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Acoustic-Mode Mobilities for "Split p-Silicon"

It has recently been demonstrated[1] by cyclotron resonance that, as predicted,[2,3,4] the degenerate valence-band edge of a silicon crystal is split by shear strains into two nondegenerate band edges at different energies, with ellipsoidal energy surfaces[5] in the neighborhood of each. For temperatures with kT small compared to the splitting, there is thus made available, for each type of strain, a "new half-semiconductor" of almost the simplest type (but with tetragonal, rather than cubic, symmetry) with corresponding new acceptor ionization energy (when the latter is small compared to the splitting) and transport constants. The mobilities, so far as they are determined by acoustic-mode phonon scattering, will be governed by a new set of deformation potentials, distinct for each type of strain.

In this note, the deformation potentials of the thermal (upper) band of the split band edge are obtained in terms of the fundamental elements, for the valence-band system, of the deformation potential operators.[6] The principal values τ_i of the relaxation-time tensor[7,8] may be calculated, for the approximation that they depend on the energy ε only,[8,9] as in Herring and Vogt's paper (Reference 8, to be denoted hereafter by "HV") by making certain approximations. In this note, HV's mathematical results are used to obtain (by substituting the new effective mass constants, given by experiment[1]) the coefficients of the expressions for the $1/\tau_i$ as bilinear functions of the fundamental elements of the deformation potential operators. The various linear transport constants are all given, within the approximations involved, by well known formulas[8] in terms of the τ_i. (The actual mobility, for example, is determined by optical-mode and impurity scattering as well as acoustic-mode scattering. For impurity scattering to be negligible one may expect to need higher temperatures than that at which cyclotron resonance was observed, and therefore larger strains for the splitting to remain large compared to thermal energies.)

The fundamental elements of the deformation potential operators are

$$(v_r|\mathfrak{D}^{st}|v_u)\equiv D^{st}_{ru}\,,$$

where[6] $\mathfrak{D}^{st}$ (Pikus and Bir's D^{st}) is the tensor component for unit axes in crystal [1, 0, 0] directions and the v_r are the basis functions, of the valence-band center, transforming like x, y, and z. For convenience we give in Table 1 the correspondence of the various notations. In the

Table 1

Pikus and Bir[10]		Adams[2]	Tiersten[11]	Matrix elements
$a+2b$	l	E_{11}	F	D^{11}_{11}
$a-b$	m	E_{12}	G	D^{11}_{22}
$\sqrt{3}\,d$	n	E_{44}	$2J$	$D^{12}_{12}+D^{21}_{12}$

notation of Pikus and Bir,[3,10] the energy of the upper band edge is[12]

$$E_0=\text{const.}+a\Delta+\sqrt{E_\varepsilon}\,. \qquad (1)$$

Differentiation of this homogeneous linear function of the strains gives the $\Xi_1,\ldots..\Xi_6$ defined by HV and hence (see their Table II) the constants Ξ_d, Ξ_u defined by Herring,[13] for the types of "valley" which we consider here. These results are tabulated in Table 2 below, for a uniaxial stress along the 1, 0, 0 axis ("1, 0, 0 case") and for a uniaxial stress along the 1, 1, 1 axis ("1, 1, 1 case"). For both cases the energy surfaces are ellipsoids of rotation and so the valleys have two distinct mass constants, $m^*_\parallel$ and $m^*_\perp$, and similarly two HV relaxation times, $\tau_\parallel$ and $\tau_\perp$.

Table 2

	1, 0, 0 case	1, 1, 1 case
Ξ_d	$a+p\cdot\frac{1}{2}\lvert b\rvert$	$a+p\cdot\lvert d\rvert/\sqrt{3}$
Ξ_u	$-p\cdot\frac{1}{2}\lvert b\rvert$	$-p\cdot\sqrt{3}\,\lvert d\rvert$

For a compression, $p=+1$; for a tension, $p=-1$.

HV's results for $\tau_\parallel$ and $\tau_\perp$, (HV49) and (HV50), may be expressed

$$1/\tau=(6kT\varepsilon^{1/2}/hc_l)\,(m_s/2\hbar^2)^{3/2}\,D^2.$$

Here m_s is the density-of-states mass, c_l is given by (HV51), τ stands for $\tau_\parallel$ or $\tau_\perp$, D for $D_\parallel$ or $D_\perp$, and D^2 is

the final factor [ ] in (HV49) or (HV50).[14] From the results of Table 2 here, we have

$$D^2=\xi a^2-(3\eta-2\xi)ag+(\xi-3\eta+9\zeta)g^2\,, \qquad (2)$$

where

$$\left.\begin{aligned} g&=p\cdot\tfrac{1}{2}|b|; && 1,0,0 \text{ case}\\ g&=p\cdot|d|/\sqrt{3}; && 1,1,1 \text{ case}\end{aligned}\right\}. \qquad (3)$$

Here ξ, η, ζ stand for $\xi_\parallel$ or $\xi_\perp$, et cetera, and the latter are given by HV in their Table VI. The coefficients in (2) have been computed for the 1, 0, 0 and 1, 1, 1 cases, using the published[1] cyclotron-resonance masses (that is, for the *compression* situations), from HV's Tables VI and VII and their Figures 2 and 3 and from the elastic constants quoted by them for silicon.[15] The results are collected in Table 3 below.

Table 3

Case	$\frac{m^*_\parallel}{m^*_\perp}$		ξ	$3\eta-2\xi$	$\xi-3\eta+9\zeta$
1, 0, 0	0.742	$\parallel$	1.32	1.83	9.18
		$\perp$	1.34	−1.37	4.36
1, 1, 1	0.319	$\parallel$	1.32	0.564	7.90
		$\perp$	1.36	−1.92	2.71

The evaluation of the scattering times $\tau_\parallel$, $\tau_\perp$, and hence the transport constants, of course requires knowledge of the values of the deformation potential elements in Table 1. It seems possible that order-of-magnitude values may be obtained from the dependence of mobilities on strain as the latter increases from zero to the magnitudes at which the limiting "split p" state is reached.

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References and footnotes

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2. E. N. Adams, Chicago Midway Laboratories Report CML-TN-P8 (1954).
3. G. E. Pikus and G. L. Bir, *Soviet Physics—Solid State* **1,** 136 (1959), and **1,** 1502 (1960).
4. See also W. H. Kleiner and L. M. Roth, *Phys. Rev. Letters* **2,** 334 (1959).
5. The effective mass constants are calculated by Pikus and Bir (Reference 3), who were evidently the first to suggest the use of cyclotron resonance to investigate the phenomenon.
6. G. D. Whitfield, *Phys. Rev. Letters* **2,** 204 (1959). The term in k_α of (5) is to be dropped in the present application.
7. W. P. Dumke, *Phys. Rev.* **101,** 531 (1956).
8. C. Herring and E. Vogt, *Phys. Rev.* **101,** 944 (1956).
9. P. J. Price, *IBM Journal* **1,** 239 (1957).
10. The a here is that defined in the first of References 3. It is thus the *negative* of the constant denoted by the same letter in the second of References 3. Correspondingly, Eq. (1) of the former gives *electron* energy while Eq. (18) of the latter gives *hole* energy.
11. M. Tiersten, to be published.
12. Eq. (1) gives *electron* (rather than hole) energy, and the resulting deformation potentials given in Table 2 are derivatives of electron energy.
13. C. Herring, *Bell System Tech. J.* **34,** 237 (1955).
14. The scattering frequency for spherical energy surfaces at the zone center of a cubic crystal is given by substituting $4E_1^2/3$ for D^2, where E_1 is the Bardeen-Shockley deformation potential.
15. See the Erratum, *Phys. Rev.* **105,** 1933 (1957).

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