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Reliability Improvement through Redundancy at Various System Levels*

Abstract: Improvement in computing machine reliability through redundancy is studied as a function of the level at which the redundancy is applied. The reliability achieved by redundancy of complete, independent machines is compared to that achieved by redundancy of smaller units.

A machine unit is termed *m* times redundant when the following conditions exist:

1. ***m* independent identical units operate simultaneously with a common input.**
2. **A failure detector is associated with each unit.**
3. **A switch is connected to the outputs of the units, so that the output is taken from some one unit until failure occurs in that unit. Then the switch steps so that the output is taken from the next redundant unit, if that unit is operating correctly. The process continues until the assigned task is completed or all *m* units fail.**

The reliability of *m* redundant units is expressed in terms of the reliability of one unit and the probabilities of correct operation of the failure detectors and switch.

It is assumed that a complete machine may be broken up into *p* units, $p=1, 2, 3, \ldots, 24$, of equal reliability. The reliability achieved by redundancy of these units is calculated as a function of *p* and *m*, $m=1, 2, 3, 4$, with single-machine reliabilities of 0.2, 0.5, 0.9 and 0.99. These results are calculated for perfect failure detection and switching devices as well as for moderately unreliable devices. The resultant system unreliability is plotted as a function of *p* on linear and on logarithmic scales.

It has been shown that the reliability of a digital computer may be considerably increased by means of redundancy of machine parts.[1-8] Three essentially different types of redundancy have been considered. In the first type,[1,2,5,6] a number of independent identical units operate simultaneously with a common input. A failure detector is associated with each unit and a switch is connected to the outputs. The output is taken from some one unit until that unit fails, at which time the switch steps to the next unit which is operating satisfactorily. This type of redundancy is applicable to units of some complexity, since a failure-detector and switch must be associated with each unit. The second type of redundancy[3,4,8] is applied to relays or other bistable devices. An array of several identical relay contacts is used to perform the function of a single contact. The array is assembled so that its reliability is greater than that of the single component. This type of redundancy is applied to the smallest components of a machine and no failure detection or switching is involved. The third type of redundancy[7,8] may be applied to any machine unit with a binary output. Three or more identical units having common input feed their outputs into a "majority organ." The output of the latter is determined by a majority of the unit outputs.

This paper is concerned solely with redundancy of the first type, involving failure detection and switching. This type of redundancy may be applied to machine units of any size large enough to make it practical to associate a failure detector and switch with each unit. It has been shown[1,5] that, for perfect failure detection and switching, maximum reliability improvement corresponds to redundancy of the smallest possible units. However, Rosenheim and Ash,[6] have pointed out the advantages of the redundancy of complete, independent machines.

*Presented at the 1958 IRE National Convention.

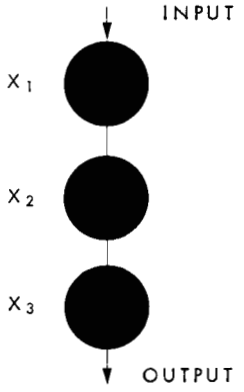


Figure 1 **Symbolic diagram of a chain.**

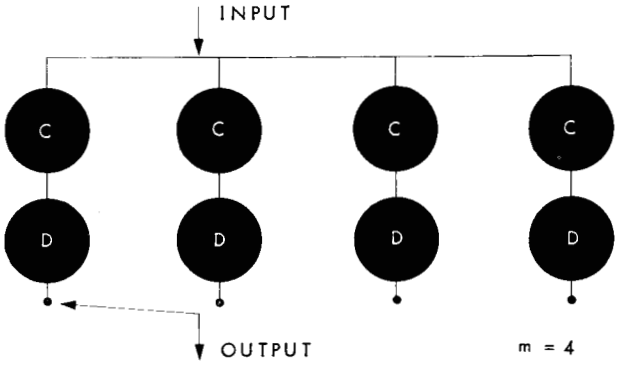


Figure 2 **System consisting of *m* redundant chains.**

The present study provides a quantitative estimate of the dependence of reliability improvement through redundancy on the following factors:

1. The initial reliability of the non-redundant machine.
2. The degree of redundancy, i.e., the number of independent identical units which operate simultaneously with a common input.
3. The level at which redundancy is applied, i.e., the relative size of the unit which is associated with a failure detector and switch.
4. The reliability of the error-detectors and switches. The effect of imperfections in the switching devices on the over-all system reliability is carefully analyzed, and the reliability improvement attainable with imperfect switching is compared with the idealized case of perfect switching.

The following terms will be used in this report:

Reliability, R, is the probability that a specified function will be adequately performed for a specified time. Thus, reliability depends both upon the unit with which it is associated and the assigned task. For any practical unit in continuous operation, R will have a value close to unity for a short operating period and will approach zero as the operating period approaches infinity.

Unreliability, $F=1-R$, is the probability that a specified function will fail to be adequately performed for a specified time. (It is identical with the cumulative distribution function of time to failure for the unit performing the specified function.)

A *component,* x_i, is the smallest machine unit with which a reliability, R_i, is associated.

A *chain* is a set of n components $x_i, \ldots, x_n$ assembled in series. The output from component x_i is the input to component x_{i+1}. Component x_1 receives the input to the chain and the output is taken from x_n. It is assumed that the components fail independently. A chain is symbolized in Figure 1.

The *reliability of a chain,* R_c, is clearly the product of the reliabilities of its components (Eq. 1). (A chain may be of any size, from a single component to a complete machine.)

$$R_c = \prod_{i=1}^{n} R_i \qquad (1)$$

Redundancy is here defined as the parallel operation of a set of independent,* identical chains. The inputs to the chains are common and the outputs are connected to a switch. A device associated with each chain indicates failure of that chain. The output is taken from some one chain as long as it does not fail. When failure occurs, the switch is immediately stepped to the output of the next chain which has not failed. We say that a chain, C, is m times redundant or that the degree of redundancy is m when m of these chains, with a failure-detector D for each chain, are thus paralleled. This is symbolized in Figure 2.

In order to compute the reliability of a system consisting of m redundant chains, we make the following assumptions:

1. The chains are ordered, $1, \ldots\ldots, m$.
2. It is demanded that the system perform a given task, i.e., that it operate for a given length of time. Each chain operates from the initial time until it fails or until the task is completed.
3. The stepping switch is connected so that its inputs are the outputs of the m chains. Its output is the output of the system. It is initially positioned to receive the output of the first chain. When it is connected to the i^{th} output $(i=1, \ldots, m-1)$ and it receives a signal from the i^{th} failure-detector, it steps to the $(i+1)^{\text{th}}$ output. When it is connected to the m^{th} output and it receives a failure signal, it switches to a device which indicates complete system failure.
4. A failure-detecting device operates in conjunction with each chain. It performs the following functions:

a) If failure occurs in the chain to which the switch is connected, a signal is immediately sent to the switch, causing it to step.

b) If failure occurs in a chain to which the switch is not connected, a signal is stored. If, at a subsequent time,

*We neglect the possibility of an input short affecting the operation of all chains connected to a common input. The probability of such an occurrence can be made very low by use of appropriate isolating elements.

the switch steps to that chain, it is signaled to step once more.

5. It is assumed that no significant time is consumed by the failure-detecting and switching operations.

The reliability of the system depends upon the reliabilities of the chains, the failure detectors, and the switches. For the detectors and switches, there are two modes of behavior with which reliabilities are associated. The first class of reliability (D_a and S_a below) is a probability that the device operates when failure occurs. This function may be performed only once for each chain, and the probability is defined for a single operation which takes place in negligible time. The second class of reliability (D_b and S_b below) is a probability that the device does not spontaneously operate during a period of time in which no failure occurs. This type of probability, like the reliability of a chain, is defined for the length of time required for the machine to complete the assigned task. Thus, we define the following probabilities:

R_c is the reliability of the chain, i.e., the probability that it performs its functions adequately for the duration of the assigned task.

D_a is the conditional probability that, when a failure occurs in a chain, the failure is detected and a signal is sent to the switch under condition 4a or 4b. (A factor of D_a is the probability that the switch control is connected to the error detector for the chain at which the switch is positioned.)

D_b is the conditional probability that, when no failure occurs in a chain for the duration of the task, no signal is transmitted to the switch when it is positioned at that chain.

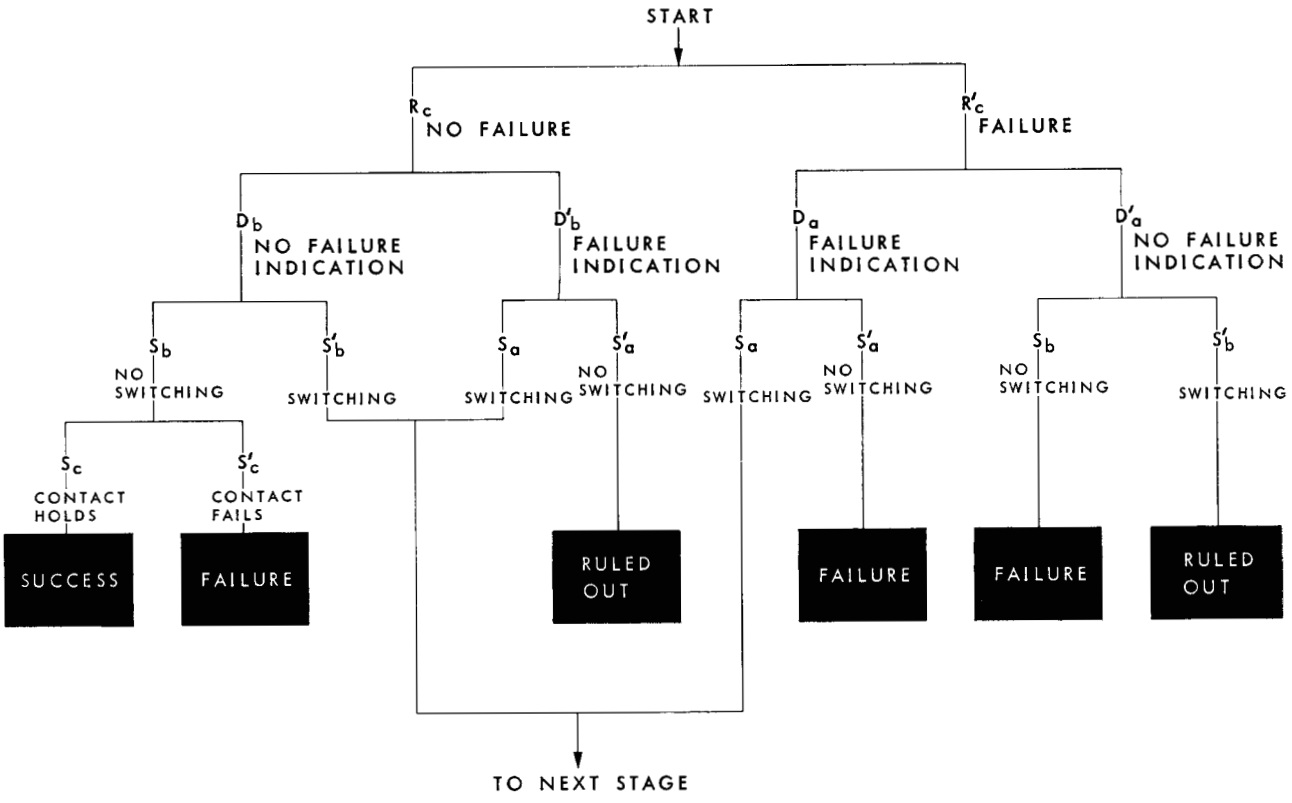
S_a is the conditional probability that, when the switch receives a failure signal, the connection at which it stands is broken and a good connection is made to the next chain.

S_b is the conditional probability that, if the switch does not receive a failure signal for the duration of the task, it does not step at any time during the run. (It is assumed that, if it does step, it makes contact on the next chain.)

S_c is the conditional probability that, if a good connection is made every time the switch steps, a good connection exists between some chain (or the device indicating system failure) and the system output at all times during the run. (It is assumed that switching occurs in zero time, so that we may think of the switch as making contact over the entire time interval except for a finite number of points in time.)

The reliability of the system of m redundant chains is defined as the probability that it performs the assigned task successfully. This occurs if, for the duration of the task, the switch constantly (except for the points in time

Figure 3 **Failure diagram of a chain.**



required for switching) makes a good connection to a chain that is functioning adequately. This can take place in m mutually exclusive ways, corresponding to final connection to the m switch contacts.

The possible modes of behavior of a chain are diagrammed in Figure 3. Successful operation through a given chain requires that the chain functions adequately (R_c), the failure detector does not signal an error (D_b), the switch does not step spontaneously while connected to this chain (S_b)* and the switch contact remains good (S_c). This has probability $R_1=R_cD_bS_bS_c$.

A stepping of the switch may occur in three ways:

a) The chain fails ($F_c=1-R_c$), the detector signals failure (D_a), and the switch steps (S_a).
b) The chain does not fail (R_c), but the detector erroneously signals failure ($D'_b=1-D_b$), and the switch steps (S_a).
c) The chain does not fail (R_c), the detector does not signal failure (D_b), but the switch steps spontaneously ($S'_b=1-S_b$).*

*We are making an approximation by using one value of S_b for the probability of no spontaneous stepping of the switch from any position. A precise analysis would use S_b as defined above only for the first chain, with successively larger values for this probability for chains 2, . . . , m. Thus, our final reliability is somewhat lower than the correct result. However, since the probability of spontaneous switching will, in all practical applications, be exceedingly small, the more precise analysis does not seem to be warranted.

Thus, the probability of one stepping of the switch is:

$$\delta=(1-R_c)D_aS_a+R_c(1-D_b)S_a+R_cD_b(1-S_b) \qquad (2)$$

There are several modes of behavior of one chain that lead immediately to system failure without any failure indication, due to a bad switch contact (S'_c), to failure of the switch to respond to an error signal (S'_a), or to failure of the detector to indicate failure (D'_a). In addition, there are modes of behavior in which the detector and switch both make errors which cancel each other out. These second-order effects will be arbitrarily ruled out.

Now the probability of successful operation with the final connection to the i^{th} switch contact is equal to the probability of $i-1$ steppings of the switch times the probability of successful operation through one chain, or $\delta^{(i-1)}R_1$.

Then the reliability of the system is the sum of these probabilities for the m switch contacts:

$$R=\sum_{i=1}^{m}\delta^{(i-1)}R_1=R_1\frac{1-\delta^m}{1-\delta}\ , \qquad (3)$$

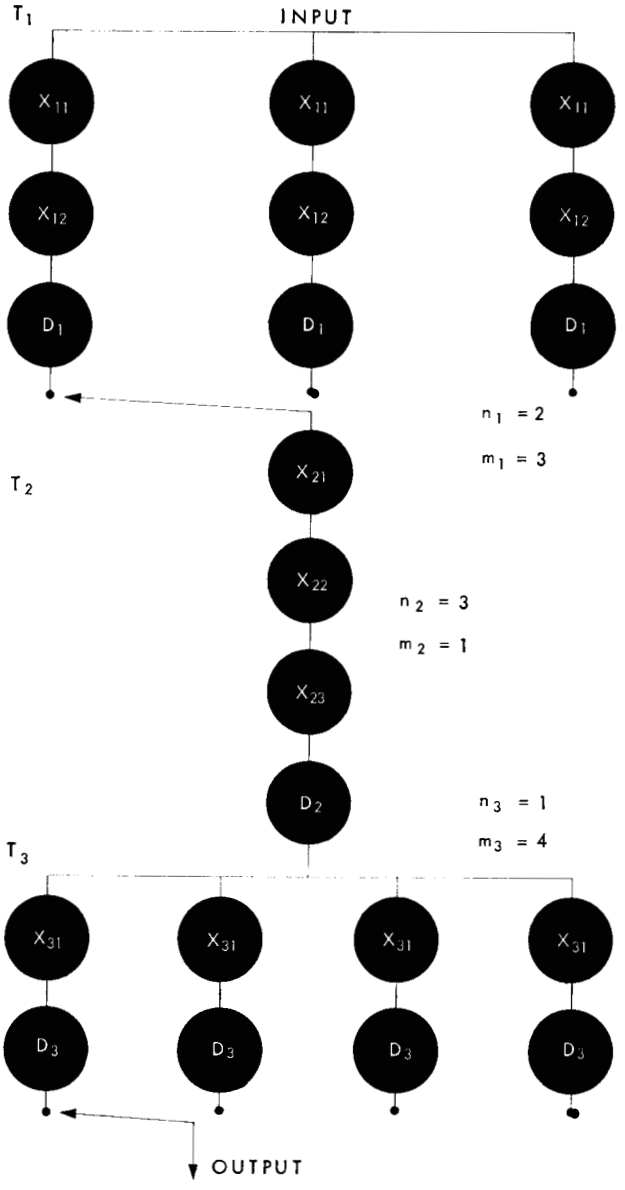


Figure 4 **A complete system.**

or

$$R=R_cD_bS_bS_c\frac{1-\{(1-R_c)D_aS_a+R_c(1-D_b)S_a+R_cD_b(1-S_b)\}^m}{1-\{(1-R_c)D_aS_a+R_c(1-D_b)S_a+R_cD_b(1-S_b)\}}\ , \qquad (4)$$

where $R_c=\prod_{i=1}^{n}R_i$.

Since $D_a\leq 1$, $S_a\leq 1$,

$$R\leq R_cD_bS_bS_c\frac{1-\{1-R_cD_bS_b\}^m}{1-\{1-R_cD_bS_b\}}=S_c\left[1-\{1-R_cD_bS_b\}^m\right],$$

so that $R\leq S_c$.

Furthermore, since $S_c\leq 1$, $D_b\leq 1$, $S_b\leq 1$,

$$R\leq 1-(1-R_c)^m \qquad (5)$$

Thus, the well-known expression corresponding to perfect failure detection and switching represents an upper limit for the reliability obtainable.

Finally, suppose a complete system (Figure 4) consists of p independent sub-systems of redundant chains, $T_1, \ldots, T_p$. *The components of the* j'^{th} chain are $x_{j1}, \ldots, x_{jn_j}$ with reliabilities $R_{j1}, \ldots, R_{jn_j}$ and the degree of redundancy of the j'^{th} chain is m_j.

The reliability of this complete system is the product of the reliabilities of the independent sub-systems:

$$R_s = \prod_{j=1}^{p} R_j, \tag{6}$$

where $$R_j = R_{c_j} D_{b_j} S_{b_j} S_{c_j} \frac{1 - \{(1-R_{c_j}) D_{a_j} S_{a_j} + R_{c_j}(1-D_{b_j}) S_{a_j} + R_{c_j} D_{b_j}(1-S_{b_j})\}^m}{1 - \{(1-R_{c_j}) D_{a_j} S_{a_j} + R_{c_j}(1-D_{b_j}) S_{a_j} + R_{c_j} D_{b_j}(1-S_{b_j})\}},$$ where $$R_{c_j} = \prod_{i=1}^{n_j} R_{ji}.$$

D_{a_j} and D_{b_j} are probabilities (defined above) associated with the failure detectors for the j'^{th} sub-system, T_j.

This formula provides for the calculation of total reliability with different degrees of redundancy for different parts of a machine. Presumably, it might be desirable to provide considerable redundancy for relatively unreliable components and little or no redundancy for more reliable components.

In the present application we consider that our initial machine, with no redundancy, has a reliability R_o. We assume that it is possible to break the machine up into p chains of equal reliability, $R_o^{1/p}$. We further assume that the failure detector for the complete machine consists of p units, each associated with a chain, such that indications of failure originating from any of these units are equally probable. Then, if D_a and D_b are probabilities associated with the failure detector for one complete machine, the corresponding probabilities for the unit associated with a chain will be $D_a^{1/p}$ and $D_b^{1/p}$. If each chain is made m times redundant, the reliability of the final system is:

$$R_s = \left[R_o^{1/p} D_b^{1/p} S_b S_c \frac{1 - \{(1-R_o^{1/p}) D_a^{1/p} S_a + R_o^{1/p}(1-D_b^{1/p}) S_a + R_o^{1/p} D_b^{1/p}(1-S_b)\}^m}{1 - \{(1-R_o^{1/p}) D_a^{1/p} S_a + R_o^{1/p}(1-D_b^{1/p}) S_a + R_o^{1/p} D_b^{1/p}(1-S_b)\}} \right]^p . \tag{7}$$

For perfect failure detection and switching, this becomes

$$R_s = [1 - (1 - R_o^{1/p})^m]^p \tag{8}$$

Clearly, this latter expression approaches unity as m gets large, for any $p \geq 1$, or, as p gets large, for any $m \geq 2$. Thus, if switching and failure detection errors are neglected, it appears as if any desired reliability could be achieved either by sufficient redundancy (large m) or by a sufficiently low level of redundancy (large p). (Of course, the size of p is limited by the total number of machine components.)

On the other hand, a study of Eq. 7 reveals that an upper limit for R_s is S_c^p. This is the probability that every switch maintains a good contact for the duration of the task, and is a decreasing function of p. This factor sets a limit on the reliability achievable by redundancy.

We have studied the dependence of $F_s = 1 - R_s$ on p and m for various values of F_o, first with all switches and failure detectors assumed perfect, and finally with what we consider low values for the probabilities of correct operation of these devices.

Tables 1-4 and Figs. 5-12 are concerned with our results for perfect switching and error detection. They give $F_s = 1 - R_s$ as a function of p, with p varying from 1 to 24 for $m = 1, 2, 3, 4$ and for $F_o = 0.8, 0.5, 0.1$, and 0.01. In Figs. 5 to 8, the data are plotted on linear scales, while in Figs. 9 to 12, they are plotted on logarithmic scales. For those applications in which the penalty for failure is moderate in magnitude, Figs. 5 to 8 present an adequate picture of the absolute improvement in reliability achieved by redundancy. However, in cases where the penalty for failure is very large, a decrease in unreliability from say 10^{-6} to 10^{-8}, may be very significant. In Figs. 9 to 12, the logarithmic scale is used to show the behavior of F_s as it approches zero on the linear scale.

In each case, the line for $m = 1$ represents the unreliability of a machine with no redundancy.

The values of F_s for p = 1 represent the unreliabilities for redundancy of independent machines. For $p = 1$, $F_s = F_o^m$. Increasing the value of p corresponds to breaking the machine down into smaller redundant units which are operated in parallel. It is clear from the shape of these curves that, for initially unreliable machines ($F_o = 0.8$, 0.5), there is some improvement in reliability for the redundancy of entire machines, but for moderate redundancy, high reliability may be achieved only by increasing p. For initially reliable machines, on the other hand, ($F_o = 0.1$, 0.01), the great increase in reliability stems from the degree of redundancy, and is only secondarily affected by the level at which redundancy is applied.

For initially reliable machines ($F_o << 1$) with perfect failure detection and switching, we can derive an approximate expression for F_s as a simple function of m and p.

Starting with

$$R_s = [1 - (1 - R_o^{1/p})^m]^p$$

or

$$F_s = 1 - \{1 - [1 - (1 - F_o)^{1/p}]^m\}^p,$$

we assume that $[1 - (1 - F_o)^{1/p}]^m$ is small compared to one, and we neglect powers higher than the first. Then

$$F_s \cong p[1 - (1 - F_o)^{1/p}]^m.$$

Now, neglecting powers higher than the first in $F_o^{1/p}$, we get

$$(1-F_o)^{1/p} \cong 1 - \frac{F_o}{p},$$

so that

$$1-(1-F_o)^{1/p} \cong \frac{F_o}{p}$$

and

$$F_s \cong \frac{F_o^m}{p^{m-1}}.$$

Thus, for machines with high initial reliability the logarithm of F_s is approximately a linear function of the logs of F_o and p.

$$\log F_s \cong m \log F_o - (m-1) \log p.$$

Figures 11 and 12, in which F_s is plotted against p on logarithmic scales, confirm the nature of this relationship. The curves are approximately straight lines of slope $(m-1)$.

Tables 5-8 and Figs. 13-16 are concerned with imperfect switching and error detection. For this condition, we have made the following assumptions:*

1. All one-operation probabilities are independent of the assigned task, and we have set absolute values for them.
2. All probabilities of correct operation for the duration of the task are related to the initial reliability of the machine, R_o, (or the unreliability, F_o).
3. The failure detecting and switching equipment is far less complicated than a whole computer, so that the unreliability associated with these devices is several orders of magnitude less than the unreliability of a machine.

Thus:

$D_a = 1 - 10^{-3}$

$D_b = 1 - 10^{-2}(1-R_o) = 1 - 10^{-2}F_o$

$S_a = 1 - 10^{-4}$

$S_b = 1 - 10^{-4}(1-R_o) = 1 - 10^{-4}F_o$

$S_c = 1 - 10^{-4}(1-R_o) = 1 - 10^{-4}F_o.$

With these values set into Eq. 7, Tables 5 to 8 were calculated for the same values of F_o, m, and p, as in Tables 1 to 4. Figures 13 to 16 show these data plotted on logarithmic scales. The dotted lines represent $1-S_c^p$, the lower limit for possible values of F_s for any p.

In those cases where the final unreliability, F_s, with perfect error detection and switching, is relatively large, the modifications introduced by considering imperfect detection and switching are of a low order. However, when F_s approaches $1-R_c^p$ in magnitude, this factor begins to dominate the modified value. Thus, the curves follow an expected pattern.

*These assumptions are purely arbitrary and are presented as an example of an application of the method developed in this paper. They do not correspond to any existing equipment.

Table 1 **F_s as a function of p and m for F_o=0.8.** *(Perfect switching)*

p	$m=1$	$m=2$	$m=3$	$m=4$
1	0.8	0.640	0.512	0.410
2	0.8	0.518	0.309	0.178
3	0.8	0.433	0.200	0.087
4	0.8	0.372	0.138	0.047
6	0.8	0.289	0.076	0.0182
8	0.8	0.237	0.047	0.0088
12	0.8	0.174	0.024	0.0030
24	0.8	0.096	0.0065	0.0004

Table 2 **F_s as a function of p and m for F_o=0.5.** *(Perfect switching)*

p	$m=1$	$m=2$	$m=3$	$m=4$
1	0.5	0.250	0.125	0.0625
2	0.5	0.164	0.050	0.0147
3	0.5	0.122	0.026	0.0054
4	0.5	0.098	0.0160	0.0026
6	0.5	0.069	0.0078	0.00085
8	0.5	0.054	0.0046	0.00038
12	0.5	0.037	0.0021	0.00012
24	0.5	0.0193	0.00055	0.00002

Table 3 **F_s as a function of p and m for F_o=0.1.** *(Perfect switching)*

p	$m=1$	$m=2$	$m=3$	$m=4$
1	0.1	10.00×10^{-3}	10.00×10^{-4}	10.00×10^{-5}
2	0.1	5.26	2.70	1.39
3	0.1	3.57	1.23	0.43
4	0.1	2.70	0.70	0.183
6	0.1	1.82	0.32	0.055
8	0.1	1.37	0.179	0.023
12	0.1	0.92	0.080	0.0070
24	0.1	0.46	0.020	0.0009

Table 4 **F_s as a function of p and m for F_o=0.01.** *(Perfect switching)*

p	$m=1$	$m=2$	$m=3$	$m=4$
1	10^{-2}	10.00×10^{-5}	10.00×10^{-7}	10.00×10^{-9}
2	10^{-2}	5.03	2.52	1.27
3	10^{-2}	3.36	1.13	0.38
4	10^{-2}	2.52	0.63	0.159
6	10^{-2}	1.68	0.28	0.047
8	10^{-2}	1.26	0.159	0.0199
12	10^{-2}	0.84	0.071	0.0059
24	10^{-2}	0.42	0.0178	0.00074

We feel that the best failure detecting and switching techniques now in existence lead to higher reliabilities than the values we have assumed. Therefore, the actual behavior of redundant circuitry organized in the way we have described should lie somewhere between the case of perfect switching and the case of imperfect switching described here.

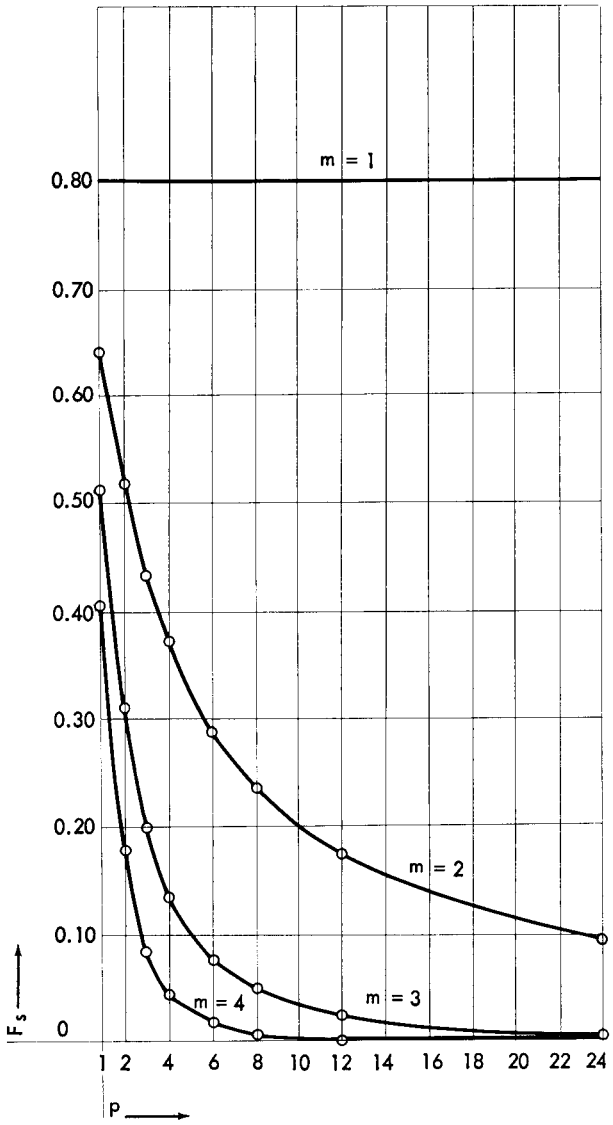


Figure 5 **F_s vs p, with $F_o = 0.8$.**

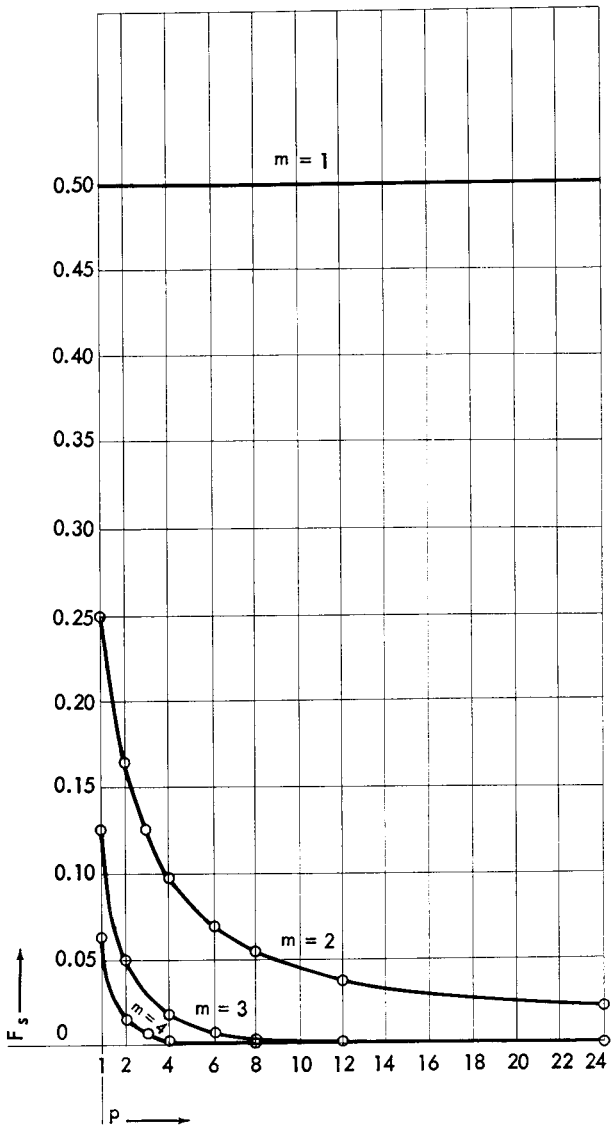


Figure 6 **F_s vs p, with $F_o = 0.5$.**

Table 5 **F_s as a function of p and m for $F_o = 0.8$.**
(Imperfect switching)

p	$m=1$	$m=2$	$m=3$	$m=4$
1	0.802	0.643	0.516	0.414
2	0.802	0.521	0.313	0.181
3	0.802	0.436	0.203	0.089
4	0.802	0.375	0.140	0.049
6	0.802	0.292	0.078	0.020
8	0.802	0.240	0.049	0.0100
12	0.802	0.176	0.025	0.0043
24	0.802	0.099	0.0088	0.0026

Table 6 **F_s as a function of p and m for $F_o = 0.5$.**
(Imperfect switching)

p	$m=1$	$m=2$	$m=3$	$m=4$
1	0.503	0.253	0.128	0.065
2	0.503	0.167	0.051	0.0156
3	0.503	0.124	0.027	0.0061
4	0.503	0.099	0.0168	0.0031
6	0.503	0.071	0.0084	0.00137
8	0.503	0.055	0.0052	0.00096
12	0.503	0.038	0.0029	0.00083
24	0.503	0.021	0.00185	0.00130

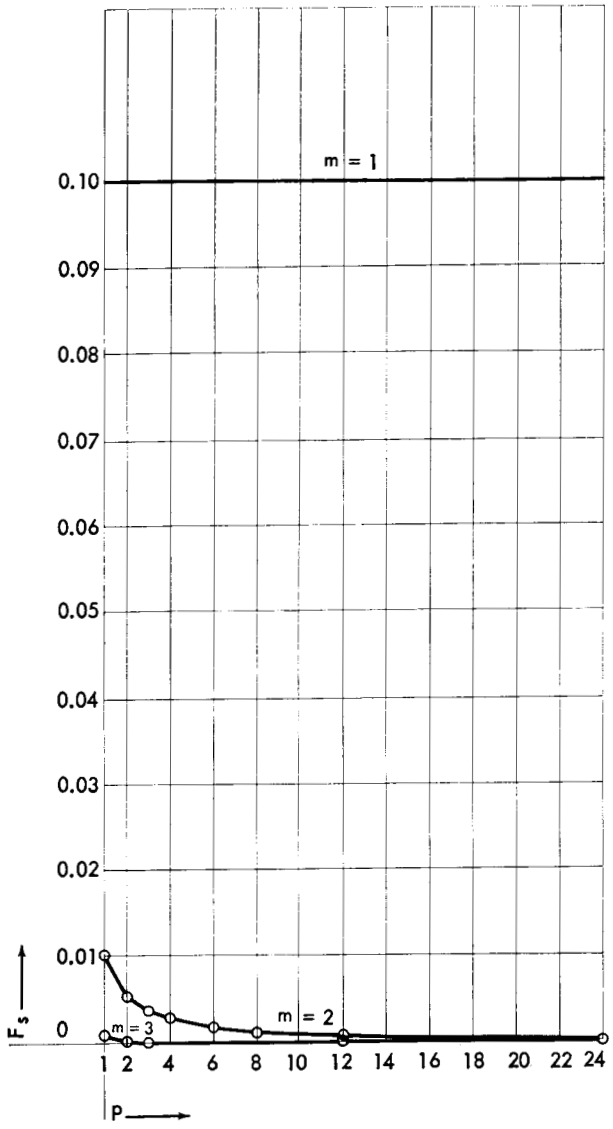


Figure 7 **F_s vs p, with $F_o = 0.1$.**

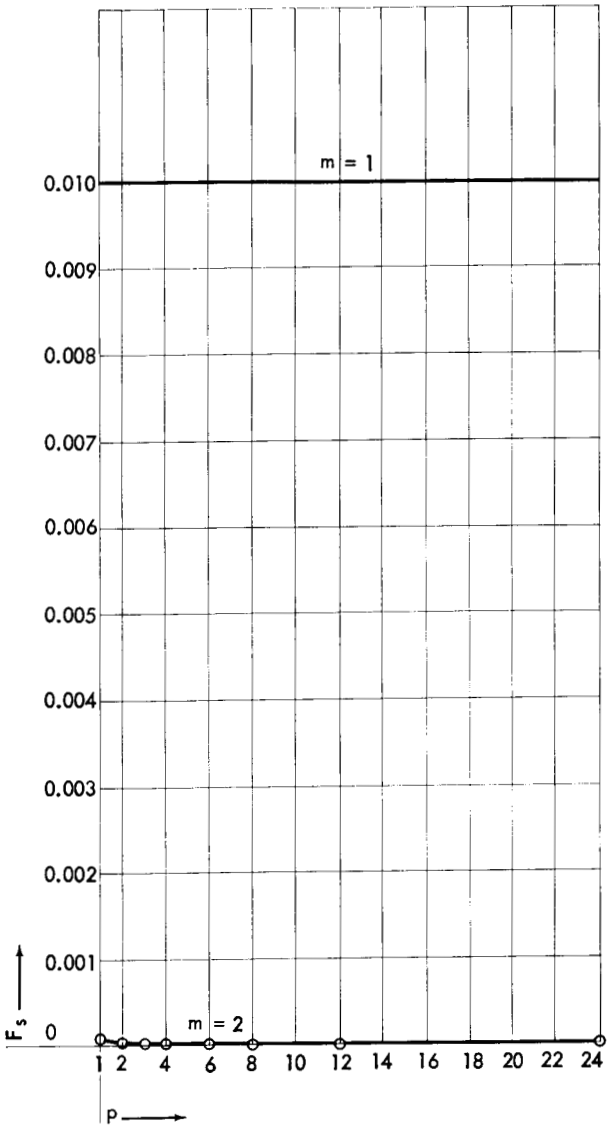


Figure 8 **F_s vs p, with $F_o = 0.01$.**

Table 7 **F_s as a function of p and m for $F_o = 0.1$.**
(Imperfect switching)

p	$m=1$	$m=2$	$m=3$	$m=4$
1	0.101	10.3×10^{-3}	11.6×10^{-4}	23.6×10^{-5}
2	0.101	5.44	3.62	9.8
3	0.101	3.72	2.09	8.8
4	0.101	2.84	1.61	9.1
6	0.101	1.95	1.31	10.1
8	0.101	1.51	1.28	11.1
12	0.101	1.08	1.54	14.2
24	0.101	0.73	2.53	25.3

Table 8 **F_s as a function of p and m for $F_o = 0.01$.**
(Imperfect switching)

p	$m=1$	$m=2$	$m=3$	$m=4$
1	1.01×10^{-2}	11.4×10^{-5}	13.1×10^{-6}	12.1×10^{-6}
2	1.01×10^{-2}	5.9	8.3	8.1
3	1.01×10^{-2}	4.2	7.5	7.4
4	1.01×10^{-2}	3.3	7.6	7.5
6	1.01×10^{-2}	2.6	8.7	8.7
8	1.01×10^{-2}	2.3	10.3	10.3
12	1.01×10^{-2}	2.2	13.9	13.9
24	1.01×10^{-2}	3.0	25.4	25.4

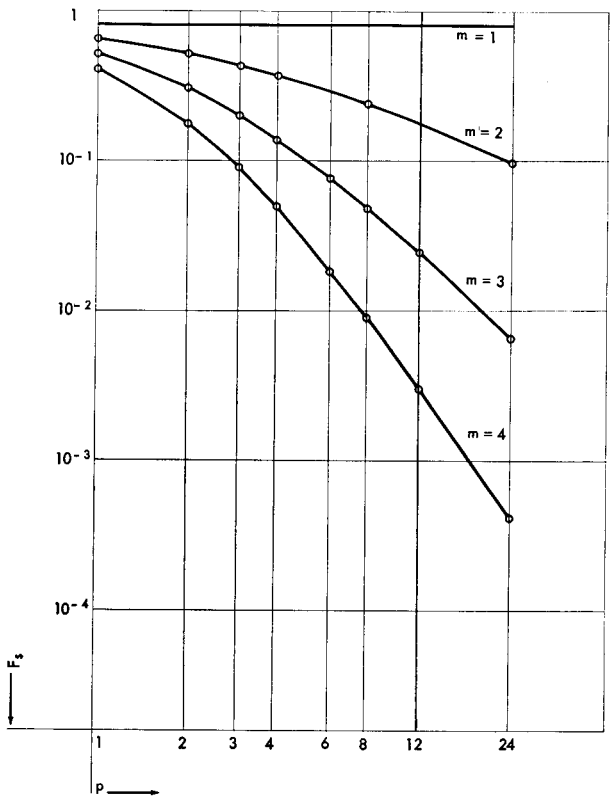


Figure 9 **F_s vs p, with $F_o = 0.8$. (Perfect switching.)**

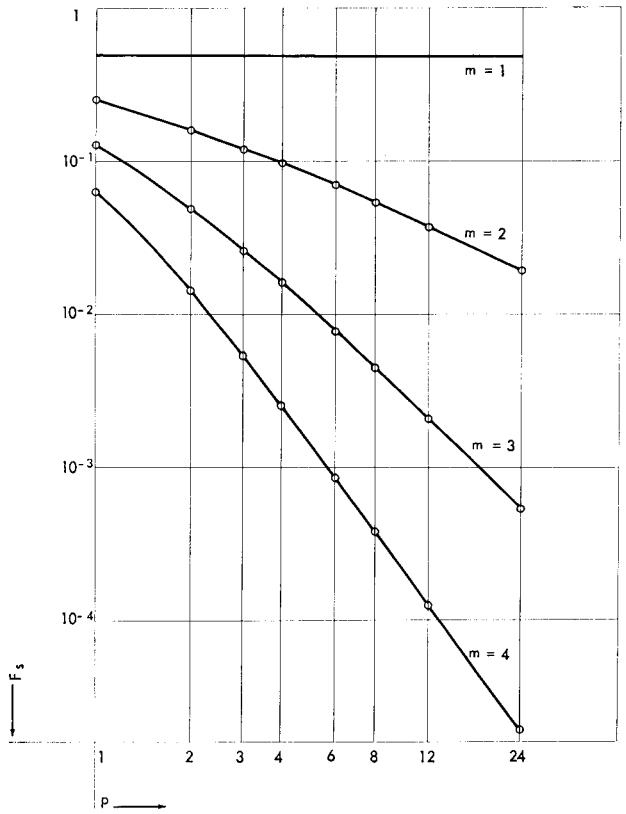


Figure 10 **F_s vs p, with $F_o = 0.5$. (Perfect switching.)**

Figure 11 **F_s vs p, with $F_o = 0.1$. (Perfect switching.)**

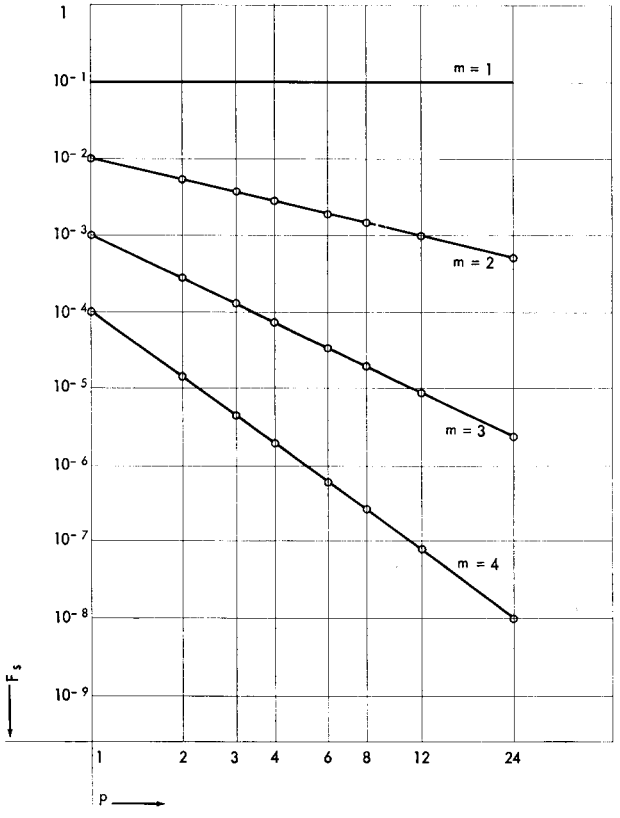
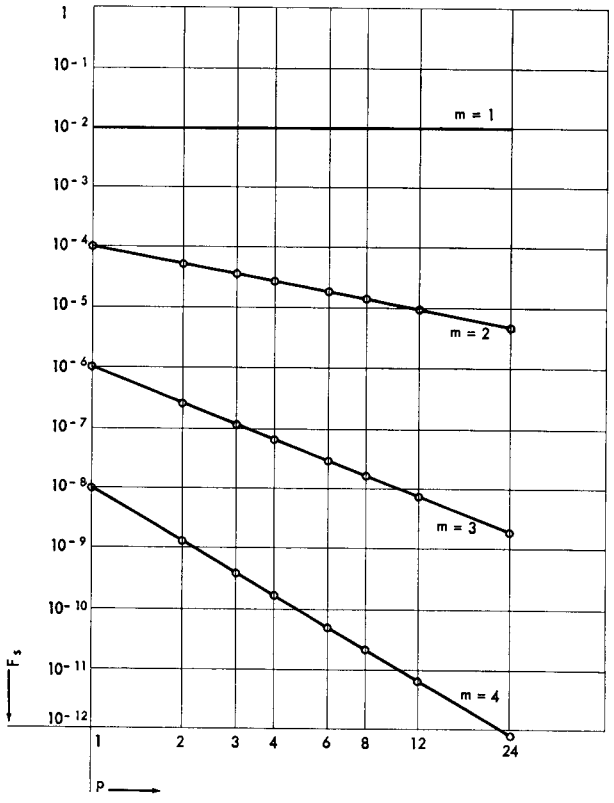


Figure 12 **F_s vs p, with $F_o = 0.01$. (Perfect switching.)**



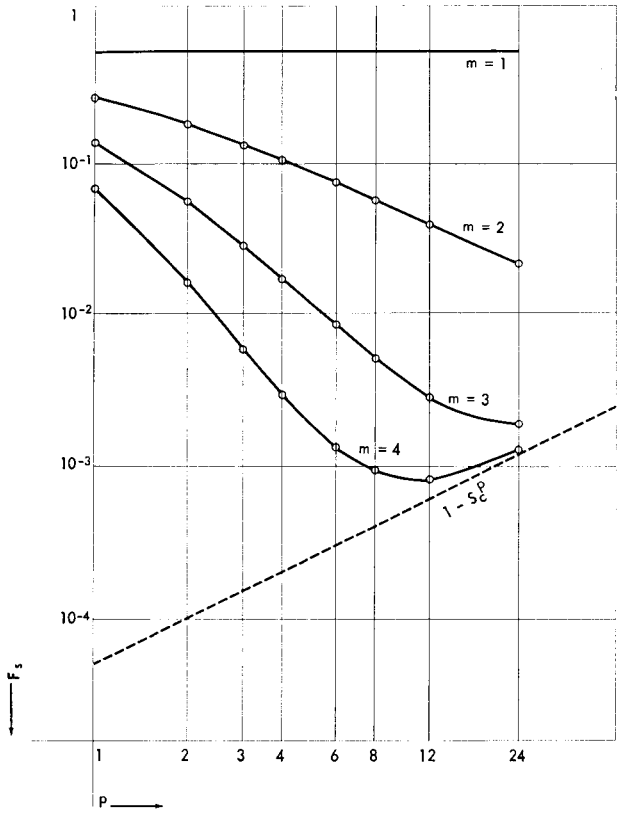


Figure 13 F_s vs p, with $F_o = 0.8$. (Imperfect switching.)

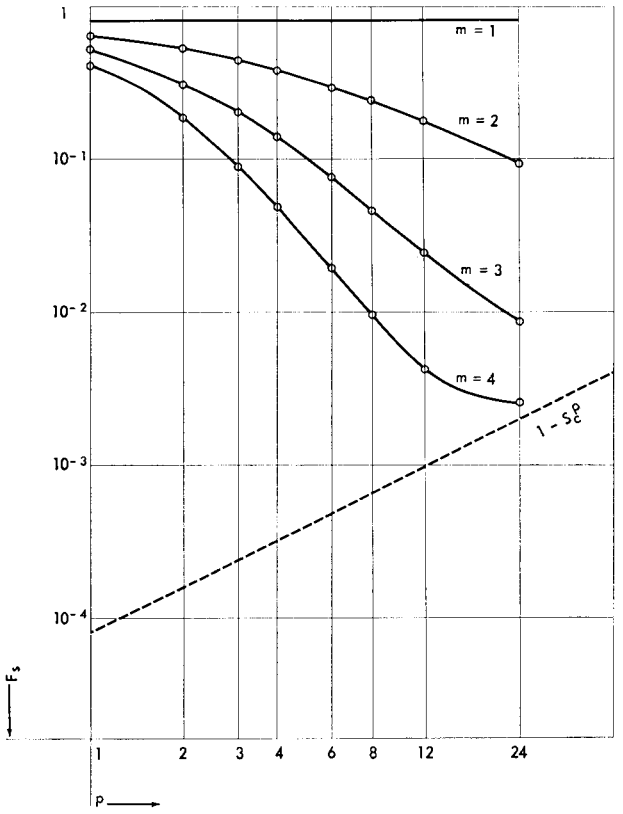


Figure 14 F_s vs p, with $F_o = 0.5$. (Imperfect switching.)

Figure 15 F_s vs p, with $F_o = 0.1$. (Imperfect switching.)

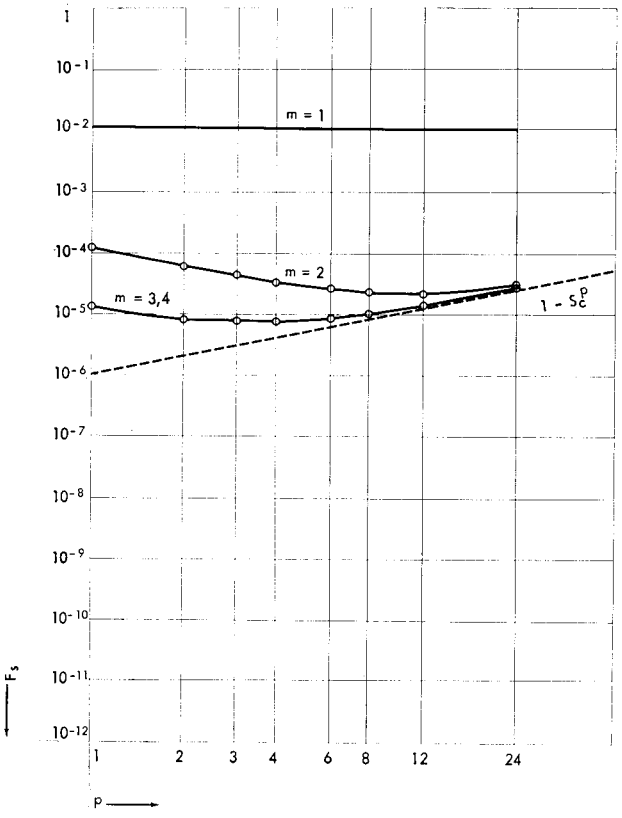
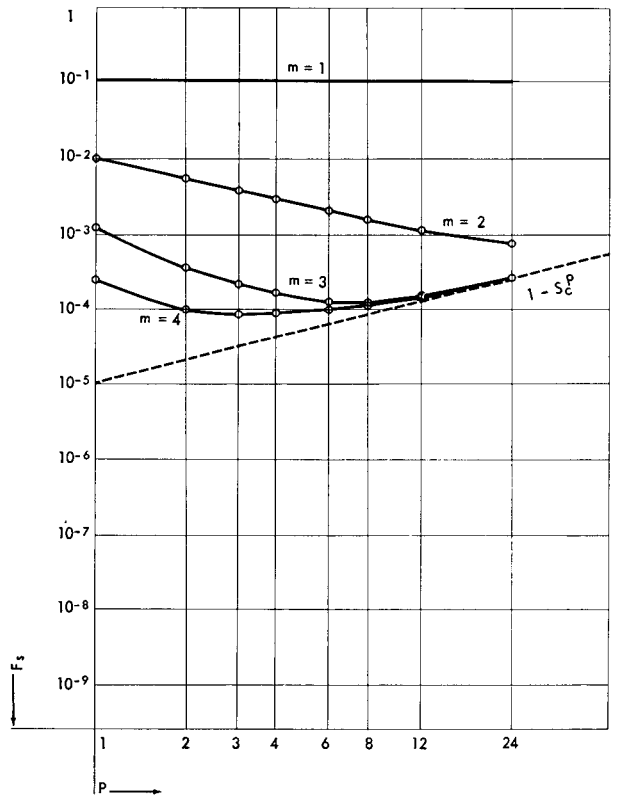


Figure 16 F_s vs p, with $F_o = 0.01$. (Imperfect switching.)



Summary of conclusions

Reliability improvement through redundancy has been analyzed with the aim of determining the optimum level at which redundancy should be applied. Clearly, from the point of view of economy, ease of maintenance, and ease of adaptation of existing equipment, it would be desirable to make the largest possible units redundant. The various curves may be used as follows: Given an estimate of the initial unreliability, F_o, of a non-redundant machine, and the tolerable unreliability, F_s, to be permitted in the final system, the user can judge from the appropriate curve the minimal combinations of degree of redundancy, m, and the number of chains, p, which will meet the specifications.

For initially unreliable machines and a moderate degree of redundancy, high reliability may be achieved only by applying the redundancy to relatively small units. However, for initially reliable machines, improvement stems primarily from the degree of redundancy rather than the level at which it is applied. Imperfect switching limits the reliability attainable in all cases. This limiting effect is most marked for initially reliable machines. With imperfect switching, the unreliability is not a steadily decreasing function of p, but has a definite minimum beyond which it increases.

Thus, any decision about the level at which a given degree of redundancy is to be applied must be carefully weighed in the light of initial reliability, cost, convenience, and the reliability of available switches and error detectors.

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