A HOST-PARASITE PROBLEM

INTRODUCTION

Various biological processes in plants and animals occur as periodic functions of time, in which the rhythm of these occurrences is governed by a biological "clock" (1). Galston and Dalberg (2), for example, recognized that an inducible enzyme system could show rhythmic properties. In the simplest case, an enzyme is induced in the presence of a substrate which itself is destroyed by the induced enzyme. The substrate is produced at a constant rate, but the increasing amount of enzyme reduces the total substrate concentration.

This substrate reduction, in turn, decreases the rate of induced enzyme formation, thus allowing the concentration of substrate to increase. Hence, a periodic behavior results between the amount of enzyme induced and the concentration of the substrate.

An almost analogous condition exists in the case of the classical "host-parasite" problem presented in this study.

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PROBLEM DESCRIPTION

A typical ''host-parasite'' situation can be described as follows: in a parasitic relationship in which the birth of a parasite (i.e., an increase in parasite population) causes the demise of its host (i.e., a decrease in host population), the rate of host population growth or decay depends on the number of living hosts and the number of encounters between host and parasite.

The following assumptions were made:

1. In the absence of parasites, the net rate of increase in host population is exponential, the food supply being unlimited.

2. In the absence of hosts, the rate of parasite decrease is also exponential.

3. The rate of encounter between hosts and parasites is constant.

SYSTEM EQUATIONS

The differential equations describing this host-parasite relationship are

$$\frac{dH}{dt} = K_1 H - K_4 HP \qquad (1)$$

$$\frac{dP}{dt} = -K_2 P + K_3 HP \qquad (2)$$

where H = host population as a function of time

P = parasite population as a function of time

K_1 = overall growth rate of hosts

K_2 = overall decay rate of parasites

K_3 = K_4 = rate of host parasite encounters

t = time

PROBLEM ANALYSIS

The specific problem is to analyze the rhythmic behavior resulting between host and parasite in the situation described, through solution of the system differential equations.

Solution of these equations on an analog computer requires numerical values. Suppose that, initially, there are 100 hosts and 200 parasites, and that the pattern of the relationship between them will be established by the time the host population reaches 2000 and/or the parasite population reaches 1000.

Suppose, also, that the hosts increase at a rate of 5% per hour (in the absence of parasites), and that the decay rate for parasites is 10% per hour (in the absence of hosts). Further, assume that there is one encounter between host and parasite per 5000 hours.

Therefore,

K_1 = 0.05, /hour
K_2 = 0.10, /hour
K_3 = 2×10^{-4}, /host-hour
K_4 = 2×10^{-4}, /parasite-hour

COMPUTER PROGRAM

The voltage-scaled computer equations are

$$\left[\frac{\dot{H}}{200}\right] = (K_1)\left[\frac{\dot{H}}{200}\right] - (1000\ K_4)\left[\frac{\left[\frac{H}{200}\right]\left[\frac{P}{100}\right]}{10}\right] \qquad (3)$$

and

$$\left[\frac{\dot{P}}{100}\right] = -(K_2)\left[\frac{P}{100}\right] + (2000\ K_3)\left[\frac{\left[\frac{H}{200}\right]\left[\frac{P}{100}\right]}{10}\right] \qquad (4)$$

Time Scaling: The potentiometer settings. . . . . (K_1), (K_2), ($1000K_4$), and ($2000K_3$). . . . in Equations (3) and (4) are reasonably low. This suggests that the problem should be speeded up by a factor of, say, 10 to 1. Accordingly, a time scale factor of $\beta = 0.1$ is introduced to the denominator of all potentiometers going into integrators.

$$\left[\frac{\dot{H}}{200}\right] = \left(\frac{K_1}{\beta}\right)\left[\frac{H}{200}\right] - 10\left(\frac{100K_4}{\beta}\right)\left[\frac{\left[\frac{H}{200}\right]\left[\frac{P}{100}\right]}{10}\right] \qquad (5)$$

and

$$\left[\frac{\dot{P}}{100}\right] = -\left(\frac{K_2}{\beta}\right)\left[\frac{P}{200}\right] + 10\left(\frac{200K_3}{\beta}\right)\left[\frac{\left[\frac{H}{200}\right]\left[\frac{P}{100}\right]}{10}\right] \qquad (6)$$

Equations (5) and (6) are the scaled computer equations from which the system model is constructed.

Please note that in addition to the 10:1 time scaling employed in this problem, there is also an <u>implied</u> time scaling which relates computer time (in seconds) to problem time (in hours). Thus, 1 second of computer time is equivalent to 10 hours of real time.

The computer diagram for this problem is shown in Figure 1. Tables showing the potentiometer and amplifier assignments are given in Figures 2 and 3.

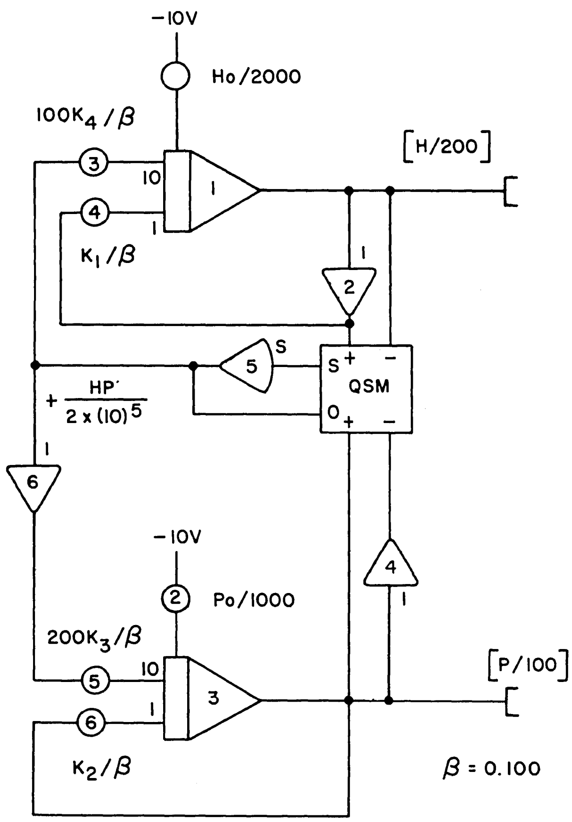

Figure 1. Computer Diagram - - Host-Parasite Problem

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TR-20 POTENTIOMETER ASSIGNMENT SHEET

PROBLEM *Host-Parasite*

POT NO.	PARAMETER DESCRIPTION	SETTING STATIC CHECK	STATIC CHECK OUTPUT VOLTAGE	SETTING RUN NUMBER 1	NOTES	POT NO.
1	$H_0/2000$			0.050	Set at 0.500 for static check	1
2	$P_0/1000$			0.200		2
3	$200 K_3/\beta$			0.400		3
4	K_1/β			0.600		4
5	$100 K_4/\beta$			0.200		5
6	K_2/β			1.000		6
7						7
8						8

Figure 2. Potentiometer Assignment Sheet

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TR-20 AMPLIFIER ASSIGNMENT SHEET

PROBLEM *Host-Parasite*

AMP NO.	FB	OUTPUT VARIABLE	STATIC CHECK: CALCULATED		STATIC CHECK: MEASURED		NOTES
			INTEGRATOR INPUT SUM	OUTPUT	INTEGRATOR INPUT SUM	OUTPUT	
1	∫	$H/200$	+0.50V*	+5.00V			Problem has been
2	Σ	$-H/200$		-5.00			time scaled so that
3	∫	$P/100$	+3.60	+2.00			1 sec. of computer
4	Σ	$-P/100$		-2.00			time is equivalent
5	HG	$+HP/200{,}000$		+1.00			to 10 clock hours
6	Σ	$-HP/200{,}000$		-1.00			$(\beta = 0.1)$.
7							

* Check amplifier gain = -1

Figure 3. Amplifier Assignment Sheet

RESULTS AND CONCLUSIONS

Figure 4 shows a plot of typical results obtained for a problem of this type. The rhythm of increase and decrease in both host and parasite populations, once established, continues as long as the same constraints are imposed. As shown, when the number of parasites is near zero (during the first 50 hours, for example), the number of hosts shows a rapid increase. This increased availability of hosts, however, provides an increased availability of parasite breeding ground and, during the second 50 hours, the parasite population makes an even more rapid increase to a peak occurring when the host population itself is near zero.

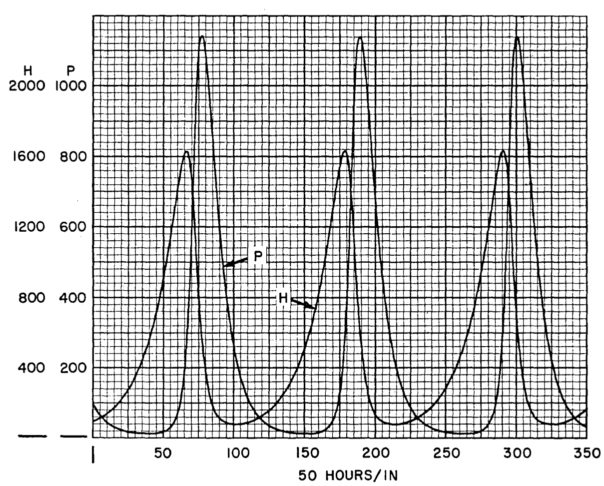

Figure 4

In addition to this typical solution of the stated problem, a number of other interesting experiments can be performed with such an analog computer model. For example, if in a particular culture medium, it is known that ambient temperature variations have an effect on either growth or decay rates, these coefficients can be varied simply through the manipulation of hand-set potentiometers.

Another possibility for experimentation, at the expense only of a slight increase in the complexity of the equations, would be the introduction of a second type of parasite with different characteristics.

It can be seen that use of an analog computer model in the solution of this problem gives the advantages of rapid solution, immediate availability of graphic results, and quick and easy parameter variations.

EQUIPMENT COMPLEMENT

The major pieces of equipment required for this simulation include:

- 2 integrators
- 1 high gain amplifier
- 2 inverters
- 1 summing amplifier
- 6 potentiometers
- 1 quarter-square multiplier
- 1 X-Y plotter.