acquired with the experimental system over both private and toll facilities is summarized.

Extensive and systematic tests performed over the switched networks of many European countries (the results of which have been partially reported in a paper presented at the NEC),[5] show that a transmission speed of 1,200 bauds can be achieved over a great majority of switched network lines, with no amplitude or phase equalization. However, in every country, at least one type of line could be found which required a reduction of speed to 750 or 600 bauds.

A tape recording of the line signal was made during each test. It was found that, in many cases, the error rate could be drastically reduced if the tapes were played back through a phase-equalizing network. The equalization used was the same for all tapes. This interesting fact leads to the conclusion that a compromise amplitude and phase equalization could substantially improve

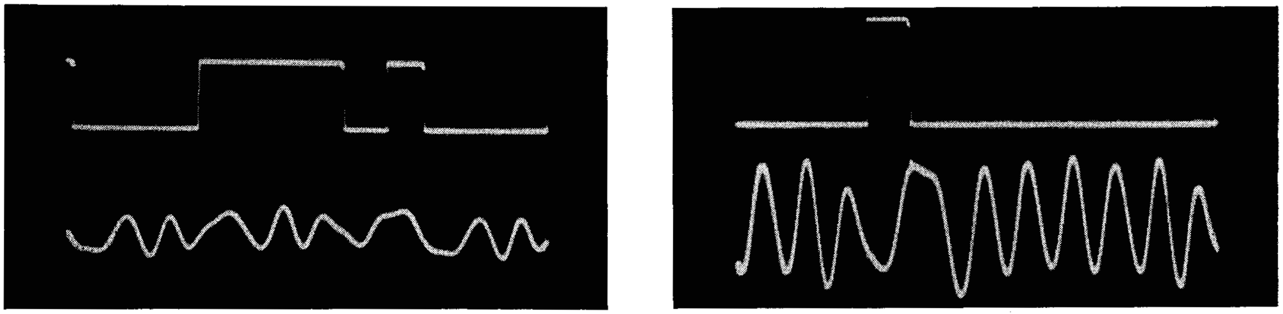
Figure 7 **Impulse amplitude distribution.**





Before phase equalization *After phase equalization*

Figure 8 **A 2400-baud dipulse received at the end of a private line. Scale: 200 μsec per division.**



Arbitrary pattern *Single bit*

Figure 9 **Signal input and binary data output from SAGE synchronous line at 2400 baud.**

the reliability of switched network transmission, and reduce the number of lines not suited at present for 1,200 baud transmission. A compromise amplitude equalization with the addition of a receiving band-pass filter, arranged as in Fig. 3, would further improve system operation.

A search was made for the noisiest lines in different European countries and data transmission tests were conducted over these lines at 1,000 and 1,200 bauds, using the pilotless system on a dial-up basis. Error statistics have shown that even with these artificially created worst conditions, less than 2 percent of information blocks, on the average, would have been in error, assuming a message length of approximately 500 bits.

Single-sideband carrier systems are usually the backbone of national and international telephone networks. A series of experiments has been performed using carrier frequency telephone links, complying with the general characteristics required by CCITT[6] recommendations, and excluding the influence of local lines and loaded cables. It was found that the experimental system worked very well at 2,000 bauds (using one cycle of the carrier per bit of information), without phase equalization. The transmitted carrier had to be synchronous with the input information for proper operation of the experimental subset. This was in contrast to the situation on the switched network at lower speeds, where one-and-a-half or two cycles per bit were used. In these tests, a lack of synchronization of the carrier with the data increased the jitter in the output slightly, but did not disable the system.

A series of experiments was performed with a great variety of private lines including loaded cables and many carrier links in tandem. It was found that successful 2,000 baud data transmission links could be established if phase equalization was used. The same phase equalization was required for most of the links.

If the line in question was synchronous in nature, that is, if the commonly found frequency spectrum shift did not exist, successful operation could be established up to a speed of 2,400 bauds. More elaborate phase equalization was required for 2,400 baud operation; its effect is shown in Fig. 8. Figure 9 shows the received signal and reconstructed binary data taken at a speed of 2,400 bauds over a standard SAGE equalized synchronous line. Had the line not been synchronous, 2,400-baud operation would not have been possible for reasons explained below.

The reliability, average error rate and efficiency of transmission was sometimes several orders of magnitude better for the private line tests than for the switched networks.

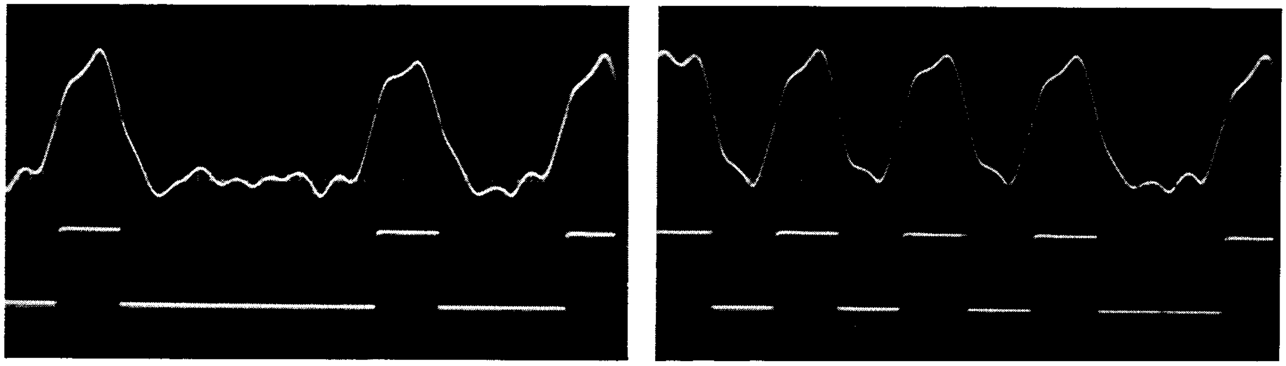
- *Radio tests*

High-frequency radio links are still the mainstay of many communications systems, both long and short range. It was therefore of interest to explore the performance of the phase reversal system for data transmission over existing radio links. The extremely good dynamic range and relative immunity to noise of the system are important features for radio communications.

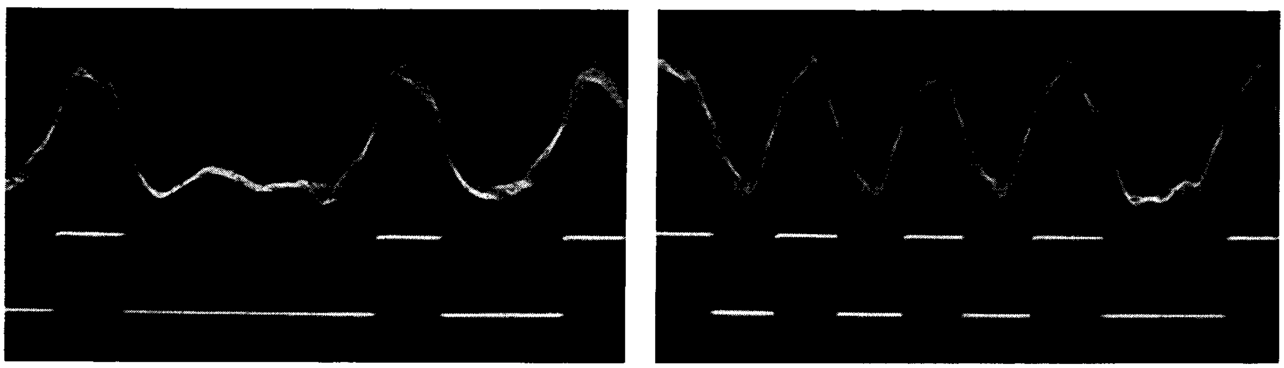
Extensive long-range (3,000 miles and more), high-speed transmission tests (performed at 1,000 and 2,000 bauds) have conclusively shown that the phase reversal system designed for telephone line applications is not workable over such distances and speeds in its present form. The main reason is the severe delay distortion introduced by the multi-path propagation mode. However, extensive experiments over a shorter distance—the IBM San Jose-Tucson 780-mile link, operating in the 6, 11 and 17 megacycle range, under the authority of the Federal Communications Commission—have shown promise of successful operation at 1,000 to 2,000 bauds. From these experiences, we can tentatively conclude that the experimental phase reversal system would successfully operate over existing high-frequency radio facilities at distances up to approximately 1,000 miles at the above speeds.

Figure 10 **Received 2000-baud vestigial sideband signals after low-pass filtering and squaring. Scale: 1 bit per division.**

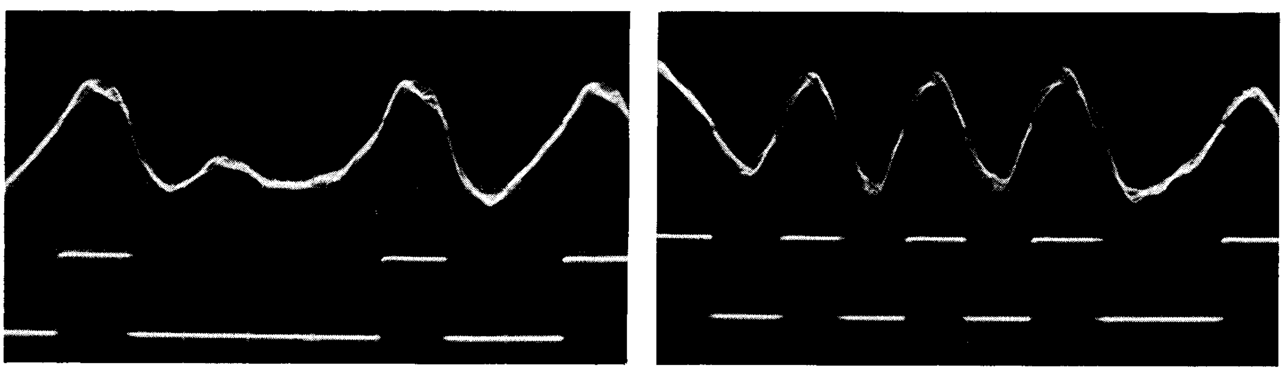
a) Two sections of band-pass filter.



b) Two sections of band-pass filter, loaded line and single sideband carrier system with 5 cps frequency shift.



c) Two sections of band-pass filter, loaded line and single sideband carrier system with 5 cps frequency shift with the addition of an L-type carrier line.

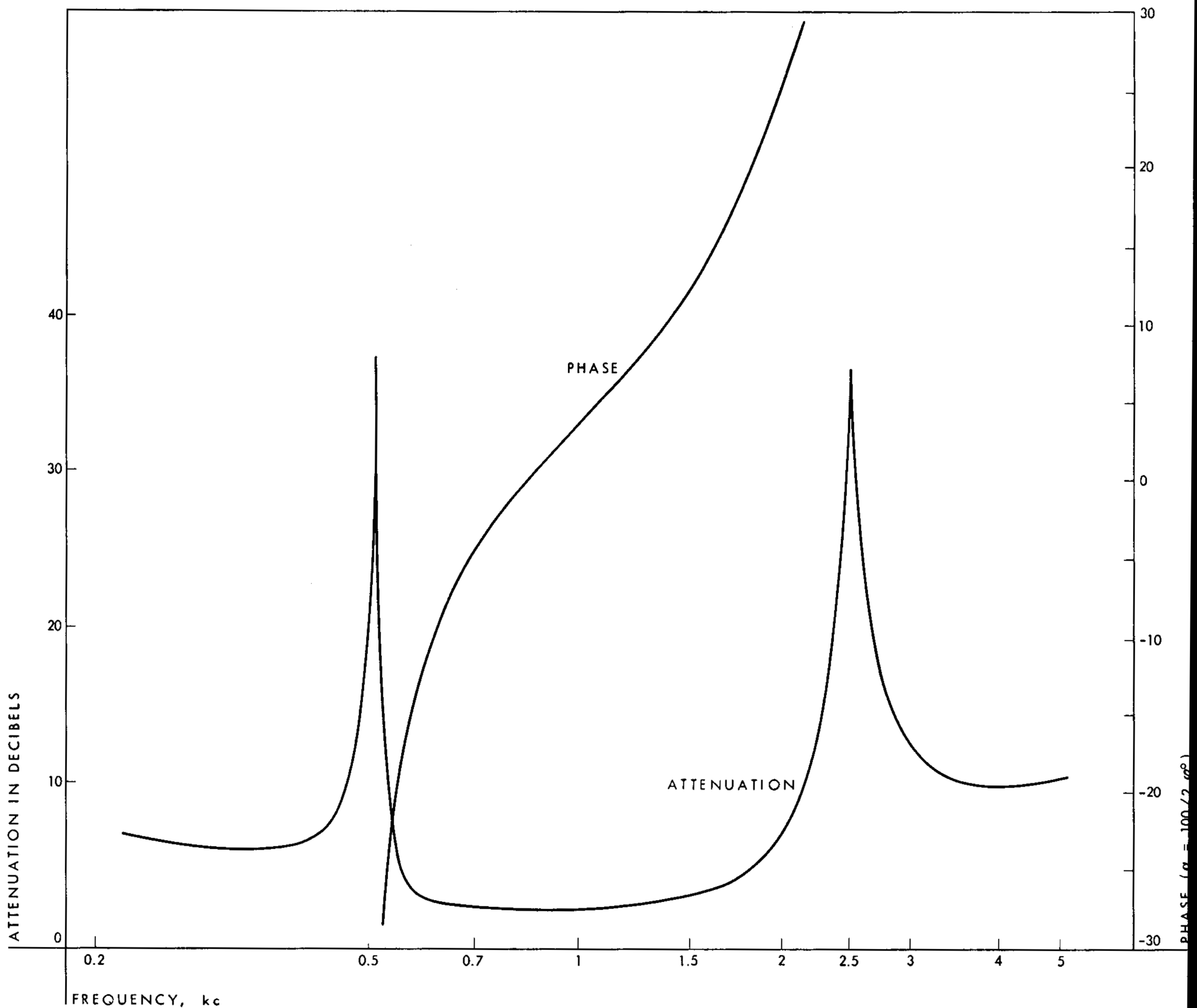


Speed limitations

It is theoretically permissible to limit the transmission band sharply up to and including the information carrier frequency, as long as the phase characteristic of such a transmission channel remains linear. One should therefore be able to transmit up to 3,000 bauds over a private line with proper phase equalization and using one cycle of the information carrier per bit.

A study of line characteristics has shown that considerable improvement in the performance of the 2,000-baud phase reversal system could be achieved if the frequency spectrum above the information carrier frequency was eliminated with a properly designed vestigial sideband filter, and the information thus confined to the good portion of the available channel. Computer simulation of loaded line characteristics and the effect of vestigial sideband filtering have shown encouraging results. Previous experience indicated that, for loaded lines, phase equalization was essential for successful operation even at 1,600 bauds. The computer simulation now indicates that 2,000-baud service could be established over loaded lines by filtering without phase equalization. Subsequent experiments were performed with such lines; successful operation was achieved at 2,000 bauds with no phase equalization. Practical and theoretical considerations have further shown that a 2,000-baud signal can be confined to a band from 500 to 2,500 cps. Figure 10a shows two data signal patterns after band-pass filtering, wave shaping, and squaring, using a band-pass filter having the frequency response shown in Fig. 11 at the transmitter and receiver. The data signal, as it appears after going through the band-pass filters, the loaded cable, and a single sideband car-

Figure 11 **Band-pass filter attenuation and phase.**



rier system, with a five-cycle frequency band shift, is shown in Fig. 10b. After the addition of an L carrier system in tandem to the above line, the signal shown in Fig. 10c resulted. The phase reversal system used for the experiments performed on lines having no frequency spectrum shift is shown in Figs. 2 and 3. As soon as frequency spectrum shift was introduced, the system became inoperable. In order to perform the experiments resulting in the patterns shown in Figs. 10b and 10c, the detection carrier had to be derived independently of the information band transmitted, for reasons explained in the following paragraphs.

Although it has been theoretically predicted and experimentally proven that a vestigial sideband operation of the phase reversal system is practical and would allow transmission speeds up to 3,000 bauds over synchronous telephone lines, a vestigial sideband system becomes inoperable over asynchronous lines if the detection carrier is not supplied by an external source. This fact can be easily explained if we examine the case of a line signal consisting of consecutive phase reversals as shown in Fig. 12. The frequency spectrum of such a signal would obviously not contain the carrier frequency, because that frequency is suppressed by the modulation process; therefore, the transmitted spectrum falling within the frequency characteristic of the telephone line would consist essentially of two frequencies, $f_c+f_{b/2}$ and $f_c-f_{b/2}$, where f_c=carrier frequency, f_b=bit rate frequency (see Fig. 13). In passing such a signal through a vestigial sideband filter, we would eliminate the upper frequency and limit our transmission signal to the single frequency, $f_c-f_{b/2}$. In going through a shifted carrier system, the single frequency would be shifted by $\pm\alpha$. There is no way to reconstruct, from this information alone, either the timing or the detection carrier frequency ($f_c\pm\alpha$) at the receiving end. In contrast, in a synchronous line, the phase reversal system can reconstruct both the detection carrier f_c and the timing from the $f_c-f_{b/2}$ frequency with no difficulty, because they are in a fixed relationship unimpaired by frequency shift.

At 2,000 bauds, f_c=2,000 cps, and $f_c+f_{b/2}$ and $f_c-f_{b/2}$ are 3,000 and 1,000 cycles respectively. In the case of frequency shift, these two frequencies, which are essential for the reconstruction of the detection carrier, can be transmitted over good private lines. At 2,400 bauds the upper frequency essential for asynchronous operation is 3,600 cps. Such a high frequency can normally not be transmitted over regular telephone lines.

Operation of the phase reversal system at 2,000 bauds is feasible and has been successfully demonstrated, but a speed of 2,400 bauds is feasible only if the telephone lines are synchronous. If asynchronous operation at 2,400 bauds is desirable with a phase reversal system, the detection carrier has to be derived from sources outside the information band.

From the above consideration and more general studies of the subject matter, one can conclude that a simple and reliable phase modulation system, having a top speed of about 2,000 bauds, can be built for data transmission over telephone lines. If higher speeds (such as 2,400 bauds and above) are required, new techniques would have to be developed that depart from some basic principles used in the experimental system described here. Multiphase and multifrequency systems with phase equalization, as well as amplitude modulation systems, have been successfully operated at 2,400 bauds and faster. A vestigial sideband phase reversal system is also feasible.

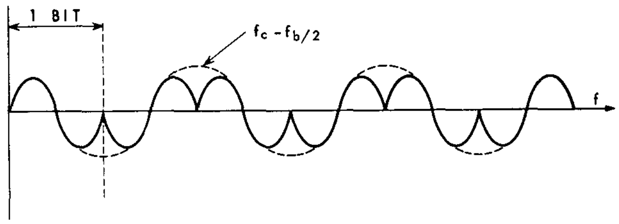


Figure 12 **Phase reversal pattern.**
(f_c=carrier frequency; f_b=bit rate frequency.)

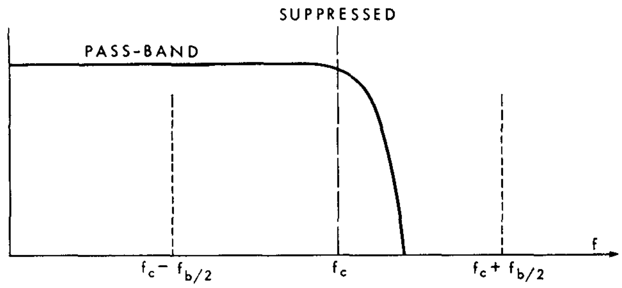


Figure 13 **Frequency spectra of phase reversal pattern and of a vestigial sideband filter.**
(f_c=carrier frequency; f_b=bit rate frequency.)

Clocking problems

This discussion will be, in general, limited to bit timing problems. Character synchronism is a subject by itself and will not be dealt with thoroughly here.

The design of a clock depends very much on the data transmission system requirements and cannot be considered in connection with the modulation system only. The choice of the clocking system used in our experimental data set was based upon the following assumptions and considerations.

1. It was assumed that a transition in the information flow can be guaranteed to occur every few characters.

2. It is desirable to achieve bit synchronism of the clock within a character time after the start of transmission.

3. The clock should be as simple as possible, while meeting other requirements. The clock chosen required only six transistors.

4. There is no need for the clock to be very much more error-free than the data, for the following reasons:

a) If the information is transmitted in blocks with error checking, and the correction is performed by retransmission, a single bit in error would require retransmission of the whole block; therefore, even if the clock rigidity is such that even a single bit error will cause loss of synchronization, there is no loss of reliability in the system.
b) If error correction by logical means is used, the clock should be able to tolerate more errors before it is disturbed than the burst correction system can correct automatically.
c) Among other advantages, a flexible timing source is indispensable for information recovery of magnetically recorded data, since it can follow the flutter and slow variations in tape speeds.

In the gathering of statistical information on error distribution, however, it is important to have a very rigid clock if it is desired to record only the data errors, unaffected by loss of clock synchronism.

The ability of the clock to sample correctly a long record with no transition depends upon the stability of its multivibrator. Although the stability of the clock chosen is sufficient to maintain synchronism within a character or two, special stabilization has to be incorporated to make it operable over larger blocks of information, without a synchronizing signal. The clock stabilizer shown in Fig. 14 can make the multivibrator as stable as any possible external reference. The multivibrator output is fed to a stable tuned amplifier taken as a reference. The output of that amplifier, shifted by 90 degrees, and squared to make the error signal independent of amplitude, is compared with the multivibrator output in a phase detector. A drift in multivibrator frequency will result in a phase shift and cause either a positive or a negative error signal at the phase detector output. This signal is fed back to one multivibrator base. The ability of the clock to achieve bit synchronism with the first transition remains unchanged. The Q of the clock tuned circuit will determine its ability to retain synchronism through noise or interruptions.

• *Start-stop operations*

The clock mechanism shown in Fig. 4 can be very easily modified for start-stop operation, where timing is determined by a transition occurring at the beginning of each character, and where the interval between characters is variable. The tuned stage provided for synchronous operation can be left out of the circuit, resulting in a three-transistor start-stop clock, able to lock in on the start transition.

Conclusions and acknowledgements

The purpose of our investigation was to explore the possibilities of a simple and reliable phase reversal system for data transmission applications. From the extensive field and laboratory tests one can conclude that such a simple system can be efficiently used for reliable data transmission over switched networks and private lines, as well as for relatively short-range, high-frequency radio circuits.

The usefulness of the phase-reversal system for data transmission services could be fully explored only because of the strong support of government communications laboratories and agencies in Switzerland, Holland, France, England, Germany, Sweden, and Norway. The support in testing of SAGE lines given by the Lincoln Laboratories of MIT is also greatly appreciated. The IBM effort had the additional support of the IBM World Trade Corporation.

The phase-reversal system and test equipment were developed by C. M. Melas, and built and tested by O. F. Meyer, in the San Jose IBM Laboratories. The theoretical and experimental error probability studies reported in the Appendix were conducted by S. E. Estes, whose effort is greatly appreciated.

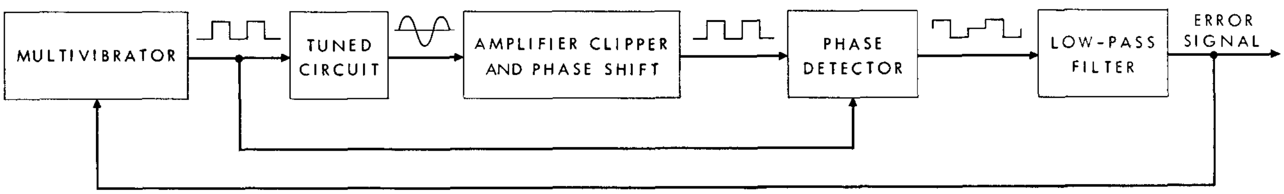
The band-pass filter design for vestigial sideband operation and the extensive experimental work related to it were performed by Werner Hofmann of the IBM Product Development Laboratories in Sindelfingen, Germany.

The computer simulation was performed by T. C. Kelly of the San Jose IBM Computation Laboratory.

Appendix: Theoretical and experimental error probability of a phase reversal system in the presence of random noise

Analytical and experimental studies were made of the average error rate of the data set as a function of received signal-to-noise ratio for additive white Gaussian noise. In the analytical study it was assumed that the received signal was undistorted, and the receiver clock and reference subcarrier, $r(t)$, were noise-free (since the clock and reference subcarrier are outputs of narrow-

Figure 14 **Multivibrator stabilizer block diagram.**



band tuned circuits, they have a substantially higher signal-to-noise ratio than the received signal, and this approximation is reasonable). The model of the receiver then takes the form shown in Fig. 15.

Two types of post-detection filtering were used: a simple low-pass filter with 12 db/octave cutoff on the present set, and periodic integration on the pilot system. Sampling was at the peak signal value in the low-pass system and at the end of the bit interval in the periodic integrator system. The average error rate may be calculated for both systems by obtaining the amplitude distribution of both signal and noise samples at the receiver output. The received signal, $s(t)$, is

$$s(t) = \pm A \sin \omega_0 t, \tag{A-1}$$

over a bit interval, τ, of two or more half-periods of $\sin \omega_0 t$; the noise, $n(t)$, is considered to have a flat spectrum between some frequency limits ω_l and ω_h with an amplitude distribution:

$$p(n) = \frac{1}{\sqrt{2\pi}\sigma} \exp -\left(\frac{n^2}{2\sigma^2}\right), \tag{A-2}$$

where σ^2 is the received noise power.

The received signal-to-noise ratio is then:

$$\frac{S}{N} = \frac{A^2}{2\sigma^2}, \tag{A-3}$$

and the input to the post detection filter is

$$s_1(t) + n_1(t) = [s(t) + n(t)]r(t). \tag{A-4}$$

In evaluating the response of the post-detection filter to an input (A-4) only the $+$ sign on $s(t)$ in (A-1) need be considered since the noise has zero mean and the system is symmetrical.

The sampler output for the periodic integrator system is

$$s_0(\tau) + n_0(\tau) = \frac{1}{\tau} \int_0^\tau [s(t) + n(t)]r(t)\,dt. \tag{A-5}$$

Equation (A-5) may be considered a convolution integral, $[s(t) + n(t)]*h(t)$, where

$$h(t) = \frac{1}{\tau} r(\tau - t) \qquad 0 \leqq t \leqq \tau \tag{A-6}$$

$$= 0 \qquad \text{elsewhere.}$$

Thus the signal output sample for the periodic integrator system is

$$s_0(\tau) = \frac{A}{\tau} \int_0^\tau |\sin \omega_0 t|\,dt, \tag{A-7}$$

and the noise sample distribution can be calculated from the power output, σ^2, of the linear system described by its impulse response, $h(t)$, in (A-6). The probability of error is then

$$P_{PI}(E) = \frac{1}{\sqrt{\pi}} \int_{[\sigma S_0(\tau)/\sigma_0 A](S/N)^{\frac{1}{2}}}^{\infty} e^{-x^2}\,dx. \tag{A-8}$$

The signal output sample for the low-pass system may be determined by finding the peak value of the response of the low-pass filter to a series of pulses, A $|\sin \omega_0 t|$ for $0 \leqq t \leqq \tau$. However, the low-pass filter has interbit interference at the sampling point which must also be calculated to give an amplitude distribution for the signal sample. The filter has a transfer function $H(\omega)$:

$$H(\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta \frac{j\omega}{\omega_n}}, \tag{A-9}$$

and the peak signal value and interference terms become a function of ζ (the damping factor) and ω_n (the cutoff frequency).

In general when random noise, $n(t)$, is multiplied by a periodic function, $r(t)$, the result is nonstationary, but the resulting wave form may be considered as the sum of a low-frequency part and a high-frequency part. If the high-frequency part is attenuated as it is by the low-pass filter, the remaining low-frequency part is stationary and Gaussian. The response of the filter to the remaining low-frequency Gaussian noise may be found to determine the noise output sample amplitude distribution.

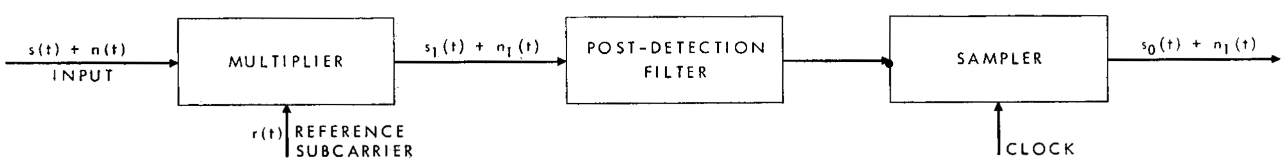
The probability of error for the low-pass system may be then calculated as:

$$P_{LP}(E) = \frac{1}{\sqrt{\pi}} \sum_i Q_i \int_{[(S^* - I_i)\sigma/A\sigma_0](S/N)^{\frac{1}{2}}}^{\infty} e^{-x^2}\,dx,$$

where I_i is the interference of a particular bit pattern, Q_i is the probability of occurrence of the pattern, σ_0^2 is the output noise power and S^* is the peak signal output sample. For the values of ζ and ω_n used, only the interference of the next previous bit was of any consequence and:

$$P_{LP}(E) = \frac{1}{\sqrt{\pi}} \left\{ Q \int_{[S^* - I)\sigma/A\sigma_0](S/N)^{\frac{1}{2}}}^{\infty} e^{-x^2}\,dx + (1 - Q) \cdot \int_{[S^* + I)\sigma/A\sigma_0](S/N)^{\frac{1}{2}}}^{\infty} e^{-x^2}\,dx \right\}. \tag{A-10}$$

Figure 15 **Analytical model of the data receiver.**



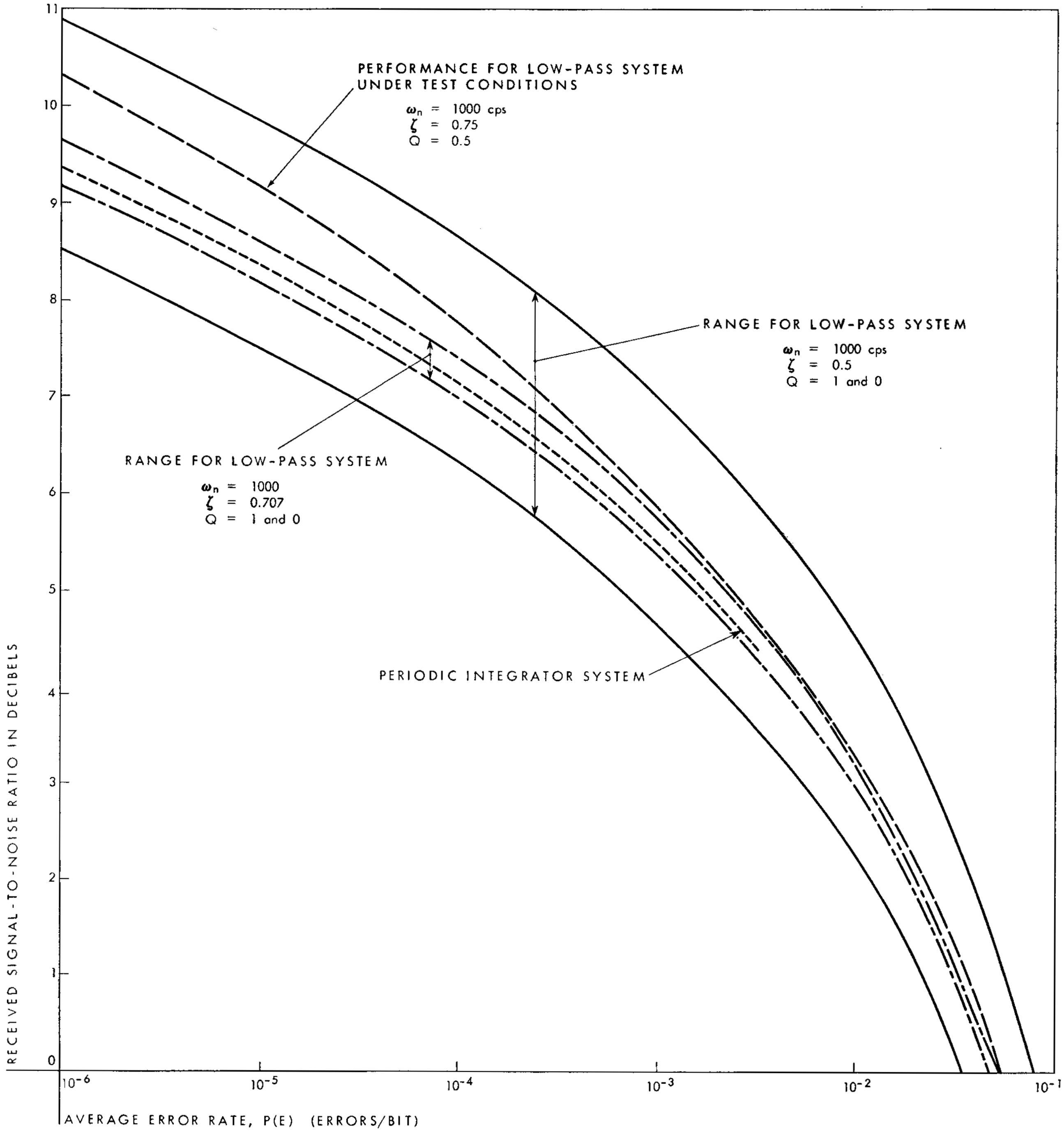


Figure 16 **Comparison of theoretical performance at 2000 bauds for different post-detection filters.**

In (A-10), Q is the probability of a phase reversal of the subcarrier at the next previous bit time and I is the interference of the next previous bit at the present sample time.

The theoretical probability of error for the periodic integration system is compared in Fig. 16 to the theoretical probability of error for the low-pass system at 2,000 bauds for $\zeta=0.5$ and 0.707 with $\omega_n=1{,}000$ cps. The low-pass system has a range of probability of error due to the interbit interference contribution of the low-pass filter which depends on the bit pattern. That range may be reduced by appropriate choice of ζ, as shown in Fig. 16 when ζ is changed from 0.5 to 0.707. Even $\zeta=0.707$ is not necessarily optimum. The periodic integration system enjoys only a small advantage over the low-pass system when the low-pass filter parameters are selected for minimum interbit interference. For no interbit interference and for no improvement in signal-to-noise ratio by post-detection filtering (A-9) and (A-10) are identical to the results derived by Cahn.[7]

The experimental performance which was determined for a repeated eight-bit character is compared to the theoretical performance in Fig. 17. The test configuration is shown in Fig. 5. The error detector compares the present character to the previous character on a bit-by-bit basis, giving an error count when the analogous bits in adjacent characters are different. The probability of error is

$$P(E)=\frac{1}{2}-\sqrt{\frac{1}{4}-\frac{N}{2n}}, \tag{A-11}$$

which is approximately $\frac{N}{2n}$ if N (the number of errors counted) is small compared to n (the total number of bits). Several experiments were run at each signal-to-noise ratio to obtain a cluster of experimental points. Since the value of $\zeta=0.5$ used in the experiment causes considerable interbit interference, the performance is a function of the bit structure. The particular eight-bit character for which the data are plotted has 6 out of 8 possible phase reversals, or $Q=0.75$. The location of the particular theoretical curve, $Q=0.75$, within the theoretical range may be seen in Fig. 16 for 2,000 bauds.

References and notes

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2. An earlier phase reversal data transmission system, recovering the detection subcarrier in a feedback loop as a function of the received information (described in Japanese Patent No. 164788, dated 7/11/44, granted to the Manchuria Telegraph and Telephone Company), came to the attention of the author in 1960.
3. See Fig. 13, p. 84, Ref. 1.
4. Pierre Mertz, "Model of Impulsive Noise for Data Transmission," *1960 IRE International Convention Record,* Part 5, p. 249.
5. C. M. Melas and E. Hopner, "Error Recording and Analysis for Data Transmission," *Proceedings of the National Electronics Conference,* **16,** 54, (October 1960).
6. Comité Consultatif International Télégraphique et Téléphonique of the International Telecommunications Union.
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Figure 17 **Comparison of theoretical and experimental performance for low-pass filtering.**