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A Radiant-Energy Heater Using an Ellipsoidal Reflector

Abstract: The effectiveness of a radiant-energy heater employing a hemi-ellipsoidal aluminum reflector and an incandescent lamp as radiation source is illustrated by application to the zone-melting of germanium. The factors affecting the design and performance of this heater are discussed.

Introduction

It is common knowledge that all the rays emitted by a point source at one focus of a perfect reflecting ellipsoid of revolution intersect at the other focus. With a reflector in the shape of a hemi-ellipsoid of revolution, this property can be utilized in concentrating into a small accessible region of space the energy radiated from an incandescent source. Such a radiant-energy heater has been previously described.[1,2]

The purpose of this paper is to point out the practicality of this heater as a laboratory device and to present the factors necessary for understanding and designing it. The effectiveness of a typical radiant heater is illustrated by its application to the zone-melting of germanium.

The main feature of the heater is that an object can be heated *locally,* even when it is completely enclosed except for a transparent window. The only other common laboratory method of heating with these properties is induction heating. In comparison with induction heating, the radiant-energy heater is simple and cheap to construct, heats dielectrics, and does not electrically or mechanically interact with the heated object or its surroundings. On the other hand, radiant-energy heaters are bulky, and therefore only a few can be used in a given area; also, any reduction in transparency of the window reduces the heating efficiency. It is noteworthy that the heat source can rapidly change temperature, and therefore permits excellent temperature control of the heated object by simple means.

Heater design and performance

The geometrical optics of a perfect hemi-ellipsoidal reflector with a source of finite extent centered at the inner focus will be considered. In order to reflect an appreciable portion of the power radiated by the source, it is necessary to have a reflector which subtends a large solid angle (more than 2π) at its inner focus. In this case, a linear magnification of the finite source cannot be defined. Since this magnification varies with the angle between an emitted ray and the symmetry axis of the reflector, no image of the source is actually formed at the outer focus. It is therefore impossible to understand the optics in terms of the behavior of paraxial rays.

For a given eccentricity and ratio of source size to semi-major axis, a certain fraction, f, of the energy radiated by the source is intercepted by the ellipsoid and reflected towards the outer focus. For a very small source with axial symmetry at the inner focus, the loci of constant intensity in a plane containing the outer focus and perpendicular to the symmetry axis are concentric circles about the outer focus. Letting d be the diameter of the circle through which half the reflected radiation passes, the performance of the heater for localized heating can then be represented by a figure of merit, f/d^2.

The variation of the figure of merit with eccentricity for a reflector with a projection-lamp filament as source can be understood by considering a small line source centered at the inner focus and lying along the axis of symmetry. From this model (see Appendix I), the figure of merit decreases monotonically as the eccentricity increases from 0.1 to 0.9, the rate of change becoming more rapid as the eccentricity increases. Note that the choice of eccentricity is also governed by the particular

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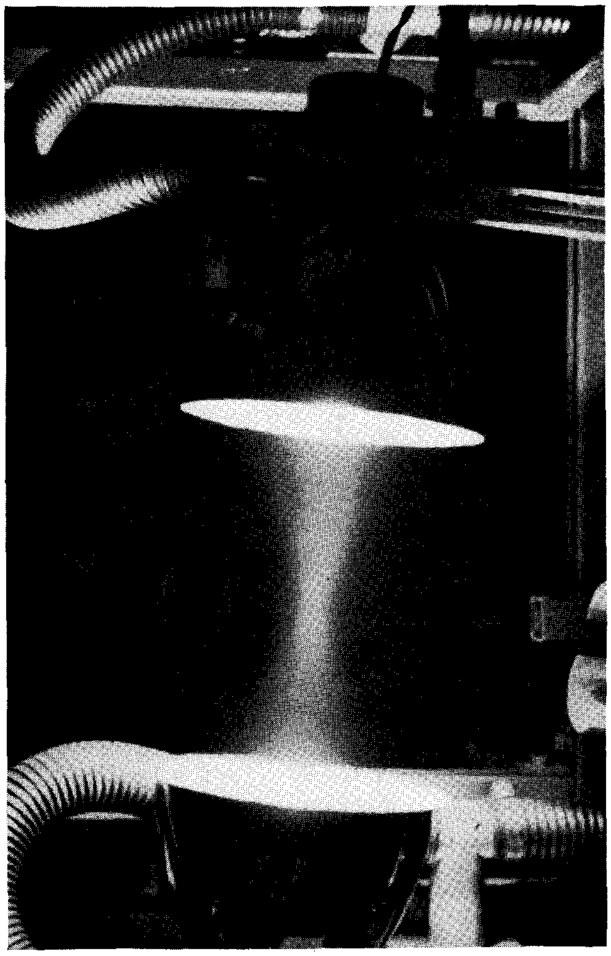


Figure 1 **Two one-kilowatt radiant-energy heaters, showing distribution of radiation flux.**

application. For example, to place three heaters about a given point, the eccentricity must be greater than 0.5, and for four heaters, it must be greater than 0.7. The lamp-size and the minimum possible value of the working distance (half the interfocal distance) will be predetermined by the application. Therefore the eccentricity should be reduced to the point where the overall dimensions of the reflector are as large as can be accommodated.

In practice, even for a point source, d is not zero because the reflector surface deviates from the ideal both in its over-all geometry and in the roughness of the surface. Most of this effect comes from the small local irregularities (surface roughness produced e.g., by spinning) rather than an over-all deviation from ellipsoidal shape.

A reflector used in the experiments described in this paper was spun out of aluminum on a wooden form and has a semi-major axis of 162 mm and an eccentricity of 0.778.

A 1000-watt General Electric incandescent lamp, either Type PH/1M/T12/P which burns base down or Type PH/1M/T12/1 which burns base up, was used as a source. These lamps have a filament about 10 × 10 mm in size which consists of a planar mat of helical coils. Two of these heaters, mounted in a vertical position diametrically opposite each other, are shown in Fig. 1. Smoke is used to show the path of the radiation flux between the two heaters. A detailed diagram of the type of heater used in Fig. 1 is shown in Fig. 2.

In Fig. 3, Curve (a) represents the energy flux distribution produced by one of these heaters along a line through the outer focus which is perpendicular both to the symmetry axis and the plane of the filament. Curve (b) represents the distribution along a line through the outer focus perpendicular to the symmetry axis and parallel to the plane of the filament. Curve (c) represents the distribution parallel to the plane of the filament along the symmetry axis. These curves were calculated from the measured equilibrium temperature attained by a 3 mm graphite cube suspended in a long evacuated quartz tube by means of Brown and Sharpe No. 36 thermocouple wires. The energy loss from the block is predominantly by radiation. (The loss of heat by conduction down the support wires rises to only 10% of the radiation loss at a distance of 60 mm from the point of maximum intensity.)

Comparing Curve (a) or (b) with Curve (c), it is apparent that the positioning of the heated object *along* the symmetry axis is less critical than adjustments *perpendicular* to it. Because of the axial asymmetry of the projection lamp filament, it would not be expected that, for a perfect reflector, the curves of constant intensity would be circles in the plane perpendicular to the symmetry axis at the outer focus. However, the half-power widths of Curves (a) and (b) are the same, indicating that the effect of the surface irregularities has tended to mask the asymmetry.

The effect of surface irregularities on the half-power diameter d can be separated from the effects of the finite size of the source by using successively smaller sources and computing d from curves similar to Fig. 3. Filaments of 10 × 10, 5 × 5, and 2 × 2 mm in size produced values of d of 14, 8 and 7 mm, respectively. From this, it is inferred that d approaches a residual value of about 7 mm for a point source. This is interpreted to be due to the reflector surface imperfections which are about 0.002 to 0.003 in. rms for a spun and polished reflector. Thus even for the 10 × 10 mm filament, a large contribution to the value of d arises from the surface imperfections which play an important role in determining the energy flux distribution.

For this heater, it is estimated that only about 28% of the input power to the source is included within the half-power circle. This is based upon a value of f of 0.94 (see Equation (3) in Appendix I), a reflectivity[3] of the aluminum reflector of 0.69, and a 13% power loss[4] in the incandescent lamp itself. Details of heater construction and adjustment are discussed in Appendix II.

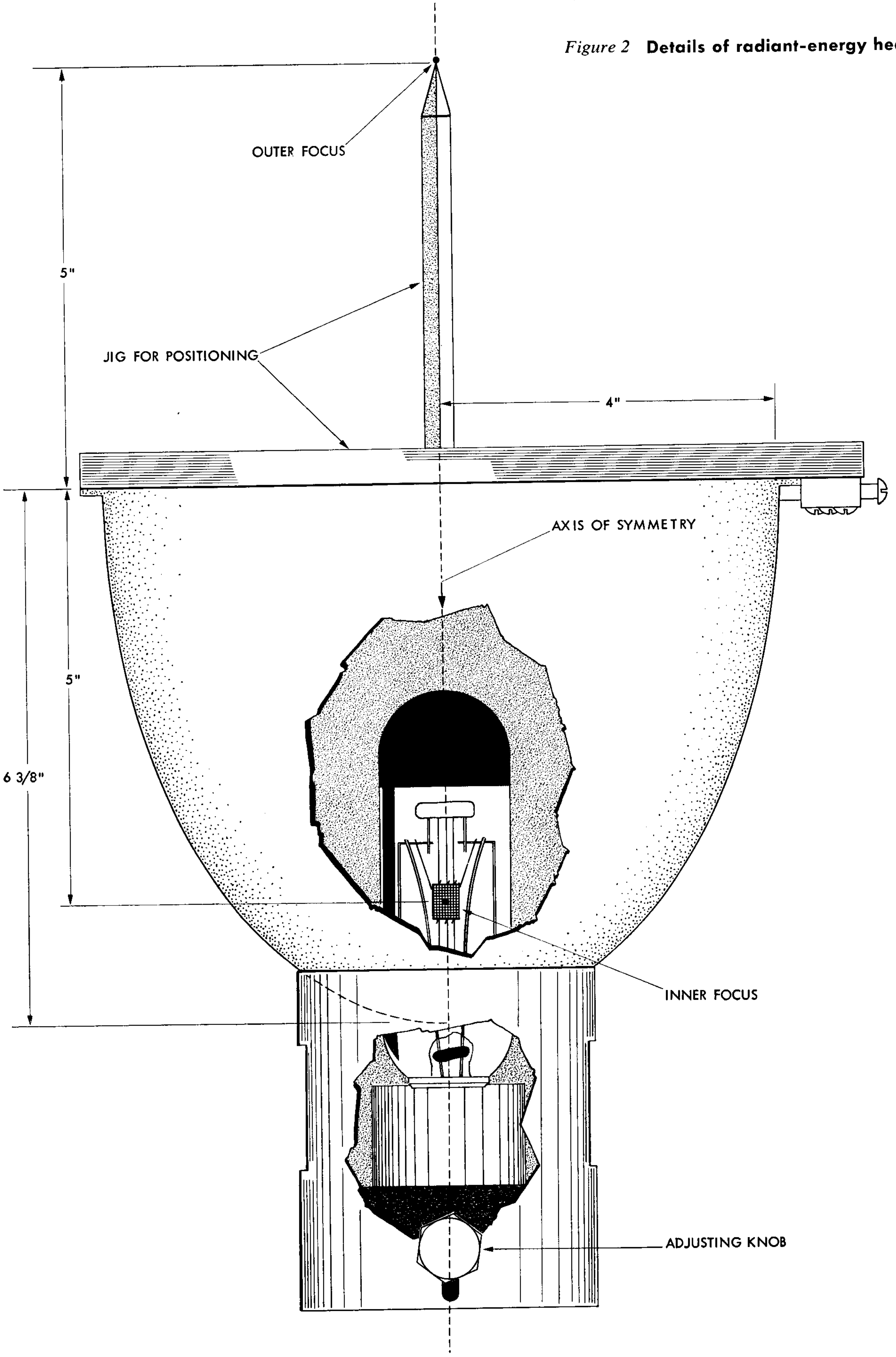


Figure 2 **Details of radiant-energy heater.**

Applications

- *Horizontal zone refiner*

In using the radiant-energy heater in a horizontal-zone refiner, a germanium ingot contained in a carbon-blacked quartz boat is mounted on thin molybdenum supports inside a quartz tube maintained at a vacuum of about 10^{-6} mm Hg. The outside temperature of the tube is held below 300°C by an air blast from a centrifugal blower. Two boat shapes have been used. The first was 80 mm long and had a trapezoidal cross section, 15 mm wide on top, 11 mm wide on bottom and 15 mm high. Four heaters, spaced 90° about this boat, were necessary to obtain uniform heating and sufficient power to completely melt a zone in the ingot.

To determine the size of the molten zone in the interior of this ingot during a pass, a quartz tube containing a thermocouple was inserted along the axis of the ingot with the junction at its geometric center. This showed that the solid-liquid interface was convex into the liquid. From this type of measurement the zone is no longer molten throughout when the outside zone length, as observed visually, is less than about 8 mm.

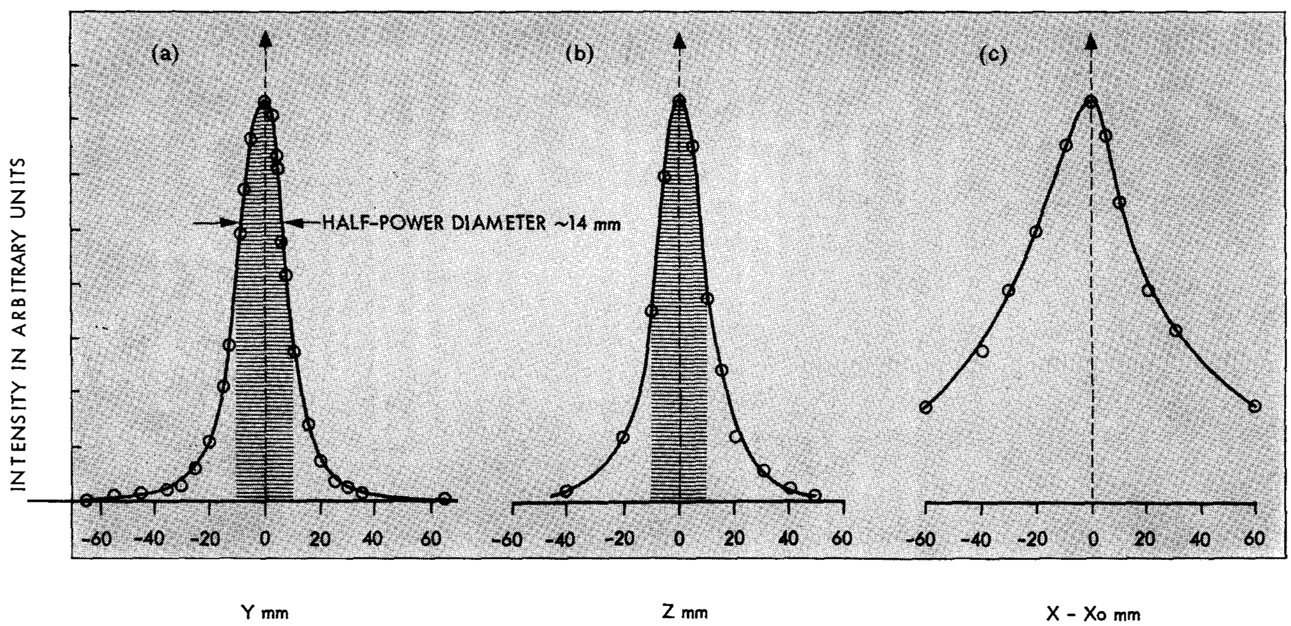
The second boat had a cross section in the shape of a half-cylinder, 19 mm in diameter and 170 mm long. Two heaters, placed above and below the ingot (see Fig. 1), sufficed to melt a zone that was 16 mm wide at the center of the surface, and 14 mm wide at the edge. Figures 4 (a) and (b) show zone length measured at the edge as a function of input power for the two boat shapes.

Because one zone reduces the heat loss of an adjacent zone, two zones can be easily maintained in an ingot in the cylindrically shaped boat with four heaters. To permit smaller zone separation, each reflector is partially cut away, 38 mm from the rim, parallel to the symmetry axis, perpendicular to the plane of the filament. This results in a loss of power of about 10% per heater. The zone separation is varied either by tilting the symmetry axes of the heaters, or by adjusting the input power. The length of one zone as a function of input power, with each zone center 57 mm from the end of the boat, is shown in Fig. 4 (c). Two additional side heaters are used in the apparatus with their heat directed at the final solid-liquid interface. This produces a planar interface, which improves the resultant crystal structure of the ingot.[5]

It is to be noted that since the lamps have a short lifetime when burned horizontally, they are introduced into the reflector from the side when the symmetry axis is horizontal, and from the end when the symmetry axis is vertical.

The deposition of both germanium dioxide and evaporated germanium from the molten zone onto the cool outer quartz tube markedly reduces heating effectiveness within two passes to the point where it is impossible to completely melt a zone. This deposition can be reduced by enclosing the boat in an inner quartz tube about 50% longer than the boat and having an inside diameter slightly larger than the boat diameter. This tube, which is in contact with the boat but is otherwise thermally isolated, runs at a sufficiently high temperature

Figure 3 **Radiant-energy flux distribution at heater's outer focus as measured:**
a) Perpendicular to the symmetry axis and filament plane.
b) Perpendicular to the symmetry axis and parallel to the filament plane.
c) Along the symmetry axis.



near the zone to inhibit the formation of the opaque film. This permits twelve or more passes to be made before cleaning becomes necessary. The use of a gas ambient to minimize filming would increase the heat loss and thus necessitate the use of more than two heaters per zone.

- *Floating zone refiner*

With a germanium rod, 250 mm long and 5 mm in diameter, enclosed in a quartz tube in a high vacuum, it is possible to produce a floating zone 5 mm long with two heaters mounted in a horizontal position diametrically opposite each other. The radiant flux from each heater is collimated by a slit 15 mm wide, and the radiation missing the germanium rod is reflected back onto it with two curved mirrors on the sides where there is no incoming radiation. The required total input power to both heaters is 1500 watts. A pass is made with this apparatus by moving the quartz tube containing the germanium rod relative to the stationary heaters. The deposition of an opaque film on the quartz tube requires cleaning after three consecutive passes.

- *Other applications*

The following applications illustrate the versatility of the ellipsoidal radiant-energy heater: heating an object between the pole faces of a magnet; heating an object under a microscope; heating barium titanate while observing its interaction with gases; vacuum evaporation of metals; zone refining of intermetallic semiconductors; and heating organic compounds in a molecular still. With a single heater, potassium niobate has been heated to the melting point in an air ambient during X-ray diffraction studies. In England, organic compounds have been zone-melted by radiant heating using parabolic reflectors.[6]

Appendix I

We shall consider qualitatively the variation of the figure of merit (f/d^2) with both eccentricity, ε, and ratio of source size to semi-major axis for a perfect hemi-ellipsoidal reflector. For simplicity, we consider a line source radiating uniformly along its length centered at the inner focus of the reflector and lying along the symmetry axis. The geometry is shown in Fig. 5.

Let y be the deviation from F' in the plane D of a reflected ray emitted from a point on the source a distance ξ from F. Then

$$y = \frac{\mu(\theta,\varepsilon)\xi}{1-h(\theta,\varepsilon)\xi/a} \qquad (1)$$

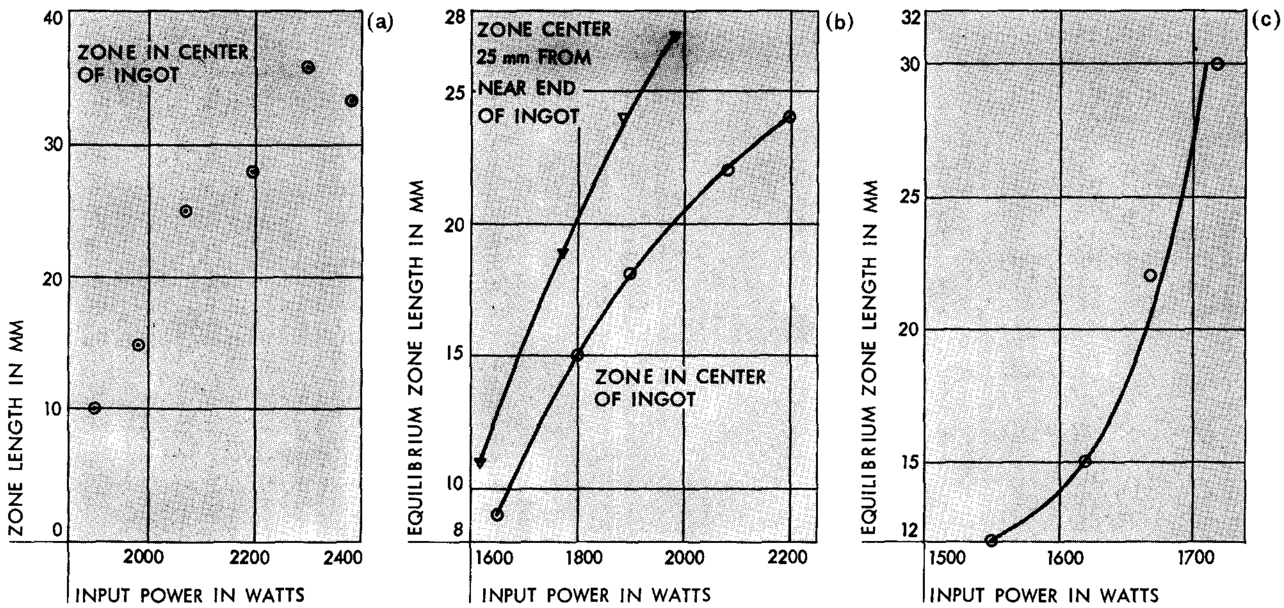
where:

$$\mu(\theta,\varepsilon) = \frac{[(1+\varepsilon^2)-2\varepsilon\cos\theta]^2\sin\theta}{(1-\varepsilon^2)[2\varepsilon-(1+\varepsilon^2)\cos\theta]}$$

$$h(\theta,\varepsilon) = (1-\varepsilon\cos\theta)\left[\frac{\cos\theta}{1-\varepsilon^2}+\frac{\sin^2\theta}{2\varepsilon-(1+\varepsilon^2)\cos\theta}\right].$$

The magnitude of the half-power diameter d will depend on how closely the reflected rays are clustered about the outer focus: that is, on the typical values of y.

Figure 4 **Length of a molten zone as a function of input power to the heaters for:**
a) A boat with trapezoidal cross section and four heaters.
b) A boat with a half-cylindrical cross section and two heaters.
c) A boat with a half-cylindrical cross section with two zones and four heaters.



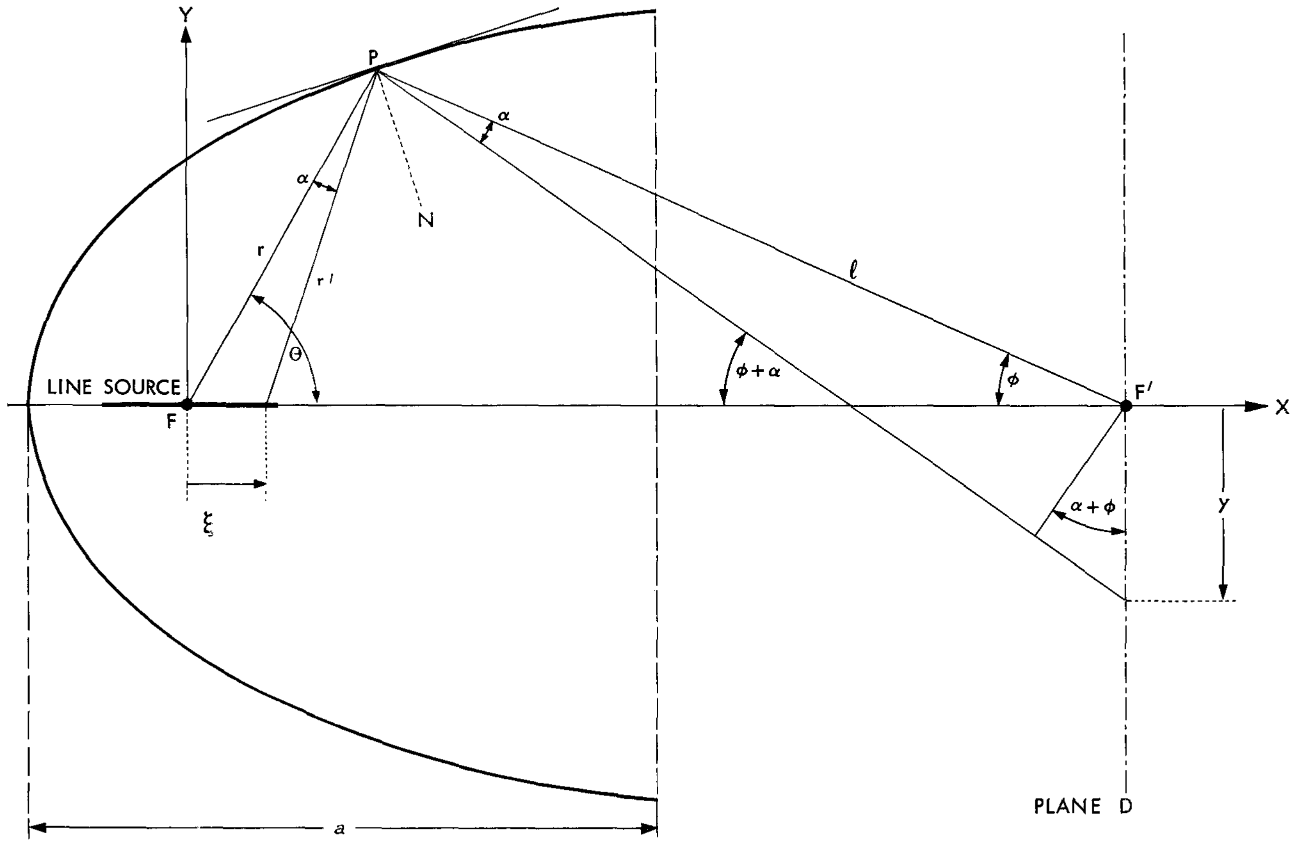


Figure 5 **Ray diagram for reflecting hemi-ellipsoid and a line source centered at inner focus.**

For a hemi-ellipsoid, $\cos\theta \leqslant \varepsilon$ and therefore $h(\theta, \varepsilon)$ in Eq. (1) is always finite.* Some representative values of the maximum value of h for a given ε are:

ε	0.1	0.5	0.8	0.9
$\lvert h\rvert_{\max}$	5	2	5	6

Thus for an eccentricity of 0.8, the error in y introduced by neglecting $h(\theta, \varepsilon)\xi/a$ in Eq. (1) will be about 15% if the largest value of ξ/a is 0.03. For the reflector described in the text, these conditions are realized. We therefore take

$$y = \mu(\theta, \varepsilon)\xi. \tag{2}$$

We divide the source into s intervals and consider r rays emitted from each interval. Then using Lambert's law for radiation exchange between an interval of source length, $\Delta\xi$, and an annular area of width Δy, a distance y_k from F' in plane D, the energy flux passing through the interval Δy is proportional to

$$\sum_r{}' \sum_s{}' \Delta\xi_s[\sin\theta(\mu, \varepsilon)]_{rs}\Delta y[\cos\phi(\mu, \varepsilon)]_{rs}$$

where: $y_k = (k+\frac{1}{2})\Delta y$; $k = 1, 2, \ldots, j$, and the primes denote summation over only those rays from the source passing through the interval Δy.

Rays emitted at both large and small values of θ make the least contribution to the intensity distribution as a result of Lambert's law. Thus the main contribution comes from rays emitted in a range of angles between about $\theta = 90°$ to $\theta = 150°$ for any ε. It can be seen from Eq. (2) that within this range of θ, all values of y increase monotonically with ε with the greatest increase occurring at large ε. From this it can be concluded that d increases monotonically with ε and increases more rapidly at large ε.

There is a maximum value of y as a function of θ for each ε, so that the energy flux is distributed over only a finite region in plane D for a given ε and ξ (as a result of the surface irregularities in the experimental reflector, there will be radiation present beyond the limit established by ε and ξ). It can also be seen from Eq. (2) that d will be independent of the absolute size of the reflector so that d increases with ξ for a given ε.

The interception factor, f, is the integrated solid angle, weighted by $\sin\theta$, subtended by the reflector at the source normalized to unity total intensity. For a hemi-ellipsoid and a line source of infinitesimal length,

*In fact, h is always finite unless the amount of ellipsoid surface covered by the reflector exceeds a hemi-ellipsoid by a certain extent which depends on ε.

$$f = 1 - [\arc\cos \varepsilon - \varepsilon\,(1-\varepsilon^2)^{\frac{1}{2}}]/\pi. \quad (3)$$

From these arguments, it is concluded that the figure of merit decreases monotonically with increasing ε and is independent of absolute reflector size. The most rapid decrease occurs for large eccentricities. It is estimated that the figure of merit decreases by an order of magnitude from an eccentricity of 0.5 to one of 0.9.

On this basis, the best performance is obtained by choosing the smallest ε consistent with the accessibility of the object to be heated, the space available for the heater, and the power required.

These results are intended as a guide for the case of an actual heater using a projection-lamp source. A calculation for a finite planar source,[7] which is a closer idealization of the projection lamp filament, is not justified because the lack of perfection of the reflector surface, at the present stage of development, has a greater influence on performance than the finiteness of the source dimensions.

Appendix II

The reflector used in the experiments described in this paper was spun from 0.064 in. 2SO aluminum sheet on a wooden form with a 3/8 in. lip at the edge for rigidity, and then polished on the inner surface by a contractor at a cost of $6.00 a unit not including the cost of the spinning form. For a slight additional cost a reflector having a smoother and high-quality surface could be made by spinning it on a steel form and using an "alzak" finish on the reflecting surface. "Alzak" has a total reflectivity of 0.79 to 0.83 for an incandescent source.[3] This compares favorably with polished silver.

For the two 1000-watt GE lamps mentioned, the radiation intensity decreases markedly after about 10 hours at full power owing to the blackening of the glass envelope. The filaments did not burn out, however, until about 20 hours or so at this power with air cooling, which is essential to achieve even this lifetime. The usable lifetime is greatly lengthened by running at reduced power.

The lamp filament must be located so that its geometric center coincides with the inner focus of the reflector. The filament can deviate from the centerline of the base in the 1000-watt GE lamp by 0.8 mm within manufacturing tolerances; but because the surface imperfections account for such a large part of the half-power diameter in the reflector, it is sufficient to allow motion for adjustment along only the symmetry axis. This adjustment is made by moving the lamp, glowing dimly at low power, until a minimum spot size is obtained on a piece of paper held at the outer focus. Then the heater as a unit is positioned with a jig (shown in Fig. 2) so that the object to be heated is at the outer focus. This jig is particularly useful if one or more pairs of heaters are to be positioned. This "focusing" method is much quicker than positioning the heater unit to obtain a maximum temperature in the object to be heated and produces very nearly as good a result.

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