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Determination of Transient Response of a Drift Transistor Using the Diffusion Equation

A mathematical model of a drift transistor has been constructed which enables us to study the time required for collector current to reach a steady value in response to a step function of current introduced at the emitter. IBM germanium drift transistors of the PNP type are being investigated. Various base widths and resistivities are being considered. The models devised are suitable for computation by electronic computers.

The basic equations used are the diffusion equation,

$$\vec{I_p} = q\mu_p p\vec{E} - qD_p \vec{\nabla p}, \qquad (*)$$

and the continuity equation,

$$\frac{\partial p}{\partial t} = \frac{p_n - p}{\tau_p} - \frac{1}{q}\nabla \vec{I_p} + g'_p \qquad (*)$$

where I_p is the current due to holes, q is the charge of an electron $= 1.5921 \times 10^{-19}$ coulombs, D_p is the diffusion constant for holes in germanium $= .02588\mu_p$, μ_p is the mobility of holes in N-type germanium, p_n is the thermal equilibrium concentration of holes in N-type material, τ_p is the lifetime of holes in N-type material, and g'_p is the rate of generation of holes in N-type material, by other than thermal effects.

A one-dimensional model of the transistor was adopted. This assumption reduces the above equations to

$$I_p = q\mu_p pE - qD_p \frac{\partial p}{\partial x} \qquad (1)$$

and

$$\frac{\partial p}{\partial t} = \frac{p_n - p}{\tau_p} - \frac{1}{q}\frac{\partial I_p}{\partial x} + g'_p \qquad (2)$$

Differentiation of (1) produces the equation,

$$\frac{\partial I_p}{\partial x} = q\mu_p \left(\frac{\partial p}{\partial x} E + \frac{\partial E}{\partial x} p\right) - qD_p \frac{\partial^2 p}{\partial x^2}$$

Substitution for $\frac{\partial I_p}{\partial x}$ in (2) results in

$$\frac{\partial p}{\partial t} = \frac{p_n - p}{\tau_p} - \mu_p \left(\frac{\partial p}{\partial x} E + \frac{\partial E}{\partial x} p\right) + D_p \frac{\partial^2 p}{\partial x^2} + g'_p$$

which can be written

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - \left(\mu_p \frac{\partial E}{\partial x} + \frac{1}{\tau_p}\right) p + \frac{p_n}{\tau_p} + g'_p \qquad (3)$$

Hole generation by other than thermal means is not considered, and therefore the g'_p term is deleted.

While in a uniform base transistor the electric field in the base is zero, in a drift transistor there is a field due to the gradient of the impurity concentration. This field is defined as

$$E = -\frac{\partial \psi}{\partial t} = -\frac{kT}{q}\frac{1}{N}\frac{\partial N}{\partial x} \qquad (4)$$

where N is the concentration of donor impurities in the base, T is Kelvin temperature, and k is Boltzmann's constant.

Donor impurities are diffused into the originally P-type germanium by exposing a surface of the germanium block to an atmosphere of vaporized impurity under controlled conditions of pressure and temperature. The donor concentration at a distance x from the diffusion surface is a function of diffusion temperature, the elapsed time and the concentration of the donor impurity at the diffusion surface. It is expressible as a complementary error function,

$$N(x, T) = N_o \operatorname{cerf} \frac{x}{2\sqrt{D_o T}} \qquad (5)$$

where x is in centimeters, N_o is the vapor concentration of the donor impurity at the surface of the germanium

*Shockley, William: *Electrons and Holes in Semiconductors,* D. Van Nostrand Company, Inc. New York, 1950, pp. 298-299.

material in units per cc, D_o is the donor diffusion constant and T is the time in hours.

In order to determine the position of the collector junction relative to the diffusion surface, we must find the point at which the concentration of diffused N-type impurity equals the net acceptor concentration of the original P-type germanium. This hole concentration is defined by the equation $p = 1/\mu_p \rho q$ where ρ is the resistivity of the original P-type germanium. The equation,

$$N_o \operatorname{cerf} \frac{x}{2\sqrt{D_o T}} = \frac{1}{\mu_p \rho q}$$

is solved for x by the computer, using an iterative process. The emitter junction is found by subtracting the base width from the value of x just computed. Having determined this junction, we must consider the effect of the depletion region surrounding the collector junction since this depletion region reduces the effective width of the base. The width of this region is a function of the collector bias and may be computed from the equation $\nabla E = \frac{q}{\varepsilon}\rho$ where E is the field due to collector bias, q is electronic charge, ρ is charge density, and ε is the dielectric constant. Solution of this equation for a one dimensional model will give a value for the width of the depletion region in both the base and collector regions.

The part of the depletion region extending into the base must then be subtracted from the base width to find the position of the collector junction under conditions of bias. This base region is then divided into n equal intervals bounded by the $n+1$ points, $x_0, x_1, \ldots, x_w (w = n$, and $x_w =$ position of the collector junction). The net impurity concentration $N(x_1) - N_a$, where N_a is the acceptor concentration of the undiffused germanium crystal, is determined for each of these points.

In equation (3) D_p is a function of μ_p, ($D_p = \frac{kT}{q}\mu_p$) where T is the Kelvin temperature of the transistor (this was assumed to be room temperature), and μ_p is a function of the net N-type impurity concentration in the base. Therefore both D_p and μ_p are functions of the x_i defined above. A curve for the dependency of μ_p upon the net donor concentration is available as a result of experimental measurements, and a polynomial was fitted to the curve. The relationship is

$$\mu_p = [6.78 \times 10^{-31} N^2(x_i) - 5.06 \times 10^{-14} N(x_i) + 1.85 \times 10^3]\ cm^2/volt\text{-}sec$$

Boundary conditions were determined as follows: At the time the square current wave is introduced at the emitter there is no hole current at either the emitter junction or the collector junction. This neglects the small collector current due to the collector bias. The hole concentration at the collector junction is defined to be zero, since the collector is assumed to be a perfect sink for holes. The holes elsewhere throughout the base are so distributed as to counterbalance the charges resulting from the diffused N-type impurity atoms. However this initial hole concentration is small and in this computation is assumed to be zero.

After the input pulse is introduced, we make the assumption that holes are crossing the emitter junction ($x = 0$) at a constant rate. Equation (3) is of the form

$$\frac{\partial p}{\partial t} = A\frac{\partial^2 p}{\partial x^2} + B\frac{\partial p}{\partial x} + C_p + D$$

where A, B, C are functions of x ($\frac{p_n}{\tau_p}$ and g'_p are not considered and are omitted in subsequent computation). Using standard difference substitutions for the derivatives, we write

$$\begin{aligned} p(x_i, t_{j+k}) = k\Bigg[&\left(\frac{A(x_i)}{h^2} + \frac{B(x_i)}{2h}\right) p(x_{i+h}, t_j) \\ &+ \left(\frac{1}{k} + C(x_i) - \frac{2A(x_i)}{h^2}\right) p(x_i, t_j) \\ &+ \left(\frac{A(x_i)}{h^2} - \frac{B(x_i)}{2h}\right) p(x_{i-h}, t_j)\Bigg] \end{aligned} \quad (6)$$

where h is the interval of distance $x_{i+1} - x_i$, and k is the increment of time. The base is divided into n intervals by the points $x, x_1, \ldots, x_n$ and the equation is solved for the hole concentrations throughout the base region at the successive time intervals $t_0, t_1, \ldots, t_m$ where $t_j = t_0 + j_k$. The size of n is dictated by the degree of accuracy desired in the solution of the differential equation while m is not assigned since it is determined by the length of time required for the collector current to reach a constant value. A different notation transforms (6) into the form

$$\begin{aligned} p_{i,j+1} = k\Bigg[&\left(\frac{A_i}{h^2} + \frac{B_i}{2h}\right) p_{i+1,j} \\ &+ \left(\frac{1}{k} + C_i - 2\frac{A_i}{h^2}\right) p_{i,j} \\ &+ \left(\frac{A_i}{h^2} - \frac{B_i}{2h}\right) p_{i-1,j}\Bigg] \end{aligned} \quad (7)$$

Using the conditions $p_{i,o} = 0$ ($i = 1, 2, \ldots, n$) and $p_{w,j} = 0$ ($j = 0, 1, 2, \ldots$), it remains only to determine the boundary conditions $p_{o,j}$ to begin the computation. From equation (1) written in the notation just adopted,

$$I_p = q\mu_{po}E_o p_{o,j} - qD_{po}\left(\frac{p_{i,j} - p_{o,j}}{h}\right) = I_e$$

$p_{o,j}$ may be determined in terms of I_e,

$$p_{o,j} = \frac{D_{p,}p_{i,j} + hI_e}{D_p + h\mu_{p_o}E_o}$$

I_c is computed by solving equation (7) for the $P_{i,j}$ and then substituting into the equation for I_p at the collector junction,

$$I_c = q\mu_{p_w} E_w p_{w,j} - qD_{p_w}\left(\frac{p_{w,j} - p_{w-1,j}}{h}\right)$$

and solving iteratively for the j at which I_c reaches a constant value.

Results were obtained for a drift transistor with a base width of 6.35 x 10^{-4} cm which had been manufactured from germanium with a resistivity of 5 ohm-cm using a diffusion time of 16 hours at a donor concentration of 1.44 x 10^{-7} per cc. A constant input of 5 milliamperes was assumed to be introduced at the emitter. A collector current of 4.97 milliamperes was computed at an elapsed time of 4.2 millimicroseconds.

This corresponds to an alpha of .994 which seems high, and a rise time of 4.2 millimicroseconds, which is small. But, if we consider that we are dealing with an idealized transistor, these results seem reasonable.

The project was suggested by K. Sih of the Physical Research Department at Poughkeepsie, and most of the preliminary investigation and computer programming were accomplished by William Stevens of the Transistor Research Section. The work is under the direction of R. Domenico.

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