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Theoretical Current Multiplication of a Cylindrical Hook Collector

Abstract: An analysis is given on the mechanisms of operation for a current-multiplying hook collector of cylindrical geometry. Mathematical equations are presented which establish the minority carrier transport efficiency within a cylindrical hook collector region; both surface and bulk recombination are assumed to be present. Also included in this analysis is the influence, upon current multiplication, of the factors determining minority carrier injection efficiency for a diffused hook emitter junction. Numerical evaluation of appropriate hook collector design equations are presented in graphical form throughout a range of geometrical parameters applicable to many practical situations.

Symbols

a = hook-collector radius (see Fig. 1)
B = carrier transport efficiency through collector region
D_n = electron diffusion constant
D_p = hole diffusion constant
E_z = z-axis component of hook-collector region drift field
I_{pc}; I_{nc} = hole current and electron current at hook-collector junction
I_{pe}; I_{ne} = hole current and electron current at hook-emitter junction
J_n = electron current density
k = Boltzmann constant
L_n = electron diffusion length
q = electron charge
s = surface recombination velocity
T = temperature (abs.)
w = width of hook-collector region (see Fig. 1)
α = hook-collector alpha
α_m = root of Eq. (14a)
β_t = total hook-current multiplication
β_s, β_r, β_γ = current multiplication from individual mechanisms of surface recombination, bulk recombination, and emitter efficiency
γ = hook-emitter injection efficiency
n = electron concentration
ψ = electrostatic potential
μ_n = electron drift mobility
τ_n = electron lifetime in hook-collector region
C_{od} = surface concentration of donor impurities
C_{oa} = surface concentration of acceptor impurities
N_d = donor impurity concentration in bulk material
X_a = diffusion depth of hook-collector junction (see Fig. 1)
X_d = diffusion depth of hook-emitter junction (see Fig. 1)

Introduction

The current-multiplying hook collector is an integral part of most four-layer (PNPN) semiconductor switching devices. The semiconductor thyratron[1] and the two-collector full adder,[2] for example, are computer logic components obtaining their unique electrical properties from current-multiplication mechanisms. Investigations on this subject were first reported by Shockley;[3] he attributed the large alpha of a point contact transistor to the presence of a hook under the collector point. The inherent applicability of current multiplication to the design of semiconductor switching devices has resulted in many analytical and experimental investigations on the operation of hook-type structures.[4]

This paper presents a theoretical analysis on the multiple-layer hook collector of cylindrical geometry, Fig. 1. Physical mechanisms contributing to the electrical properties of this device are characterized by appropriate boundary value problems; solutions of such problems yield mathematical equations applicable to its design. From such analytical methods a quantitative theory is established for the cylindrical hook collector, permitting a computation of current multiplication resulting from specific physical and geometrical parameters. Further-

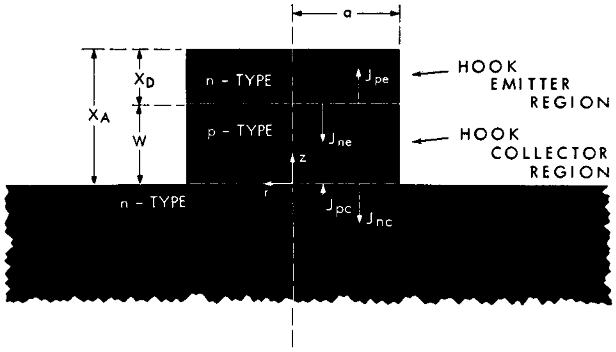


Figure 1 **Analytical model of a cylindrical hook collector.**

more, from this theoretical analysis, a design technique is established whereby the hook collector electrical characteristics are made relatively insensitive to changes in minority carrier lifetime and surface recombination velocity.

Mathematical computations required for the design of a hook collector must be conducted upon a high speed electronic computer. To facilitate such computations, therefore, computer programs are available—upon request—for the numerical evaluation of appropriate equations presented in this paper.[5]

The hook collector

Figure 1 illustrates the semiconductor device considered in this analysis. A cylindrical hook collector—consisting of a *p*-type hook collector region and an *n*-type hook emitter region—is assumed to be located upon an *n*-type base. The hook structure is similar to an NPN transistor except that no electrical contact is made to the common, or hook collector, region. The collector junction is biased in its low-current state of operation by the application of an external biasing voltage between the hook emitter and base regions; this junction, therefore, provides a sink for electrons within the hook collector region and for holes within the base.

Operation of a hook collector is easily understood by considering the electron and hole currents illustrated in Fig. 1. Minority carriers (holes) within the *n*-type base, enter the collector junction, J_{nc}, thereby resulting in an accumulation of electrostatic charge upon the hook collector region. The potentials caused by this accumulated charge forward bias the emitter junction; this bias establishes the electron current J_{ne}. Neglecting, for the moment, emitter junction hole current J_{pe}—an equilibrium condition is attained when the rate of charge accumulation, due to hole current J_{pc}, equals the rate of charge loss due to electron recombination current. A collector region of good transport efficiency, therefore, exhibits an electron current J_{nc} which is substantially greater than the hole current J_{pc}. From such operating mechanisms the total hook collector electric current, I_c, exceeds the electric current due to holes alone and therefore provides an apparent current multiplication.

Operation of this semiconductor device is dependent upon the minority carrier transport efficiency, B, of a cylindrical hook collector region and also the electron injection efficiency, γ, of an emitter junction. These parameters are expressed by the relations

$$B=\frac{I_{nc}}{I_{ne}} \tag{1a}$$

$$\gamma=\frac{I_{ne}}{I_{ne}+I_{pe}}. \tag{1b}$$

From Eqs. (1) the alpha of a hook structure is

$$\alpha=\gamma B=\frac{I_{nc}}{I_{ne}+I_{pe}}. \tag{2}$$

Eq. (2) can be written in a form comparable to the common emitter current gain of a conventional junction transistor,

$$\beta=\frac{\alpha}{1-\alpha}=\frac{I_{nc}}{I_{pe}+I_{ne}-I_{nc}}. \tag{3}$$

From Fig. 1, the total hook collector hole current I_{pc}, the total collector region recombination current $(I_{ne}-I_{nc})$, and the total hook emitter hole current, I_{pe}, all contribute to the rate of charge accumulation at the collector region. Stability is attained when the collector region electrostatic potential remains constant; this equilibrium condition is characterized by

$$I_{pc}=I_{pe}+I_{ne}-I_{nc}. \tag{4}$$

From Eq. (3) and Eq. (4), the magnitude of current multiplication is

$$\beta=\frac{I_{nc}}{I_{pc}}. \tag{5}$$

It is frequently convenient to separate the factors contributing to Eq. (5). This separation is obtained by designating β_s, β_r, and β_γ as the current multiplication resulting from the individual mechanisms of surface and bulk recombination within the collector region, and from hook emitter electron injection efficiency. Introducing this separation into (4), the total current multiplication, β_t, of a hook collector is approximated[6] by

$$\frac{1}{\beta_t}=\frac{1}{\beta_s}+\frac{1}{\beta_r}+\frac{1}{\beta_\gamma}. \tag{6}$$

Influence of surface recombination upon current multiplication

The hook collector region is assumed to be a solid cylinder of semiconductor material containing an axial distribution of acceptor impurities. This impurity distribution, introduced by diffusion techniques, results in an electric field directed toward the hook emitter junction. Electron current within this cylinder, therefore, is caused by the combined mechanisms of diffusion and drift—it is thus characterized by

$$J_n=-qD_n\,\mathrm{grad}\,n+q\mu_n n\,\mathrm{grad}\,\psi. \tag{7}$$

A simplification has been introduced by assuming an axially directed electric field, $E_z=-\partial\psi/\partial z$, of constant magnitude between the emitter and collector junctions; this magnitude is equal to the electric field at the emitter junction face.[7]

Neglecting bulk recombination, the distribution of excess electrons throughout the hook collector region, $n(r;z)$, is governed by the differential equation

$$0=\frac{\partial^2 n}{\partial r^2}+\frac{1}{r}\frac{\partial n}{\partial r}+\frac{\partial^2 n}{\partial z^2}+\varepsilon\frac{\partial n}{\partial z}\qquad \varepsilon=\frac{qE_z}{kT}. \tag{8}$$

A separation of variables within Eq. (8) is obtained by assuming

$$n(r;z)=X(r)Y(z). \tag{9}$$

Substitution of Eq. (9) into Eq. (8) yields the relations

$$\frac{\partial^2 X}{\partial r^2}+\frac{1}{r}\frac{\partial X}{\partial r}+\lambda^2 X(r)=0 \tag{10a}$$

$$\frac{\partial^2 Y}{\partial z^2}+\varepsilon\frac{\partial Y}{\partial z}-\lambda^2 Y(z)=0, \tag{10b}$$

where λ is an arbitrary constant of separation. It is important to note that the physical nature of this boundary value problem restricts solutions of Eq. (10a) to Bessel functions. Since the minority carrier distribution must be well behaved at the origin ($r=0$), Neumann functions are excluded from this solution. We have, therefore, from Eq. (9) and Eqs. (10) the general minority-carrier distribution equation

$$n(r;z)=\exp(\varepsilon z/2)\{C_1\cosh(\phi z)+C_2\sinh(\phi z)\} \tag{11a}$$

$$\phi^2=\left(\frac{\varepsilon}{2}\right)^2+\lambda^2. \tag{11b}$$

Eq. (11) is a cylindrical harmonic function used to characterize the excess minority carrier distribution (electrons) throughout this hook collector region. Application of Eq. (11) to specific boundary value problems yields an infinite summation of such harmonic functions in which the constants are selected to satisfy the required boundary conditions.

From Fig. 1—and also from previous considerations—the boundary equations assumed to adequately characterize a cylindrical hook collector are:

$$\frac{\partial n}{\partial z}+\frac{S}{D_p}n=0\quad r=a;\ 0<z<w \tag{12a}$$

$$n(r;z)=0\quad 0<r<a;\ z=0 \tag{12b}$$

$$n(r;z)=n_e\quad 0<r<a;\ z=w. \tag{12c}$$

Eq. (12a) is a Cauchy type boundary equation which establishes a minority carrier recombination velocity of (s) at the exposed hook collector surface. Eq. (12b) and Eq. (12c) locate the collector and emitter junctions. Eq. (12b) eliminates all excess minority carriers from one end of the solid cylinder, thereby providing a minority carrier sink. Further, Eq. (12c) establishes the emitter as a minority carrier source by requiring a constant density of excess electrons, n_e, across the entire emitter junction face.

A minority carrier distribution equation—subject to the boundary conditions imposed by Eqs. (12)—is, from Appendix I, given by

$$n(r;z)=2n_e\exp[(\varepsilon/2)(w-z)]\sum_{m=1}^{\infty}\frac{\sinh\left(\delta_m\frac{z}{a}\right)}{\alpha_m\sinh\left(\delta_m\frac{w}{a}\right)}\left\{\frac{J_1(\alpha_m)J_0\left(\alpha_m\frac{r}{a}\right)}{J_0^2(\alpha_m)+J_1^2(\alpha_m)}\right\}, \tag{13}$$

where

$$\alpha_m J_1(\alpha_m)-\frac{Sa}{D_n}J_0(\alpha_m)=0 \tag{14a}$$

$$\delta_m^2=\left(\frac{\varepsilon a}{2}\right)^2+\alpha_m^2. \tag{14b}$$

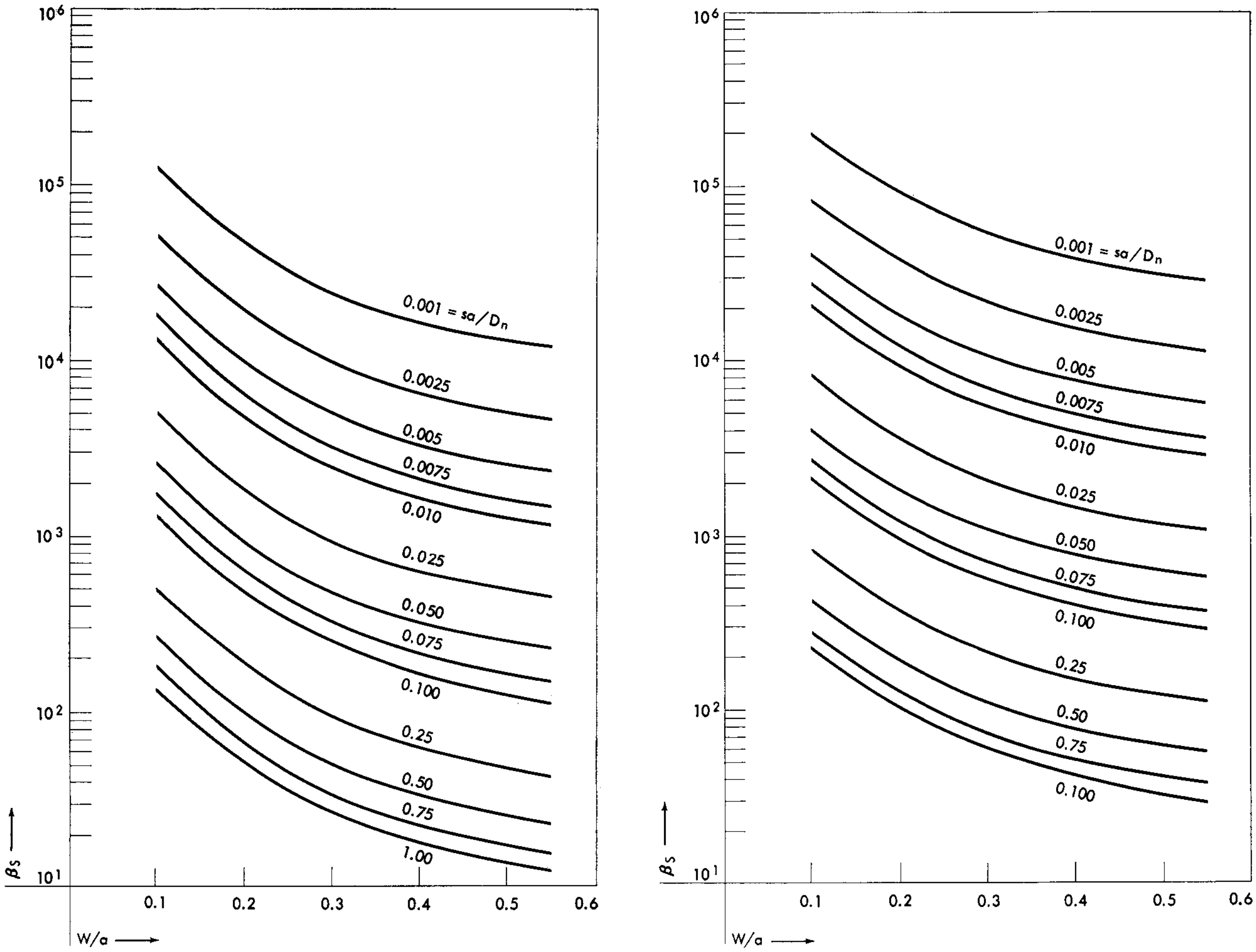
The summation of Eq. (13) is conducted over ascending roots, α_m, of Eq. (14a) and there is an infinite number of such roots for each assumed magnitude of surface recombination velocity. This expression, Eq. (13), establishes the minority carrier distribution throughout a solid cylinder of semiconductor material which is characterized by the stated boundary conditions.

Substituting Eq. (13) into Eq. (7) yields the axially directed electron current, $J_{nz}(r;z)$, within this cylindrical structure,

$$J_{nz}(r;z)=-\frac{2qn_e}{w}\exp[(\varepsilon/2)(w-z)]\sum_{m=1}^{\infty}\frac{\left[\delta_m\cosh\left(\delta_m\frac{z}{a}\right)+\frac{\varepsilon w}{2}\sinh\left(\delta_m\frac{z}{a}\right)\right]J_1(\alpha_m)J_0(\alpha_m)}{\alpha_m\sinh\left(\delta_m\frac{w}{a}\right)\left[J_0^2(\alpha_m)+J_1^2(\alpha_m)\right]}. \tag{15}$$

From Eq. (15), the minority carrier transport efficiency, B_s, can now be established. A radial integration of Eq. (15) yields a total electron current, $I_n(z)$, within the hook collector region; from this integrated expression we have the total collector junction electron current at $z=0$, $I_n(0)$, and also the total electron current at the emitter junction ($z=w$). The minority carrier transport efficiency, B_s, is therefore given by

$$B_s=\frac{I_{nc}}{I_{ne}}=\exp(\varepsilon w/2)\,\frac{\displaystyle\sum_{m=1}^{\infty}\frac{\delta_m J_1^2(\alpha_m)}{\alpha_m^2[J_0^2(\alpha_m)+J_1^2(\alpha_m)]\sinh\left(\delta_m\frac{w}{a}\right)}}{\displaystyle\sum_{m=1}^{\infty}\frac{\left[\delta_m\operatorname{ctnh}\left(\delta_m\frac{w}{a}\right)+\frac{\varepsilon w}{2}\right]J_1^2(\alpha_m)}{\alpha_m^2[J_0^2(\alpha_m)+J_1^2(\alpha_m)]}}. \tag{16}$$



a) Normalized drift-field parameter $qE_z a/kT=10$ *b) Normalized drift-field parameter* $qE_z a/kT=30$

Figure 2 **Theoretical current multiplication when** $\tau_n = \infty$ **and** $\gamma = 1$ **(Eq. 17).**

Having assumed an emitter injection efficiency of unity, current multiplication within this semiconductor device is given by

$$\beta_s = \frac{B_s}{1 - B_s}. \tag{17}$$

The large number of independent variables associated with Eq. (17) has prevented a complete graphical representation of the hook-collector multiplication parameter, β_s. To illustrate this parameter, therefore, graphs are presented for a geometrical range characterizing many practical semiconductor devices, Fig. 2. A further numerical evaluation of Eq. (17) will be necessary for the design of hook collectors having geometrical parameters outside the range of these graphs.[5]

Influence of minority carrier lifetime upon current multiplication

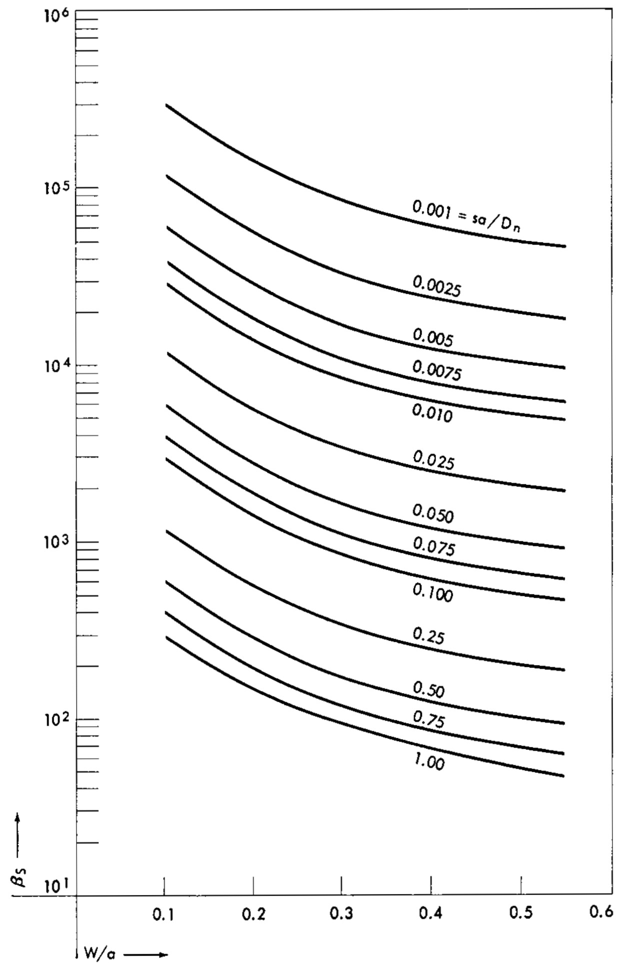
In the derivation of Eq. (8) a divergence-free minority-carrier current was assumed within the hook collector region. Such an assumption implies the presence of an infinite minority carrier lifetime. Throughout this portion of the hook collector analysis, therefore, minority carrier current is governed by the continuity equation

$$\frac{n}{\tau_n} = -\frac{1}{q}\,\mathrm{div}\,J_n \tag{18}$$

in conjunction with Eq. (7). The current multiplication parameter β_r, encountered when $s=0$ and $\gamma=1$, is mathematically established from a simplified, one-dimensional approximation of Eq. (18). Having eliminated surface recombination current within the collector region ($s=0$), we also eliminate all radially directed minority carrier current within a structure of circular symmetry. Eq. (18), therefore, can be reduced to one spatial variable yielding the differential equation

$$\frac{d^2 n}{dz^2} + \varepsilon \frac{dn}{dz} - \frac{n}{L_p^{\,2}} = 0. \tag{19}$$

The axially directed drift field appearing in Eq. (19), E_z, has been established by the same approximations used in Eq. (8)—E_z is assumed of constant magnitude.



c) Normalized drift-field parameter $qE_za/kT=50$

A solution of Eq. (19), subject to the boundary conditions specified by Eq. (12b) and Eq. (12c), is given by

$$n(z)=n_e\frac{\sinh\left(\theta\frac{z}{w}\right)}{\sinh\theta}\exp\left[(\varepsilon/2)(w-z)\right] \tag{20}$$

where

$$\theta^2=\left(\frac{\varepsilon w}{2}\right)^2+\left(\frac{w}{L_n}\right)^2. \tag{21}$$

Substituting Eq. (20) into Eq. (7) yields a one-dimensional distribution of minority carrier current within this hook-collector region

$$J_{nz}(z)=-\frac{qD_nn_e}{w}\left\{\frac{\theta\cosh\left(\theta\frac{z}{w}\right)+\frac{\varepsilon w}{2}\sinh\left(\theta\frac{z}{w}\right)}{\sinh(\theta)}\right\}\exp\left[(\varepsilon/2)(w-z)\right]. \tag{22}$$

From Eq. (22) we can now establish the hook-collector current multiplication, β_r, of this structure. Again assuming that β_s and B_γ are infinite, we have, from Eq. (1a), Eq. (3) and Eq. (22), the relation

$$\beta_r=\frac{J_{nz}(0)}{J_{nz}(0)-J_{nz}(w)}=\frac{\theta\exp(\varepsilon w/2)}{\theta[\cosh(\theta)-\exp(\varepsilon w/2)]+\theta\sinh\theta}. \tag{23}$$

Eq. (23), unlike Eq. (17), has a limited number of independent variables and is therefore graphically represented throughout a range applicable to most practical situations. This graph, Fig. 3, provides the bulk recombination term, β_r, required in the computation of total current multiplication, β_t, within a hook collector.

Influence of hook emitter injection efficiency upon current multiplication

M. Tanenbaum and D. E. Thomas[8] have considered the problem of minority carrier injection for a multiple-diffused emitter junction. A semiconductor structure of this type results from the diffusion of impurity atoms—donor and acceptor—into an n-type semiconductor material containing a specified concentration, N_d, of uncompensated donor atoms. During the first diffusion cycle a given concentration of acceptor atoms, C_{oa}, is maintained at the semiconductor surface, thereby forming a p-type layer of depth X_a within the material. Similarly, during the second diffusion cycle a concentration of donor atoms, C_{od}, is maintained at this surface; compensation of acceptor atoms results in an n-type layer of depth X_d, where $X_d<X_a$. This experimental technique yields an NPN structure having an impurity atom distribution, $N(x)$, which is given by the general expression

$$N(z)=N_d+C_{od}\operatorname{erfc}\left(\frac{z}{L_d}\right)-C_{oa}\operatorname{erfc}\left(\frac{z}{L_a}\right). \tag{24}$$

From this impurity distribution equation, electron and hole currents appearing at the hook emitter junction are given[8] by

$$J_{ne}=qn_i^2D_n\frac{\exp(qv/kT)-1}{\int_e|N(z)|dz}$$

$$J_{pe}=qn_i^2D_n\frac{\exp(qv/kT)-1}{\int_e|N(z)|dz}. \tag{25}$$

The integrations of Eqs. (25) are conducted throughout the collector and emitter regions of this semiconductor device. Further, from Eq. (1b), Eq. (3), and Eqs. (25) we have the required current multiplication parameter

$$\beta_\gamma=\frac{\gamma}{1-\gamma}=\frac{D_n}{D_p}\frac{\int_e|N(z)|dz}{\int_c|N(z)|dz}. \tag{26}$$

Eq. (26) establishes the magnitude of current multiplication obtained from a hook collector exhibiting no

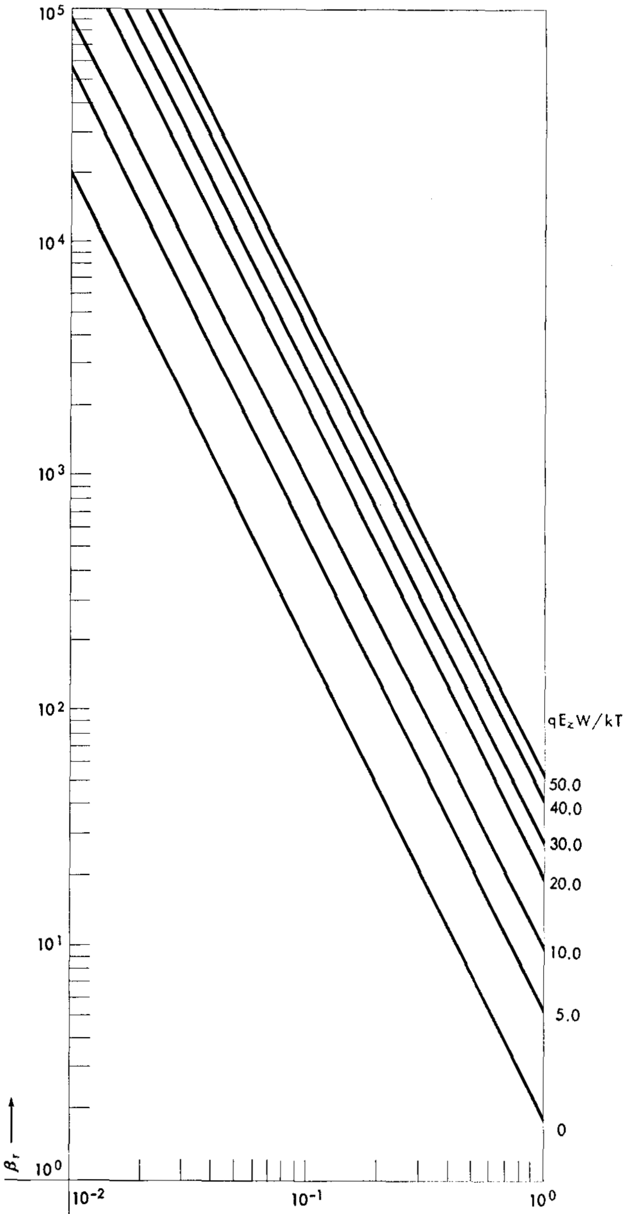


Figure 3 **Theoretical current multiplication when $\gamma=1$; $s=0$ (Eq. 23).**

minority carrier loss within its hook collector region ($\beta_s=\beta_r=\infty$). The large number of independent variables associated with Eq. (26) has prevented its graphical representation throughout a range applicable to all semiconductor devices. To illustrate β_γ, therefore, a range of parameters have been selected which includes a large number of practical situations, Fig. 4. Further numerical evaluation of this expression will be necessary for the design of hook collector structures not characterized by these graphs.

Conclusions

Theoretical investigations into the operation of a current-multiplying hook collector has yielded mathematical equations applicable to the design of this semiconductor device. Equations (17), (23), and (26), respectively, establish the parameters β_s, β_r, and β_γ which are required in the computation of total hook-collector current multiplication β_t, Eq. (6).

Important to the practical aspects of semiconductor device design is its sensitivity to changes in minority carrier lifetime and surface recombination velocity. Eq. (6) indicates a technique whereby this sensitivity can be decreased. Introducing the requirement

$$\left(\frac{1}{\beta_s}+\frac{1}{\beta_r}\right)<\frac{1}{\beta_\gamma} \tag{27}$$

—for large changes of lifetime and recombination velocity—decreases the sensitivity of current multiplication, β_t, to changes in these parameters. Application of this design technique to practical hook collector structures has, for example, yielded the following theoretical characteristics:

β_t	s	τ_n
20	0	∞
18	10^4	10^{-7}

Electron injection efficiency of the emitter junction, γ, maintains a limit upon the magnitude of current multiplication; this limit masks variations resulting from changes of transport efficiency through the hook collector region.

Acknowledgments

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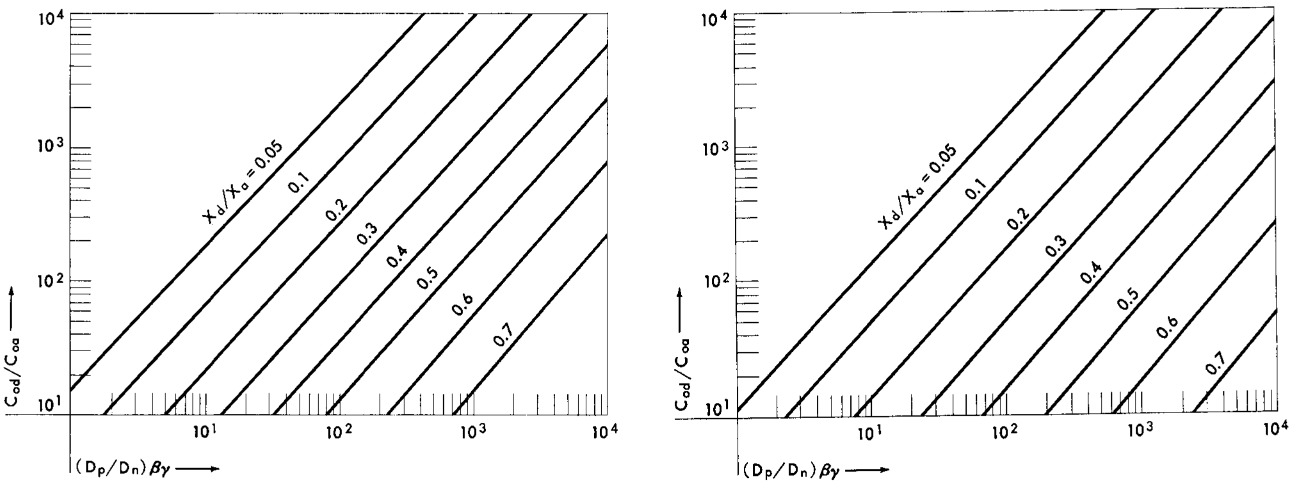
Appendix I

The general minority carrier distribution equation satisfying Eq. (8) has the form

$$n(r;z)=\exp(-\varepsilon z/2)\{C_1\cosh(\phi z)+C_2\sinh(\phi z)\}\,J_0(\lambda r), \tag{I-1a}$$

where

$$\phi^2=\left(\frac{\varepsilon}{2}\right)^2+\lambda^2. \tag{I-1b}$$



a) Impurity atom ratio $C_{oa}/N_d=10^2$

b) Impurity atom ratio $C_{oa}/N_d=10^3$

Figure 4 **Theoretical current multiplication when $\tau_n=\infty$; s=0 (Eq. 26).**

The constants of Eq. (I-1) must be adjusted to satisfy the boundary equations outlined in Eq. (12). This is accomplished by first substituting Eq. (I-1) into Eq. (12a), yielding the relation

$$n(r;z)=\exp\,(-\varepsilon z/2)\sum_{m=1}^{\infty}\left\{C_{1m}\cosh\left(\delta_m\frac{z}{a}\right)+C_{2m}\sinh\left(\delta_m\frac{z}{a}\right)\right\}J_0\left(\alpha_m\frac{r}{a}\right), \qquad \text{(I-2)}$$

where

$$\alpha_m J_1(\alpha_m)-\frac{sc}{D_n}J_0(\alpha_m)=0 \qquad \text{(I-3a)}$$

$$\delta_m^2=\left(\frac{\varepsilon a}{2}\right)^2+\alpha_m^2. \qquad \text{(I-3b)}$$

The summation of Eq. (I-2) is conducted over ascending roots of Eq. (I-3a).

Upon substituting Eq. (I-2) into the boundary equations (12b) and (12c) we obtain

$$n(r;z)=\exp\,[(\varepsilon/2)(w-z)]\sum_{m=1}^{\infty}\psi_m\frac{\sinh\left(\delta_m\frac{z}{a}\right)}{\sinh\left(\delta_m\frac{w}{a}\right)}J_0\left(\delta_m\frac{r}{a}\right), \qquad \text{(I-4)}$$

where

$$={}^{\partial}u\sum_{m=1}^{\infty}\psi_m J_0\left(\alpha_m\frac{r}{a}\right). \qquad \text{(I-5)}$$

Eq. (I-5) is a Dini series type of approximation for the boundary conditions assumed at the emitter junction. It can be shown[9] the constant ψ_m is given by

$$\psi_m=\frac{2n_e\int_0^a rJ_0\left(\alpha_m\frac{r}{a}\right)dr}{a^2[J_0^2(\alpha_m)+J_1^2(\alpha_m)]}=\frac{2n_eJ_1(\alpha_m)}{\alpha_m[J_0^2(\alpha_m)+J_1^2(\alpha_m)]}. \qquad \text{(I-6)}$$

Substituting Eq. (I-6) into Eq. (I-2) yields

$$n(r;z)=n_e\exp\,[(\varepsilon/2)(w-z)]\sum_{m=1}^{\infty}\frac{\sinh\left(\delta_m\frac{z}{a}\right)}{\alpha_m\sinh\left(\delta_m\frac{w}{a}\right)}\left\{\frac{J_1(\alpha_m)J_0\left(\alpha_m\frac{r}{a}\right)}{J_0^2(\alpha_m)+J_1^2(\alpha_m)}\right\}. \qquad \text{(I-7)}$$

Eq. (I-7) fully describes the distribution of excess minority carriers (electrons) throughout a solid cylinder of *p*-type semiconductor material characterized by the boundary conditions of Eqs. (12).

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