G 15 D PROGRAMS AND SUB-ROUTINES

Bendix Computer

DIVISION OF BENDIX AVIATION CORP.
5630 Arbor Vitae Street - Los Angeles 45, California

CONTENTS

SUBROUTINES

NUMBER		
1101 . . .	Input - Output	. . . 1101000
1202 . . .	Sine, Cosine, etc.	. . . -8zyx9u8
1203 . . .	Tan^{-1}	. . . yluzl57
1204 . . .	Linear Interpolation	. . . -8luvu30
1205 . . .	Complex Arithmetic	. . . -x20w821
1207 . . .	$Log_e x$	. . . y2wzy86
1208 . . .	e^x	. . . y749vyw

THE USE OF SUBROUTINES WITH THE BENDIX G-15

This paper is intended as a guide for following the Specification Sheets of the subroutines published for use with the Bendix G-15.

1. Execution:

This refers to the command line in which the subroutine must be stored for its execution. Care should be taken that when a subroutine is to be used, it must be stored (temporarily, at least) in the command line from which it is to be executed.

2. Input:

a. Most subroutines specify that the input to the subroutine, such as an argument (for instance, an angle of which a trigonometric function is desired) be placed in a pre-determined storage location, such as a word or words of the fast-access lines or two-word registers, before transfering to the subroutine.

b. A "Return Command" may be required in the AR on entering the subroutine. This command will be executed by the subroutine when its function is completed, to return control to the main routine. (The Mark and Return feature of changes of command line in the G-15D are not used in subroutines published for that model. All Return Commands should be written as Marked Transfers.)

3. Entry:

This refers to the word location in the subroutine command line to which control should be transfered. Give a command to transfer control to the command line of the subroutine, with this location as its "N". (For G-15D, a Marked Transfer.)

4. Exit:

The Specification Sheets will give the word time at which the Return Command will be executed. The "N" of the Return Command should specify the location and (for G-15A, the source) (for the G-15D, the characteristic) the command line of the main routine to which control is to be returned.

5. Scaling:

Numbers in the G-15 are generally scaled so that they appear as fractions (less than 1.0) within the machine. Some subroutines may specify whether their inputs (and/or outputs) are to be scaled with either a binary or decimal scaling factor.

6. Short Line Use:

Many of the Subroutines will make use of the fast access lines. Generally such use will be noted in the Specification Sheets. Information stored there by the programmer prior to subroutine entry will have been destroyed by the execution of the subroutine. The information if retention is desired, should be placed in storage locations not used by the subroutine.

Bendix Computer

Los Angeles 45, California

G-15D

Prepared by Robert J. Margolin

Date: 3-13-57

PROGRAM PROBLEM: EXAMPLE - USE OF SUBROUTINE

Line

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
00		02	06	6	20	25		$x \cdot 10^{-2}$ = 20.02 $\xrightarrow{tva}$ ID_1
06		07	09	0	00	24		$10^2 \cdot 2^{-7}$ = L0 ⟶ MQ_1
(07)								[w800000]
09		10	21	0	24	31		Mult. T = 10
21		22	25	0	20	21		$x \cdot 10^{-2}$ = 20.02 ⟶ 21.02 to be saved
26	u	29	29	2	25	20		$x \cdot 2^{-7}$ = PN_1 $\xrightarrow{tva}$ 20.00
29	u	30	36	0	08	01		Log Subroutine = L8 ⟶ L1
35		37	58	0	00	20		Binary Scale Factor at 2^{-28} = L0→20.01
(37)								[-0000007]
58		59	u6	0	00	28		Return Command = L0 ⟶ AR
*(59)		03	03	0	21*	31		[Return Comm. executed from L01.01 N.C from L 0]
u6		00	00	1	21	31		N.C. from L1
03		07	08	0	20	28		2^{-5} $\log_e X$ = 20.03 ⟶ AR

* For G-15A, word 59 should be written:

L	P	T or L_k	N	C	S	D	BP	NOTES
(59)		03	03	0	20	31		

FORM BCD 108-5.0

SAMPLE PROGRAM -- USE OF SUBROUTINE

PROBLEM: To find the natural log of x.

GIVEN: Log subroutine in Line 8

x less than 100; $x \cdot 10^{-2}$ in 20.02

Word Position of Command	EXPLANATION
00	Loading the odd side of ID from an even location, 20.02, takes two word times, 02 and 03, and is done double precision. This clears the even side of ID and all of PN. x is at a decimal scaling factor of 10^{-2}.
06	The conversion factor, $10^{2} \cdot 2^{-7}$, is transferred into the MQ during word position 07.
09	Since all binary digits of the multiplier after the fifth are zero, only the first five bits need processing, requiring multiplication for ten word times. Multiplication by the conversion factor has eliminated the decimal scaling factor and introduced a binary scaling factor, 2^{-7}, leaving x in a form suitable for the log subroutine.
21	$x \cdot 10^{-2}$, which is needed for later calculations, is moved from 20.02, where it would be destroyed by the subroutine, to 21.02.
26	$x \cdot 2^{-7}$ is moved from the odd side of PN, via AR, to an even word, 20.00. Since an odd word time, 27, and an even word time, 28, which are not parts of the same double precision word are involved, the command is written as a single precision block command.

EXPLANATION

Word Position of Command	
29	A block command for 108 word times transfers the Log Subroutine from Line 8 into Line 1 for execution.
36	The subroutine requires that the exponent of the binary scaling factor of x be placed in 20.01 as an additional input. This exponent is itself expressed at a binary scaling factor of 2^{-28}.
58	The "Return Command" is placed in the AR prior to entering the subroutine. It will be executed from L01.01 as an exit from the subroutine. Since its "N" is 03, control will be returned to L00.03 in the main program.
u6	L01.00 is the entry point for the Log Subroutine, single precision. The "N" of this command is 00.
03	$\text{Log}_e x$ at a binary scaling factor of 2^{-5} is placed in the AR, from which it may be typed or used for further calculation.

DESCRIPTION

This subroutine consists of a Binary to Decimal subroutine, a Decimal to Binary subroutine an an output subroutine. The first two are standard conversion routines with normal entries and exits, as described on their specification sheets.

The output routine scales the number then converts it and types. If the scaling is performed by multiplication, the scale factor (s.f.) must be stored with a negative sign, e.g. - (s.f.) in cell 21.00. If the scaling is by division, (s.f.) is stored in 21.00. This in no way affects the sign of the number being processed. If a new format is to be used, it is loaded into $ID_{0,1}$ and the routine is entered at word time u0 or u6. Otherwise entry is at word time 05 or 06. Typeout may be executed from either line 19 or from AR as shown on specification sheet. Line 19 typeout allows computation not using line 19 to proceed.

SPECIFICATIONS

Type. . . . . Subroutine (G-15D)

Mode of Operation . . . . . . Fixed Point, Single Precision

Execution . . . . Line 02

Entry . . . .

Typeout from	Unchanged format	New format*
L19	05	u6
AR	06	u0

Exit. . . . . At word time 37

Data Input. . . . . $\pm |x| \rightarrow 21.01$ Scale Factor $\rightarrow$ 21.00

Ret. Comm. $\rightarrow$ AR *Format $\rightarrow ID_{0,1}$

Data Output . . . . Typeout under control of format from line 19 or from AR.

Execution Time. . . . . Line 19: 3 Rev. to initiation of typing
Computation may proceed while typing.

AR: Typing completed before exit from subroutine.

Error Stops . . . . Bell rings if improper scale factor used.

Short Lines Used. . . . . 21.00 and 21.01 only

Check Sum Cell Used u7

SPECIFICATIONS

Type. . . . . Subroutine (G-15D)

Mode of Operation . . . . . . Fixed Point, Single Precision

Execution . . . . Command Line 02

Entry . . . . Word time 61

Exit. . . . . Word time 63

Scaling . . . . Convert x as a fraction

Input . . . . x (binary) in ID_1

Return command in AR

Output. . . . . $\pm |x| \frac{\pm}{\pm}$ (decimal) in AR (7 digits and sign)

Execution Time. . . . . 1 revolution

Error Alarms. . . . . None

Short Lines Used. . . . . None

Check Sum Cell Used u7

REMARKS

This routine, the Decimal to Binary and the Output routine are together as a unit in the same line.

SPECIFICATIONS

Type. . . . . Subroutine (G-15D)

Mode of Operation . . . . . . Fixed Point, Single Precision

Execution . . . . Command line 02

Entry . . . . Word time 46

Exit. . . . . Word time 47

Scaling . . . . Fraction or integer

Input . . . . x (decimal) in ID_1 (7 digits and sign)

Return command in AR

Output. . . . . $\pm |x| \pm$ (binary fraction) in MQ_0

$\pm |x| \pm$ (binary integer) in AR

Execution Time. . . . . 5 revolutions

Error Alarms. . . . . None

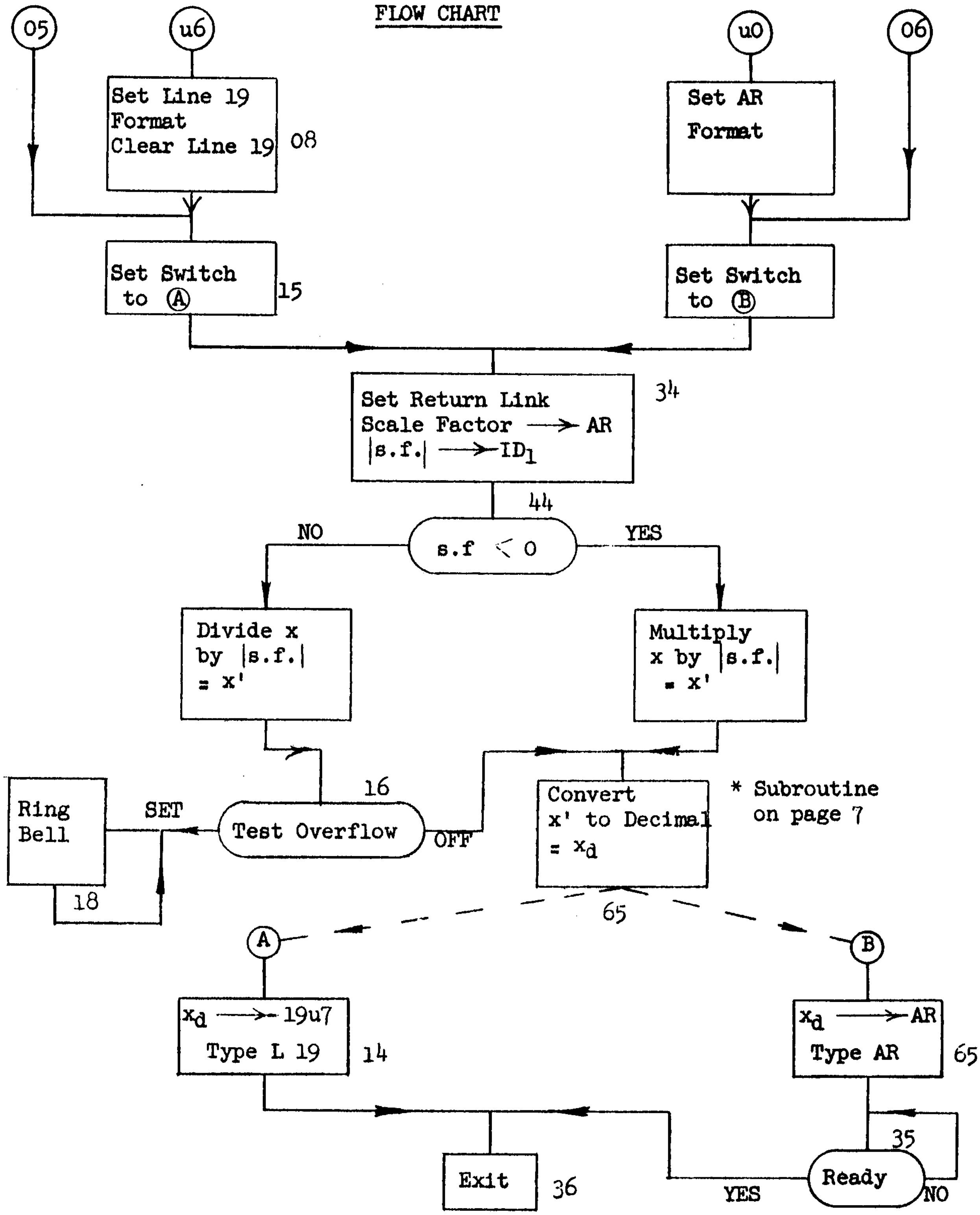
Short Lines Used. . . . . None

Check Sum Cell Used u7

REMARKS

This routine, the Binary to Decimal and the Output routine are together as a unit in the same line.

FLOW CHART



* All Commands are on page 6, with this exception only.

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Los Angeles 45, California

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G-15D

Prepared by Ed Williams (#1101)

Date: 1-25-57

PROGRAM PROBLEM: INPUT-OUTPUT ROUTINE - OUTPUT SUBROUTINE

Line 02

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
u6		02	08	4	25	02		L19 format ← A
05		08	08	0	23	31		Clear ← A^1
08		08	08	0	28	31		
09	u	10	15	0	26	19		Clear L19
15		45	56	0	02	25		(A) Switch
u0		02	06	4	25	03		AR format ← B
06		19	56	0	02	25		(B) Switch ← B^1
56		65	34	0	25	02		Set switch
34		36	37	2	21	02		link, sf → AR
37		39	44	2	28	25		$\lvert sf \rvert \rightarrow ID_1$
44		45	48	0	22	31		sf < 0
49		53	55	0	21	24		Yes $x \rightarrow MQ_1$
55		56	04	0	24	31		$x \cdot \lvert sf \rvert = x^1$
04		05	24	0	26	25		$x^1 \rightarrow ID_1$
48		49	57	0	21	26		No $x \rightarrow PN_1$
57		57	16	5	25	31		$x^1 = x \div \lvert sf \rvert$
16		17	17	0	29	31		overflow
17		18	24	6	24	25		No $x^1 \rightarrow ID_1$
24		25	61	0	02	28		link → AR - convert
(A) 65		u7	14	0	28	19		$x_d \rightarrow$ 19u7
14		36	36	0	09	31		Type L19
(B) 65		35	35	0	08	31		Type AR
35		35	35	0	28	31		Ready
[36						]		Exit
18		19	18	0	17	31		Ring bell

FORM BCD -5.0

Bendix Computer

Los Angeles 45, California

G-15D

Prepared by Ed Williams (#1101)

Date: 1-25-57

PROGRAM PROBLEM: INPUT-OUTPUT ROUTINE - BINARY TO DECIMAL

Line 02

				L	P	T or L_k	N	C	S	D	BP	NOTES
0	1	2	3									
4	5	6	7	61		63	64	0	28	02		Return link
8	9	10	11	64		65	75	0	25	28		x ⟶ AR
12	13	14	15	75		76	84	0	02	29		x^1 = x + roundoff
16	17	18	19	84		90	90	0	23	31		Clear
20	21	22	23	90		91	92	0	28	25		x^1 ⟶ ID_1
24	25	26	27	92		93	95	0	02	24		[v6xv680] ⟶ MQ_1
28	29	30	31	95		06	u2	0	24	31		} d_1
32	33	34	35	u2		u5	u5	3	23	31		
36	37	38	39	u5		06	10	0	24	31		} d_2
40	41	42	43	10		13	13	3	23	31		
44	45	46	47	13		06	20	0	24	31		} d_3
48	49	50	51	20		23	23	3	23	31		
52	53	54	55	23		06	30	0	24	31		} d_4
56	57	58	59	30		33	33	3	23	31		
60	61	62	63	33		06	40	0	24	31		} d_5
64	65	66	67	40		43	43	3	23	31		
68	69	70	71	43		06	50	0	24	31		} d_6
72	73	74	75	50		53	53	3	23	31		
76	77	78	79	53		06	60	0	24	31		d_7
80	81	82	83	60		61	63	0	26	28		|x| ± ⟶ AR
84	85	86	87	63								Exit
88	89	90	91									
92	93	94	95	u7								9u3wy94
96	97	98	99									
u0	u1	u2	u3									
u4	u5	u6										

FORM BCD ..5.0

Bendix Computer Los Angeles 45, California

G-15D

PROGRAM PROBLEM: INPUT-OUTPUT ROUTINE - DECIMAL TO BINARY

Prepared by Ed Williams (#1101)

Date: 1-25-57

Line 02

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
46		47	54	0	28	02		Link return
54		58	59	0	02	26		Clear PN_0
59		61	62	0	25	24		$x_d \longrightarrow MQ_1$
62		67	69	1	02	25		$10^6 \xrightarrow{(1)} ID_1$
69		08	78	0	24	31		d_1
78		79	81	1	02	25		d_2
81		08	91	0	24	31		
91		97	99	1	02	25		d_3
99		08	26	0	24	31		
26		27	29	1	02	25		d_4
29		08	38	0	24	31		
38		39	71	1	02	25		d_5
71		08	80	0	24	31		
80		83	85	1	02	25		d_6
85		08	94	0	24	31		
94		u1	07	1	02	25		d_7
07		08	28	0	24	31		
28		29	68	0	26	28		$x_{int} \longrightarrow AR$
68		73	77	1	02	25		$10^7 \longrightarrow ID_1$
77		57	82	5	25	31		$x_f \rightarrow MQ_0$
[47						]		Exit
82		83	86	1	28	25		$x_{int} \xrightarrow{(1)} ID_1$
86		88	89	1	24	28		$x_f \longrightarrow AR$
89		96	98	3	02	29		$x_f - 2^{-28}$
98		u0	66	1	28	24		$x_f^1 \longrightarrow MQ_0$
66		67	47	1	25	28		$x_{in} \longrightarrow AR$

FORM BCD 9-5.0

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Los Angeles 45, California

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G-15D

Prepared by Ed Williams (#1101)

Date: 1-30-57

PROGRAM PROBLEM: OUTPUT ROUTINE

Line 02

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or Lk	N	C	S	D	BP	NOTES
U3								0zzzzzz
U4								zzzzzzz
11								00zzzzz
12								zzzzzzz
21								000zzzz
22								zzzzzzz
31								0000zzz
32								zzzzzzz
41								00000zz
42								zzzzzzz
51								000000z
52								zzzzzzz
76								000000x
93								v6xv680
45		U7	14	0	28	19		(A)
19		35	35	0	08	31		(B)
25		64	65	0	28	28		link
67		(0z42400)						
79		(0186u00)						
97		(0027100)						
27		(0003Y80)						
39		(0000640)						
83		(00000U0)						
U1		(0000010)						
73		(0989680)						
58 96		(0000000) 0000001						

FORM BCD 28-5.0

SPECIFICATIONS

Type: . . . . Subroutine for G-15D

Mode of Operation: . . . . Fixed point, single precision

Execution: . . . . From Command Line 1

Entry: . . . . At word time 093

Exit . . . . Ret. Comm. from Line 1 at word time 098

Scaling: . . . . θ must be in circles; θ circles = $\frac{\theta \text{ degrees}}{360^\circ} = \frac{\theta \text{ radians}}{2\pi}$, where $-1 < \theta < 1$

Data Input: . . . . θ ⟶ 20.02

Ret. Comm. ⟶ AR

Data Output: . . . . 1/2 sin θ = 20.00
θ = 20.02

Ret. Comm. = Loc. 098 of Line 1
20.01 is undisturbed

Execution Time: . . . . 203 msec (7 rev.)

Error Stops: . . . . None

REMARKS

Command in location 050 can be changed to clear (Source 23, Destination 31) to prevent obtaining an answer -0.

All of the short lines are used in this subroutine.

METHOD

The argument, θ , must be in circles.

$$\theta \text{ circles} = \frac{\theta \text{ degrees}}{360^\circ} = \frac{\theta \text{ radians}}{2\pi}$$

Consider θ circles $= \pm\left[\frac{K}{4} + y\right]$, where K is the number of right angles mod. 4, and y is the acute angle where $0 \leq y < \frac{1}{4}$.

Since $\cos\theta = \sin\left[\frac{\pi}{2} \pm \theta\right]$, we use the same subroutine for both functions.

The following table shows the instructions used to develop the proper sign of X and the proper X for the polynomial evaluation.

$$\frac{1}{2}\sin\frac{\pi}{2}X = X\sum_{i=0}^{5} b_i \left(-x^2\right)^i$$

where: $-1 < X < 1$ and where $\left|\frac{a_i}{2}\right| = b_i$ are the positive constants.

Reference: MTAC, VIII, No. 47 July 1954

SPECIFICATIONS

Type: . . . . Subroutine for G-15D

Mode of Operation: . . . . Fixed point, single precision

Execution: . . . . From Command Line 1

Entry: . . . . At word time 097

Exit: . . . . Ret. Comm. from Line 1 at word time 098

Scaling: . . . . θ must be in circles; θ circles = $\frac{\theta \text{ degrees}}{360^\circ} = \frac{\theta \text{ radians}}{2\pi}$, where $-1 < \theta < 1$

Data Input: . . . . $\theta \longrightarrow$ 20.02

Ret. Comm. $\longrightarrow$ AR

Data Output: . . . . $1/2 \cos \theta$ = 20.01
θ = 20.02
20.00 is undisturbed
Ret. Comm. = Loc. 098 of Line 1

Execution Time: . . . . 203 msec (7rev.)

Error Stops: . . . . None

REMARKS

Command in location 050 can be changed to clear (Source 23, Destination 31) to prevent obtaining an answer -0.

All of the short lines are used in this subroutine.

METHOD

The argument, θ , must be in circles.

$$\theta \text{ circles} = \frac{\theta \text{ degrees}}{360^\circ} = \frac{\theta \text{ radians}}{2\pi}$$

Consider θ circles $= \pm \left[\frac{K}{4} + y\right]$, where K is the number of right angles mod. 4, and y is the acute angle where $0 \leq y < \frac{1}{4}$.

Since $\cos\theta = \sin\left(\frac{\pi}{2} \pm \theta\right)$, we use the same subroutine for both functions.

The following table shows the instructions used to develop the proper sign of X and the proper X for the polynomial evaluation.

$$\frac{1}{2}\sin\frac{\pi}{2}X = X\sum_{i=o}^{5} b_i \left(-x^2\right)^i$$

where: $-1 < X < 1$ and where $\left|\frac{ai}{2}\right| = b_i$ are the positive constants.

Reference: MTAC, VIII, No. 47 July 1954

			1	2	3	4	5
INSTRUCTION	→		$\theta \xrightarrow{\pm}$ AR+	$\lvert AR \rvert \rightarrow$ AR	AR → AR+	AR → ID_1	$\lvert ID_1 \rvert \rightarrow$ AR
RESULT →	θ QUADRANT	θ: k | y | Sign	0+θ	$\lvert 0+\theta \rvert = \theta'$	$2\theta'$	Sets Sign In I.P. ff	θ''
SIN θ	1	00y0	00y0	00y0	0y00	+	0y00
	1	11y1	00$\bar{y}$1	00$\bar{y}$0	0$\bar{y}$00	+	0$\bar{y}$00
	2	01y0	01y0	01y0	1y00	+	1y00
	2	10y1	01$\bar{y}$1	01$\bar{y}$0	1$\bar{y}$00	+	1$\bar{y}$00
	3	10y0	10y0	10y0	0y01	-	0y00
	3	01y1	10$\bar{y}$1	10$\bar{y}$0	0$\bar{y}$01	-	0$\bar{y}$00
	4	11y0	11y0	11y0	1y01	-	1y00
	4	00y1	11$\bar{y}$1	11$\bar{y}$0	1$\bar{y}$01	-	1$\bar{y}$00
			$\frac{1}{4}+\theta$	$\lvert \frac{1}{4}+\theta \rvert = \theta'$			
COS θ	1	00y0	01y0	01y0	1y00	+	1y00
	1	11y1	01y1	01$\bar{y}$0	1$\bar{y}$00	+	1$\bar{y}$00
	2	01y0	10y0	10y0	0y01	-	0y00
	2	10y1	10$\bar{y}$1	10$\bar{y}$0	0$\bar{y}$01	-	0$\bar{y}$00
	3	10y0	11y0	11y0	1y01	-	1y00
	3	01y1	11$\bar{y}$1	11$\bar{y}$0	1$\bar{y}$01	-	1$\bar{y}$00
	4	11y0	00y1	00y0	0y00	+	0y00
	4	00y1	00$\bar{y}$0	00$\bar{y}$0	0$\bar{y}$00	+	0$\bar{y}$00

			6	7	8	9	10
INSTRUCTION →			AR → AR+	TEST FOR-0	TEST FOR-0	AR → ID_0	ID_0 → 21.02
RESULT →	θ QUADRANT	θ (k y Sign)	2θ"	NOT SET	NOT SET	2θ" or $\overline{2θ''}$	PICK ff SIGN ± x
SIN θ	1	00y0	y000		0	y000	y000
	1	11y1	$\bar{y}$000			$\bar{y}$000	$\bar{y}$000
	2	01y0	y001	$\frac{1}{2}$		$\bar{y}$001	$\bar{y}$000
	2	10y1	$\bar{y}$001			y001	y000
	3	10y0	y000		0	y000	y001
	3	01y1	$\bar{y}$000			$\bar{y}$000	$\bar{y}$001
	4	11y0	y001	$-\frac{1}{2}$		$\bar{y}$001	$\bar{y}$001
	4	00y1	$\bar{y}$001			y001	y001
COS θ	1	00y0	y001	$\frac{1}{2}$		$\bar{y}$001	$\bar{y}$000
	1	11y1	$\bar{y}$001			y001	y000
	2	01y0	y000		0	y000	y001
	2	10y1	$\bar{y}$000			$\bar{y}$000	$\bar{y}$001
	3	10y0	y001	$-\frac{1}{2}$		$\bar{y}$001	$\bar{y}$001
	3	01y1	$\bar{y}$001			y001	y001
	4	11y0	y000		0	y000	y000
	4	00y1	$\bar{y}$000			$\bar{y}$000	$\bar{y}$000

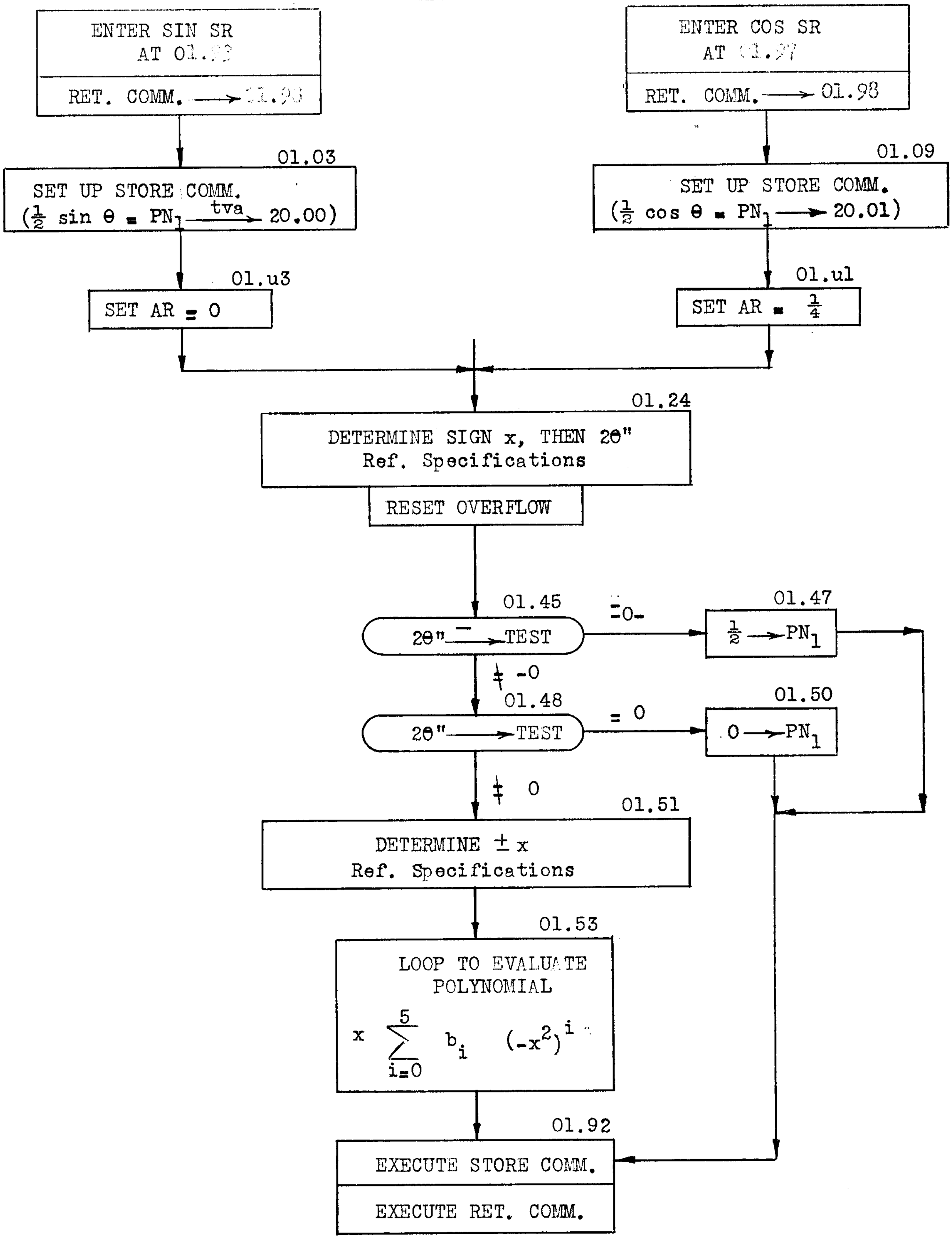
NOTES:

$\bar{y} = 1-y$

STEP 7 IF NOT SET, MEANS ON y-AXIS FOR SIN θ, x-AXIS FOR COS θ.

STEP 8 IF NOT SET, MEANS ON x-AXIS FOR SIN θ, y-AXIS FOR COS θ.

FLOW CHART



FORM BCD 108-7.0

SPECIFICATIONS

Type: . . . . Subroutine for G-15D

Mode of Operation: . . . . Fixed point, single and double precision

Execution: . . . . From Command Line 1

Entry: . . . . At word time 094

Exit: . . . . Ret. Comm. from line 1 at word time 098

Scaling: . . . . $N < 1$; $\sqrt{N} < 1$.

Data Input: . . . . $N \xrightarrow{+} PN_{0,1}$

Ret. Comm. ⟶ AR

Data Output: . . . . $\sqrt{N}$ double precision = $PN_{0,1}$;
$\sqrt{N}$ single precision = 20.03
N = 21.00; 21.01

Ret. Comm. = Loc. 098 of Line 1

Execution Time: . . . . 261 msec (9 rev.)

Error Stops: . . . . If -N (double precision) is entered, routine will ring bell and halt.

REMARKS

Programmer must test for -N (single precision) since routine does not detect this.

All of the short lines are used in this subroutine.

METHOD

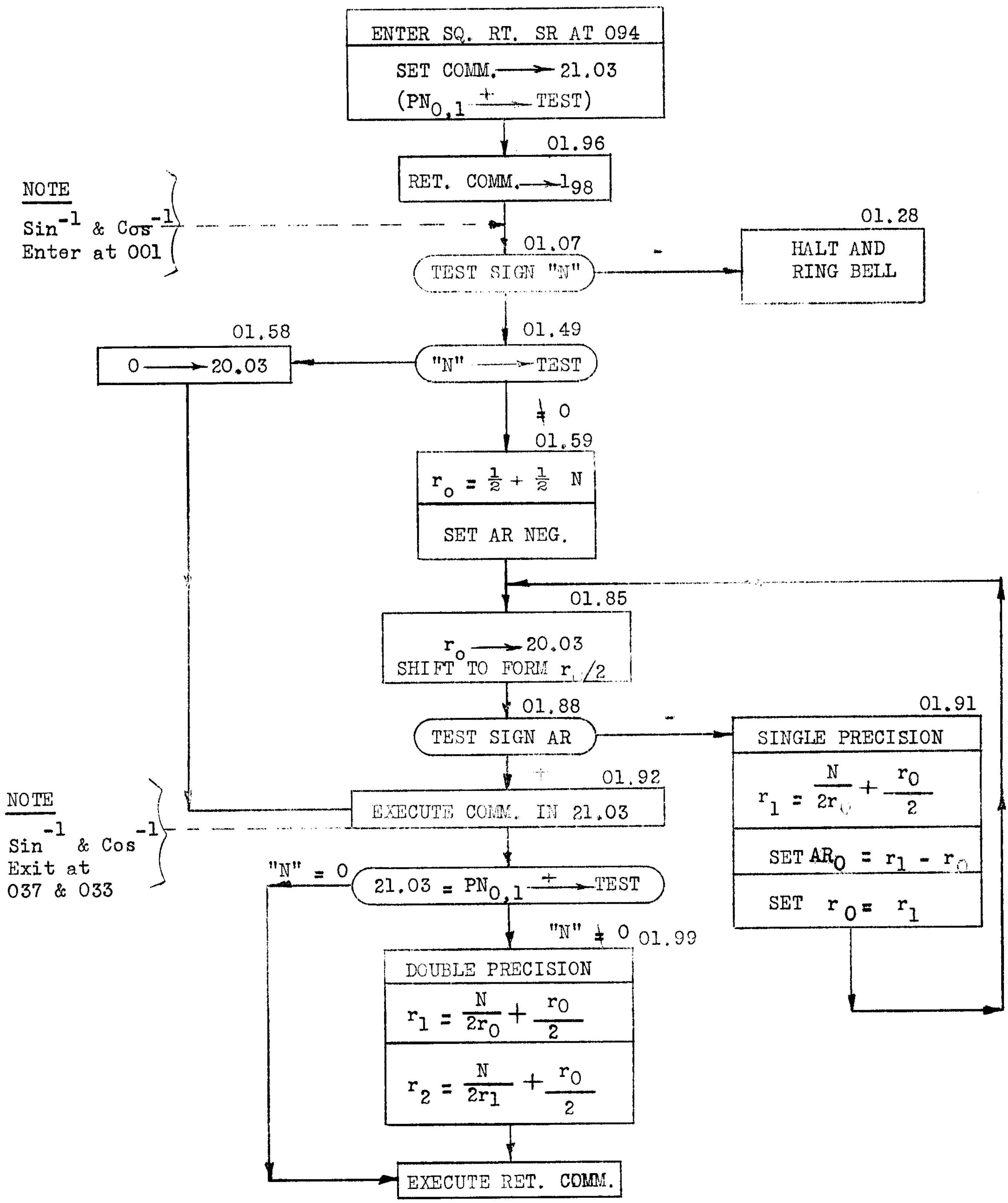
The routine finds the double precision square root of the number, N. But it is written so that the arcsine and arccosine routines can use it to find a single precision square root. So first, the single precision square root is found as follows:

(1) $r_0 = \frac{1}{2} + \frac{1}{2} N$ is used as the first approximation, where $N < 1$. r_0 will be greater than the square root. Then we use the formula.

(2) $r_1 = \frac{N}{2} + \frac{r_0}{2}$. and iterate until $(r_1 - r_0 + 2^{-28}) \geq 0$

Now that we have the single precision square root, two double precision divisions using formula (2) are sufficient to form the double precision square root.

FLOW CHART


ENTER SQ. RT. SR AT 094
SET COMM. → 21.03
(PN0,1 ± → TEST)
01.96
RET. COMM. → 198
NOTE
Sin-1 & Cos-1
Enter at 001
01.07
TEST SIGN "N"
01.28
HALT AND RING BELL
01.49
"N" → TEST
01.58
0 → 20.03
≠ 0
01.59
ro = ½ + ½ N
SET AR NEG.
01.85
ro → 20.03
SHIFT TO FORM ro/2
01.88
TEST SIGN AR
01.91
SINGLE PRECISION
r1 = N/2ro + ro/2
SET AR0 = r1 - r0
SET r0 = r1
01.92
EXECUTE COMM. IN 21.03
NOTE
Sin-1 & Cos-1
Exit at
037 & 033
"N" = 0
21.03 = PN0,1 ± → TEST
"N" ≠ 0
01.99
DOUBLE PRECISION
r1 = N/2r0 + r0/2
r2 = N/2r1 + r0/2
EXECUTE RET. COMM.

SPECIFICATIONS

Type: . . . . Subroutine for G-15D

Mode of Operation: . . . . Fixed point, single precision

Execution: . . . . From Command Line 1

Entry: . . . . At word time 078

Exit: . . . . Ret. Comm. from AR at word time 054

Scaling: . . . . $\phi = \sin^{-1} S = \frac{\phi \text{ degrees}}{360^\circ} = \frac{\phi \text{ radians}}{2\pi}$

$-\frac{1}{4} < \sin^{-1} S \leq \frac{1}{4}$

Data Input: . . . . $\frac{1}{2} S \longrightarrow 20.00$

Ret. Comm. $\longrightarrow$ AR

Data Output: . . . . $\frac{1}{2} S = 20.00$

$\frac{1}{2} \cos \phi = 20.01$

$\phi = \sin^{-1} S = 20.02$

Ret. Comm. = 21.02

Execution Time: . . . . 754 msec (26 rev.)

Error Stops: . . . . If $\left[\frac{1}{4} - \left(\frac{S}{2}\right)^2\right]$ is negative, routine will ring bell and halt. If $\left(\frac{1}{2} S + \sqrt{\frac{1}{4} - \left(\frac{S}{2}\right)^2}\right) > 1$, routine will ring bell and halt.

REMARKS

Tan^{-1} (y/x) subroutine must be stored in Line 04. After use, $\tan^{-1}$ SR is left in Line 1.

All of the short lines are used in this subroutine.

METHOD

Since $\sin^{-1} S = \tan^{-1} \frac{S}{\sqrt{1-S^2}}$; first $\sqrt{\frac{1}{4} - \left(\frac{S}{2}\right)^2}$ is evaluated by the square root routine, then assigning $\frac{1}{2} S = y$ and $\sqrt{\frac{1}{4} - \left(\frac{S}{2}\right)^2} = X$, we enter the $\tan^{-1} \left(\frac{y}{x} \right)$ routine to find ϕ.

SPECIFICATIONS

Type: . . . . Subroutine for G-15D

Mode of Operation: . . . . Fixed point, single precision

Execution: . . . . From Command Line 1

Entry: . . . . At word time 073

Exit: . . . . Ret. Comm. from AR at word time 054

Scaling: . . . . $\phi = \cos^{-1} C = \frac{\phi \text{ degrees}}{360^\circ} = \frac{\phi \text{ radians}}{2\pi}$

$0 \leq \cos^{-1} C \leq \frac{1}{2}$

Data Input: . . . . $\frac{1}{2} C \longrightarrow 20.01$

Ret. Comm. $\longrightarrow$ AR

Data Output: . . . .

$\frac{1}{2} \sin \phi = 20.00$
$\frac{1}{2} C = 20.01$
$\phi = \cos^{-1} C = 20.02$
Ret. Comm. = 21.02

Execution Time: . . . . 754 msec (26 rev.)

Error Stops: . . . . If $\left[\frac{1}{4} - \left(\frac{C}{2}\right)^2\right]$ is negative, routine will ring bell and halt. If $\left(\frac{1}{2} C \pm \sqrt{\frac{1}{4} - \left(\frac{C}{2}\right)^2}\right) > 1$, routine will ring bell and halt.

REMARKS

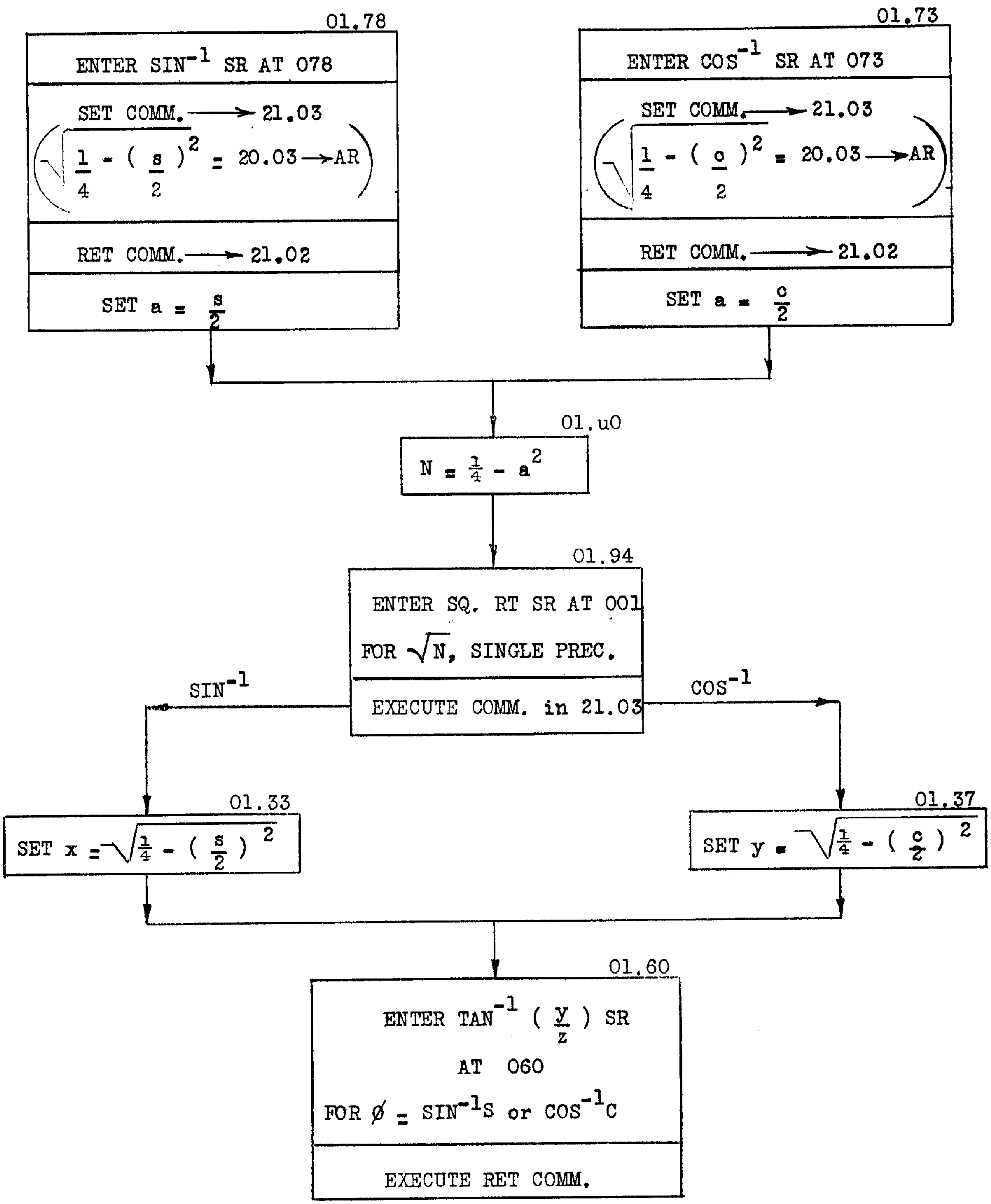
Tan^{-1} (y/x) subroutine must be stored in Line 04. After use, $\tan^{-1}$ SR is left in Line 1.

All of the short lines are used in this subroutine.

METHOD

Since $\cos^{-1} C = \tan^{-1} \frac{\sqrt{1-C^2}}{C}$; first $\sqrt{\frac{1}{4} - (c/2)^2}$ is evaluated by the square root routine, then assigning $\sqrt{\frac{1}{4} - (c/2)^2} = y$ and $\frac{1}{2} C = X$, we enter the $\tan^{-1} \left(\frac{y}{X} \right)$ routine to find $\emptyset$.

FLOW CHART


01.78
ENTER SIN^{-1} SR AT 078
SET COMM. → 21.03
($\sqrt{\frac{1}{4} - (\frac{s}{2})^2}$ = 20.03 → AR)
RET COMM. → 21.02
SET a = $\frac{s}{2}$
01.73
ENTER COS^{-1} SR AT 073
SET COMM. → 21.03
($\sqrt{\frac{1}{4} - (\frac{c}{2})^2}$ = 20.03 → AR)
RET COMM. → 21.02
SET a = $\frac{c}{2}$
01.u0
N = $\frac{1}{4} - a^2$
01.94
ENTER SQ. RT SR AT 001
FOR $\sqrt{N}$, SINGLE PREC.
EXECUTE COMM. in 21.03
SIN^{-1}
COS^{-1}
01.33
SET x = $\sqrt{\frac{1}{4} - (\frac{s}{2})^2}$
01.37
SET y = $\sqrt{\frac{1}{4} - (\frac{c}{2})^2}$
01.60
ENTER TAN^{-1} ($\frac{y}{z}$) SR
AT 060
FOR Ø = SIN^{-1}S or COS^{-1}C
EXECUTE RET COMM.

Bendix Computer

Los Angeles 45, California

G-15 D

PROGRAM PROBLEM: SINE, COSINE, SQUARE ROOT, ARCSINE, ARCCOSINE

Prepared by S. H. Lewis #1202

Page 1 of 5

Date: 3-13-57

Line 01

				L	P	T or L_k	N	C	S	D	BP	NOTES
0	1	2	3									
4	5	6	7	00								b_0 = w90Zxuu
8	9	10	11	02								b_1 = 52uyZ3u
12	13	14	15	04								b_2 = 0u335yy
16	17	18	19	06								b_3 = 00996v5
20	21	22	23	08								b_4 = 0005439
24	25	26	27	21								b_5 = 0001yl
28	29	30	31	93		98	u3	0	28	01		SINE {Ret. Comm.} = (AR) → 01.98
32	33	34	35	u3	u	u5	u6	0	01	28		0 = (01.u4) → AR
36	37	38	39	u4								0
40	41	42	43	u6		03	24	0	01	20		{Comm.} = (01.03) → 20.03 →
44	45	46	47	03	u	97	98	2	26	20		{½Sin θ = (PN) →tva 20.00} From AR At T=094
48	49	50	51	97	u	99	ul	0	28	01		COSINE {Ret. Comm.} = (AR) → 01.98
52	53	54	55	ul	u	u3	09	0	01	28		¼ = (1.u2) → AR
56	57	58	59	u2								¼ = 4000000
60	61	62	63	09		11	24	0	01	20		{Comm.} = (01.11) → 20.03 →
64	65	66	67	11		97	98	0	26	20		{½Cos θ = (PN_1) → 20.01} From AR At T=094
68	69	70	71	24		26	32	1	20	29		θ = (20.02) →+ AR + ←
72	73	74	75	32	u	34	34	2	28	28		|AR| → AR
76	77	78	79	34	u	36	36	0	28	29		(AR) → AR +
80	81	82	83	36	u	38	38	0	28	25		(AR) → ID_1
84	85	86	87	38	u	40	40	2	25	28		|ID_1| → AR
88	89	90	91	40	u	42	42	0	28	29		(AR) → AR +
92	93	94	95	42		44	44	0	29	31		Overflow → Test
96	97	98	99	44	u	46	47	3	28	27		NOT SET (AR) →− Test
u0	u1	u2	u3	45	u	47	47	3	28	27		SET (AR) →− Test
u4	u5	u6		47		71	87	0	01	26		(AR) = −0 ½ = (01.71) → PN_1

FORM BCD 108-5.0

Bendix Computer Los Angeles 45, California

G-15 D

Prepared by S. H. Lewis #1202 Date: 3-13-57

PROGRAM PROBLEM: SINE, COSINE, SQUARE ROOT, ARCSINE, ARCCOSINE Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
71								$\frac{1}{2}$ = 8000000
48	u	50	50	0	28	27		(AR≠0 (AR) → TEST
50		52	87	0	23	31		(AR)=0 Clear 34572ZZ
51	u	53	53	1	28	25		(AR≠0 (AR) →+ ID_0
53	u	55	56	0	25	21		x = (ID_0) → 21.02
56		58	60	6	21	25		x = 21.02 →tva ID_1
60	u	62	63	0	25	24		x = (ID_1) → MQ_1
63		52	13	0	24	31		Mult.
13		15	19	0	01	21		{Comm.} = (01.15) → 21.03
15		08	10	3	01	28		{b_i →− AR} From AR At T=034
19		21	23	0	01	24		b_5 = (01.21) → MQ_1
23	u	26	26	4	26	21		x^2 = ($PN_{0,1}$) → 21.00, 01
26		28	31	4	21	25		x^2 =(21.00,01) → $ID_{0,1}$ ←
31		56	90	0	24	31		Mult.
90	u	92	92	0	21	28		{Comm.} = (21.03) → AR
92		94	94	0	31	31		N. C. From AR {b_i →− AR}
10	u	12	12	2	26	29		$\lvert PN_1 \rvert$ → AR+
12	u	14	14	1	28	24		(AR) →+ MQ_1
14	u	16	16	0	21	28		{Comm.} = (21.03) → AR
16	u	18	18	3	01	29		{T = 002} →+ AR+
17	u	02	00	0	00	00		{T = 002}
18	u	20	20	0	28	21		{Modified Command} = (AR) → 21.03
20	u	22	22	0	28	29		(AR) → AR+
22		24	25	0	29	31		Overflow → Test (Set)
25	u	28	29	6	21	25		Not Set x = (21.02) →tva ID_1
29		56	87	0	24	31		Mult.

FORM BCD 108-5.0

Bendix Computer

Los Angeles 45, California

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G-15D

Prepared by S. H. Lewis #1202

Date: 3-13-57

PROGRAM PROBLEM: SINE, COSINE, SQUARE ROOT, ARCSINE, ARCCOSINE

Line 01

				L	P	T or L_k	N	C	S	D	BP	NOTES
0	1	2	3									
4	5	6	7	87		91	92	0	20	28		[Store Comm.] = 20.03 ⟶ AR
8	9	10	11	94	u	96	96	0	01	21		Sq. Rt. [Comm.] = (01.95) ⟶ 21.03
12	13	14	15	95	u	97	98	1	26	27		[($PN_{0,1}$) $\xrightarrow{+}$ Test] From AR At T=094
16	17	18	19	96		98	01	0	28	01		[Ret. Comm.] = (AR) ⟶ 01.98
20	21	22	23	01	u	03	05	1	26	20		(PN_0) $\xrightarrow{+}$ 20.02
24	25	26	27	05	u	07	07	0	20	28		(20.02) ⟶ AR
28	29	30	31	07		09	27	0	22	31		T_1·AR ⟶ Test
32	33	34	35	27	u	30	49	5	26	21		Normal: N = ($PN_{0,1}$) $\xrightarrow{+}$ 21.00,01
36	37	38	39	28		29	54	0	17	31		Error: Ring Bell
40	41	42	43	54		56	28	0	16	31		Halt
44	45	46	47	49	u	52	58	5	26	27		N = ($PN_{0,1}$) $\xrightarrow{+}$ Test ←
48	49	50	51	58	u	60	90	1	26	20		N=0: 0 = (PN_1) $\xrightarrow{+}$ 20.03
52	53	54	55	59		61	62	0	21	25		N≠0: N_1 = (21.01) ⟶ ID_1
56	57	58	59	62		64	65	0	23	31		Clear (Even)
60	61	62	63	65		02	68	0	26	31		Shift
64	65	66	67	68	u	70	70	2	25	28		$\vert ID_1 \vert$ ⟶ AR
68	69	70	71	70	u	72	72	0	01	29		$\frac{1}{2}$ = (01.71) ⟶ AR+
72	73	74	75	72	u	74	74	0	28	25		r_0 = (AR) ⟶ ID_1
76	77	78	79	74	u	76	76	3	28	28		(AR) $\xrightarrow{-}$ AR
80	81	82	83	76		80	82	4	21	26		N = (21.00,01) ⟶ $PN_{0,1}$ From P19
84	85	86	87	82	u	84	85	0	25	20		r_0 = (ID_1) ⟶ 20.03
88	89	90	91	85		02	88	0	26	31		Shift
92	93	94	95	88		90	90	0	22	31		T_1 AR ⟶ Test
96	97	98	99	90	u	92	92	0	21	28		(AR) ≥ 0 [Comm.] = (21.03) ⟶ AR
u0	u1	u2	u3	91		54	41	5	25	31		(AR) < 0 Divide
u4	u5	u6		41	u	43	52	0	24	28		$N/2r_0$ = (MQ_0) ⟶ AR

Bendix Computer Los Angeles 45, California

G-15D

PROGRAM PROBLEM: SINE, COSINE, SQUARE ROOT, ARCSINE, ARCCOSINE

Prepared by S. H. Lewis #1202

Date: 3-13-57

Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
52	u	64	64	0	25	29		$r_0/2$ = (ID_1) ⟶ AR +
64	u	66	66	0	28	25		r_1 = (AR) ⟶ ID_1
66	u	68	76	3	20	29		r_0 = (20.03) ⟶ AR + (−) To P 18
92		94	94	0	31	31		NC From AR
99		v2	30	5	25	31		N ≠ 0 Divide
30		32	35	4	24	26		$N/2r_0$ = $(MQ_{0,1})$ ⟶ $PN_{0,1}$
35	u	38	39	4	25	30		$r_0/2$ = $(ID_{0,1})$ ⟶ $PN_{0,1}$ +
39	u	42	43	4	26	25		r_1 = $(PN_{0,1})$ ⟶ $ID_{0,1}$
43		02	46	0	26	31		Shift
46		48	55	4	21	26		N = (21.00,.01) ⟶ $PN_{0,1}$
55		v2	61	5	25	31		Divide
61	u	64	69	4	24	26		$N/2r_1$ = $(MQ_{0,1})$ ⟶ $PN_{0,1}$
69	u	72	98	4	25	30		$r_1/2$ = $(ID_{0,1})$ ⟶ $PN_{0,1}$ +
73		75	77	0	01	21		ARCCOS {Comm.} = (1_{75}) ⟶ 21.03
75	u	96	33	0	20	28		$\left[\sqrt{\frac{1}{4}-(\frac{c}{2})^2}\right]$ = (20.03) ⟶ AR From AR AT T=094
77	u	79	[illegible]	0	28	21		{Ret. Comm.} = (AR) ⟶ 21.02
80		82	84	0	23	31		Clear (Even)
84	u	86	86	0	20	25		$\frac{c}{2}$ = (20.01) ⟶ ID_1
78	u	[illegible]	81	0	01	21		ARCSIN {Comm.} = (01.79) ⟶ 21.03
79	u	96	37	0	20	28		$\left[\sqrt{\frac{1}{4}-(\frac{s}{2})^2}\right]$ = (20.03) ⟶ AR From AR AT T=094
81	u	83	83	0	28	21		{Ret. Comm.} = (AR) ⟶ 21.02
83	u	86	86	6	20	25		$\frac{s}{2}$ = (20.00) ⟶(tva) ID_1
86	u	88	89	0	25	24		(ID_1) ⟶ MQ_1
89		56	67	0	24	31		Mult.
67		70	u0	5	26	20		$(PN_{0,1})$ ⟶(+) (20.02,03) -w664754
u0		u2	u5	6	01	26		$\frac{1}{4}$ = (1_{102}) ⟶(tva) PN_1

FORM BCD 108-5.0

Bendix Computer

Los Angeles 45, California

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G-15 D

Prepared by S. H. Lewis #1202

Date: 3-13-57

PROGRAM PROBLEM: Sine, Cosine, Square Root, Arcsine, Arccosine

Line 01

L	P	T or L_k	N	C	S	D	BP	NOTES
u5	u	00	01	7	20	30		(20.02, .03) $\xrightarrow{-}$ $PN_{0,1}$+
33		36	57	0	28	20		ARCCOS $\sqrt{\frac{1}{4}-(\frac{c}{2})^2}$ = (AR) → 20.00
37		41	57	0	28	20		ARCSIN $\sqrt{\frac{1}{4}-(\frac{s}{2})^2}$ = (AR) → 20.01
57	u	58	60	0	04	01		Tan^{-1}S. R. = (Line 4) → L1

0 1 2 3
4 5 6 7
8 9 10 11
12 13 14 15
16 17 18 19
20 21 22 23
24 25 26 27
28 29 30 31
32 33 34 35
36 37 38 39
40 41 42 43
44 45 46 47
48 49 50 51
52 53 54 55
56 57 58 59
60 61 62 63
64 65 66 67
68 69 70 71
72 73 74 75
76 77 78 79
80 81 82 83
84 85 86 87
88 89 90 91
92 93 94 95
96 97 98 99
u0 u1 u2 u3
u4 u5 u6

FORM BCD 108-5.0

SPECIFICATIONS

Type: . . . . Subroutine for G-15D

Mode of Operation: . . . . Fixed point, single precision

Execution: . . . . From Command Line 1

Entry: . . . . First command location = 053

Exit: . . . . From AR at word time 054

Scaling: . . . . x and y must be given with equal scale factors such that $|x| + |y| < 1$. Z is obtained with the same scale factor.

$$\theta = \tan^{-1}\frac{y}{x} = \frac{\theta\ (\text{degrees})}{360^\circ} = \frac{\theta\ (\text{radians})}{2\pi}.$$

$$-\tfrac{1}{2} < \theta \leq \tfrac{1}{2}.$$

Data Input: . . . . $y \longrightarrow 20.00$
$x \longrightarrow 20.01$

Exit command $\longrightarrow$ AR

Data Output: . . . .

20.00)	y	21.00	$Z^2 = x^2 + y^2$
20.01)	x	21.01	
20.02)	0		
20.03)	z	21.02	Exit command

Execution Time: . . . . 551 msec Maximum (19 rev.)

Error Stops: . . . . Ring bell and halt if $|x| + |y| \geq 1$

REMARKS

$\theta = \tan^{-1}\frac{y}{x}$ May be obtained without $Z = \sqrt{x^2 + y^2}$ in 13 1/6 revolutions (366 msec) if the commands in 064 and 069 are replaced by 064, -066, 069, 0, 21, 28 + and 069, + 071, 071, 0, 31, 31+.

This subroutine must be stored in Line 4 when used with the subroutine for arcsine and arccosine, unless the SOURCE in command 057 of the latter is changed.

METHOD

Expressing all angles as fractions of a circle let $\phi = \tan^{-1} \left|\frac{y}{x}\right|$,

and let $R = \tan(\phi - \frac{1}{8}) = \frac{\tan\phi - 1}{\tan\phi + 1} = \frac{|y| - |x|}{|y| + |x|}$. Then $\phi = \frac{1}{8} + \tan^{-1} R$,

where $-1 \leq R \leq 1$. (if $x = y = 0$, then $\phi = Z = 0$).

$\tan^{-1} R$ is approximated to single precision accuracy throughout the range of R by a polynomial. (Ref. MATC, vol. VII No. 47, July, 1954)

$$\tan^{-1} R = R \sum_{i=0}^{9} b_i (-R^2)^i, \text{ where:}$$

$b_0 = .15915\ 4943$	$b_5 = .01327\ 9175$
$b_1 = .05305\ 1549$	$b_6 = .00907\ 5407$
$b_2 = .03182\ 7760$	$b_7 = .00482\ 5354$
$b_3 = .02269\ 4252$	$b_8 = .00166\ 8861$
$b_4 = .01739\ 4550$	$b_9 = .00027\ 1190$

$\theta = \tan^{-1} \frac{y}{x}$ is derived from $\phi = \tan^{-1} \frac{|y|}{|x|}$.

x	y	Quad.	θ
+	+	1	ϕ
-	+	2	$\frac{1}{2}-\phi$
-	-	3	$-(\frac{1}{2}-\phi)$
+	-	4	$-\phi$

Finally, $Z = \sqrt{x^2 + y^2}$ is obtained by iteration to convergence of Z.

$Z_{i+1} = \frac{Z^2}{2Z_i} + \frac{Z_i}{2}$, where $Z_o = |x| + |y|$. Since $Z \leq Z_i \leq Z_o \leq Z \leq \sqrt{2}$,

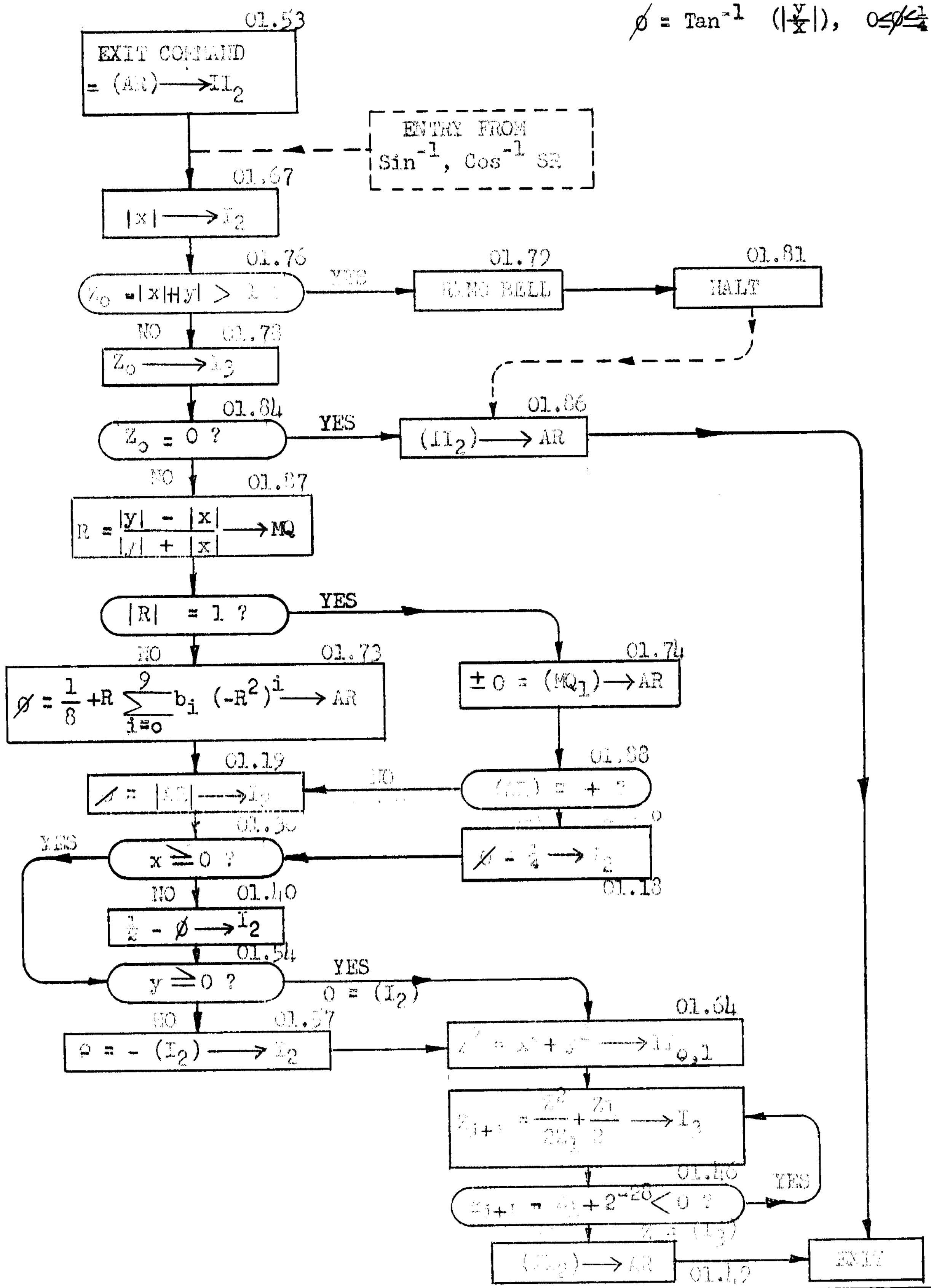
not more than four iterations are required.

FLOW CHART

$\theta = \text{Tan}^{-1}\left(\frac{y}{x}\right), -\frac{1}{2} < \theta \leq \frac{1}{2}$

$Z = \sqrt{x^2 + y^2}$

$\phi = \text{Tan}^{-1}\left(\left|\frac{y}{x}\right|\right), 0 \leq \phi \leq \frac{1}{4}$



Bendix Computer Los Angeles 45, California

G-15D
PROGRAM PROBLEM: Rectangular Coordinates to Polar (G-15D)

Prepared by D. Stein #1203

Page 1 of 4
Date: 3-13-57
Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
53	u	55	60	0	28	21		[Ret. Comm.] = (AR) → 21.02
60	u	62	62	2	20	28		|x| = |20.01| → AR
62		65	65	0	23	31		Clear
65		67	67	0	29	31		Overflow → test
67		70	71	0	28	20		not set: |x| = (AR) → 20.02
68		70	71	0	28	20		set: |x| = (AR) → 20.02
71	u	73	76	2	20	29		|y| = |20.00| → AR+
76		78	78	0	29	31		Overflow → test
78	u	80	82	0	28	20		normal: Z_0 = |x| + |y| = (AR) → 20.03
79		80	81	0	17	31		Overflow: Ring Bell
81		83	86	0	16	31		Halt
82	u	84	84	0	28	25		|x| + |y| = (AR) → ID_1
84		86	86	0	28	27		(AR) → Test
86		90	52	0	21	28		|Z| ≯ 0: [Ret. Comm.] = (21.02) → AR
87	u	89	89	2	20	28		|Z| ≯ 0: |y| = |20.00| → AR
89	u	91	92	3	20	29		|x| = (20.02) −→ AR+
92		94	96	1	28	20		|y| − |x| = (AR) +→ 20.02
96		98	u1	6	20	26		|y| − |x| = (20.02) tva→ PN_1
u1		58	70	5	25	31		Divide
70	u	72	73	1	24	27		(MQ_1) → Test
73	u	75	77	0	24	20		|R| < 1: R = (MQ_0) → 20.02
74	u	76	88	0	24	28		|R| = 1: ± 0 = (MQ_1) → AR
77	u	80	80	6	20	25		R = (20.02) tva→ ID_1
80	u	82	83	0	25	24		R = (ID_1) → MQ_1
83		56	58	0	24	31		Mult.
58		63	3	0	01	21		[Comm] = (01.63) → 21.03

Bendix Computer
Los Angeles 45, California

G-15D
PROGRAM PROBLEM: Rectangular Coordinates to Polar (G-15D)

Prepared by D. Stein #1203 Date: 3-13-57
Line

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
63		16	17	3	01	28		[$b_i \xrightarrow{-}$ AR] From AR At T-106
u3		u5	34	0	01	24		b_9 = (01.u5) → MQ_1
34		36	38	5	26	21		R^2 = $(PN_{0,1}) \xrightarrow{+}$ 21.00, 01
38		40	45	4	21	25		R^2 = (21.00,01) → $ID_{0,1}$
45		56	u2	0	24	31		Mult.
u2	u	u4	u4	0	21	28		[Comm.] = (21.02) → AR
u4		u6	u6	0	31	31		NC From AR [$b_i \xrightarrow{-}$ AR]
17		19	20	2	26	29		$\lvert PN_1 \rvert$ → AR+
20	u	22	26	1	28	24		(AR) $\xrightarrow{+}$ MQ_1
26	u	28	28	0	21	28		[Comm.] = (21.03) → AR+
28	u	30	30	3	01	29		[T = 002] = (01.29) $\xrightarrow{-}$ AR+
29		02	00	0	00	00		[T = 002]
30	u	32	32	0	28	21		[Modified Command] = (AR) → 21.03
32	u	34	35	0	28	29		(AR) → AR+
35		37	37	0	29	31		Overflow → Test (Set)
37	u	40	41	6	20	25		[Not Set] R = (20.02) $\xrightarrow{tva}$ ID_1
41		56	98	0	24	31		Mult.
98	u	u0	u0	0	26	28		$R \sum b_i (-R^2)^i$ = (PN_1) → AR
u0	u	u2	01	1	28	28		(AR) $\xrightarrow{+}$ AR
01		11	19	0	01	29		$\frac{1}{8}$ = (01.11) → AR+
88		90	18	0	22	31		$T_1 \cdot$ AR → Test ($\lvert AR \rvert$ = 0)
18		22	24	0	01	20		[Not set] $\theta = \frac{1}{4}$ (01.22) → 20.02
19		22	24	2	28	20		[Set] θ = AR → 20.02
24	u	26	36	0	20	28		x = (20.01) → AR
36		38	39	0	22	31		$T_1 \cdot$ AR → Test
39	u	41	54	0	20	28		[$x \geq 0$] y = (20.00) → AR

FORM BCD 108-5.0

Bendix Computer Los Angeles 45, California

G-15D PROGRAM PROBLEM: Rectangular Coordinates to Polar (G-15D)

Prepared by D. Stein #1203 Date: 3-13-57

Line

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
040		42	043	3	20	28		x < 0 θ = (20.02) $\xrightarrow{-}$ AR
043	u	045	048	0	01	29		½ = (01.44) ⟶ AR+
000								b_0 = 28vy60y
002								b_1 = 0x94w95
004								b_2 = 0825xx3
006								b_3 = 05wZ4u6
008								b_4 = 0473Z82
010								b_5 = 0366439
012								b_6 = 0252w41
014								b_7 = 013w3w0
016								b_8 = 006x5yx
105								b_9 = 0011w5x
011								$\frac{1}{8}$ = 2000000
022								$\frac{1}{4}$ = 4000000
044								$\frac{1}{2}$ = 8000000
048		050	051	0	28	20		(AR) ⟶ 20.02
051	u	053	054	0	20	28		y = (20.00) ⟶ AR
054		056	056	0	22	31		T_1 · AR ⟶ Test
056		059	064	0	23	31		y ≥ 0 Clear
057	u	059	059	3	20	28		y < 0 (20.02) $\xrightarrow{-}$ AR
059		061	061	0	23	31		Clear (Even)
061	u	063	064	1	28	20		θ = (AR) ⟶ 20.02
064	u	066	066	0	20	25		x = (20.01) ⟶ ID_1
066	u	068	069	0	25	24		x = (ID_1) ⟶ MQ_1
069	u	056	023	0	24	31		Mult.
023	u	026	027	5	26	21		x^2 = $(PN_{0,1})$ $\xrightarrow{+}$ 21.00, 01

FORM BCD 108-5.0

Bendix Computer

Los Angeles 45, California

G-15D

PROGRAM PROBLEM: Rectangular Coordinates to Polar (G-15D)

Prepared by D. Stein #1203

Date: 3-13-57

Line

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
27		28	42	6	20	25		$y \equiv (20.00) \xrightarrow{tva} ID_1$
42	u	44	47	0	25	24		$y \equiv (ID_1) \longrightarrow MQ_1$
47		56	u6	0	24	31		Mult.
u6		00	07	4	21	30		$x^2 \equiv (21.00,01) \longrightarrow PN_{0,1}+$
07		08	15	5	26	21		$z^2 \equiv (PN_{0,1}) \xrightarrow{+} 21.00,01$
15		19	31	0	20	25		$Z_0 \equiv (20.03) \longrightarrow ID_1$
31		33	33	0	23	31		Clear (Even)
33		02	50	0	26	31		Shift
50		52	55	4	21	26		$z^2 \equiv (21.00,01) \longrightarrow PN_{0,1}$ ←
55		54	03	5	25	31		Divide
03	u	05	05	0	24	28		$z^2/2Z_0 \equiv (MQ_0) \longrightarrow AR$
05		07	09	0	25	29		$Z_0/2 \equiv (ID_1) \longrightarrow AR+$
09		11	13	0	28	25		$Z_1 \equiv (AR) \longrightarrow ID_1$
13		15	21	3	20	29		$Z_0 \equiv (20.03) \longrightarrow AR+$
21		23	25	0	25	20		$Z_1 \equiv (ID_1) \longrightarrow 20.03$
25		02	46	0	26	31		Shift
46		48	49	0	22	31		$T_1 \cdot AR \longrightarrow$ Test (Set) →
49	u	51	52	0	21	28		not set Ret. Comm. $\equiv (21.02) \longrightarrow AR$
52		54	54	0	31	31		NC from AR

FORM BCD 108-5.0

LINEAR INTERPOLATION
TITLE

Prepared by: D. Stein

Approved by: [signature]

Date: 3-13-57

No. 1204

SPECIFICATIONS

Type: . . . . Subroutine for G-15D

Mode of Operation: . . . . Fixed point, single precision

Execution: . . . . Command Line 1

Entry: . . . . At word time 059

Exit: . . . . Ret. Comm. from Line 1 at word time 067

Scaling: . . . . The argument and all tabular values of x are at one scale. All tabular values of y are at one scale, not necessarily the same scale as the x values.

Data Input: . . . . $x \longrightarrow$ ID.

Ret. Comm. $\longrightarrow$ AR

Pickup Comm. (deferred, T = "T", T_N = 081, CH = 0, S = "L", D = 25, double precision) $\longrightarrow$ 21.02 where "L" and "T" specify the line and location of y.

Data Output: . . . . y = AR
x = 21.01
y_i = 20.00
x_i = 20.01
y_{i+1} = 20.02
x_{i+1} = 20.03

Ret. Comm. = Loc. 067 of Line 1

Execution Time: . . . . . If $X_k < X < X_{k+1}$, then approximately 3+2k revolutions = 90+58k ms.

Error Stops: . . . . If $x < x_1$, routine will ring bell and halt.

REMARKS

Short Line Use: . . . . . . Routine uses 20.00, 01, 02, 03 and 21.01, 02, 03.

Table must be stored such that $x_i < x_{i+1}$.

Programmer must test for $x < x_n$.

METHOD

The table is stored in Line "L" starting at even location "T" as follows:

$$y_1, x_1, y_2, x_2, y_3, x_3, \ldots \ldots y_n, x_n$$

where:

$$n \leq 54; \; y_i = (L_{T+2i-2}), \; x_i = (L_{T+2i-1})$$

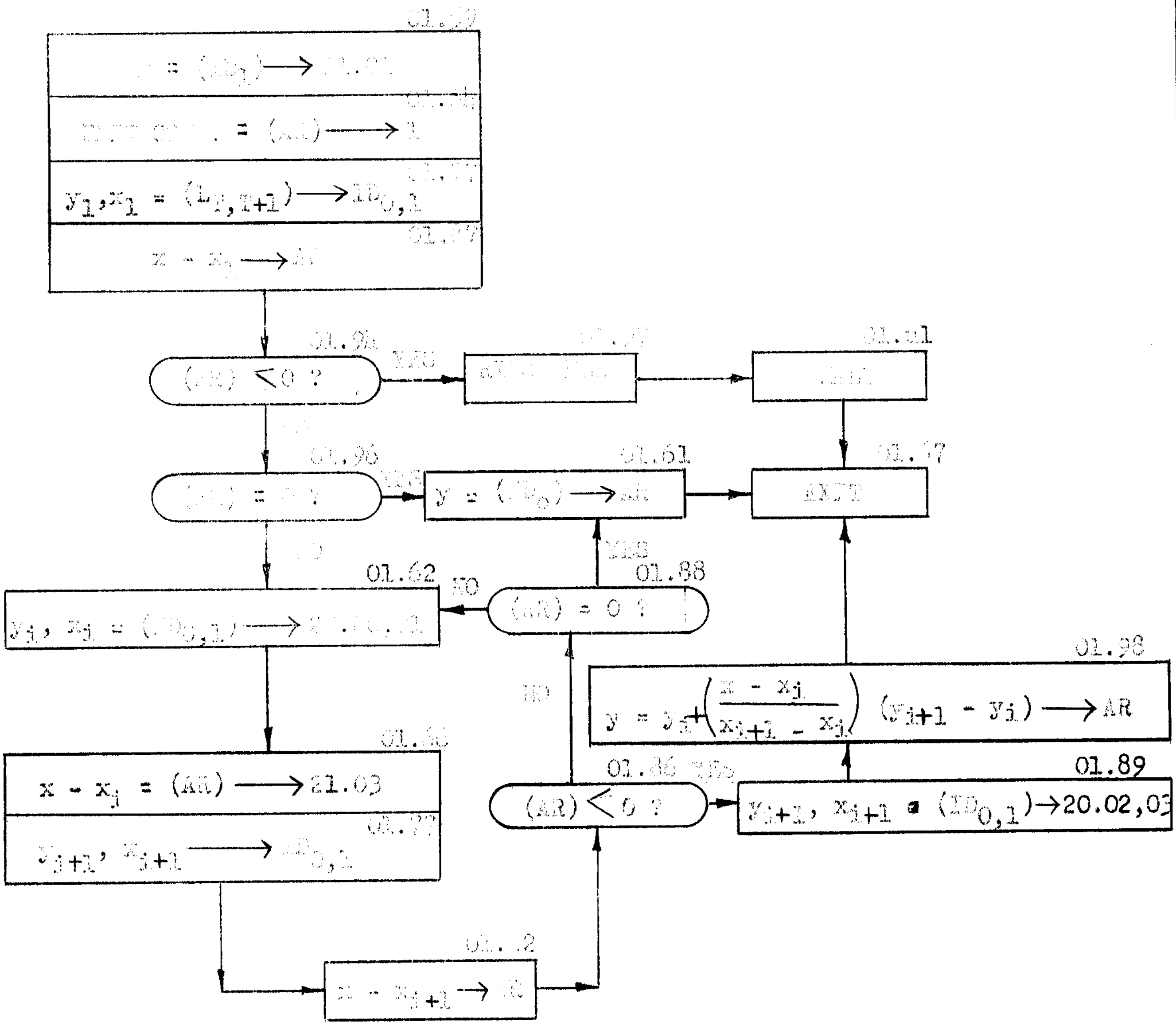
The argument, x, is compared with each successive value, starting with x_1, until $x_i \leq x \leq x_{i+1}$. If x is not equal to x_i or x_{i+1}, then:

$$y = y_i \left[\frac{x - x_i}{x_{i+1} - x_i} \right] (y_{(i+1)} - y_i)$$

TITLE

Date: 3-13-57

No. 1204



Bendix Computer

Los Angeles 45, California

Page 1 of 2

Prepared by D. Stein, S. H. Lewis   # 1204

Date: 3-13-57

Line 01

G-15D

PROGRAM PROBLEM: Linear Interpolation

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
59		61	64	0	25	21		$x = ID_1 \longrightarrow 21.01$
64		69	68	0	28	01		Ret. Comm. = AR ⟶ 01.67
68		70	75	0	21	28		Pickup Comm. = 21.02 ⟶ AR
75		79	77	0	31	31		NC From AR
(77) AR		T	81	4	L	25		$y_1, x_1, = L_{T,T+1} \longrightarrow ID_{0,1}$
81		83	85	0	01	29		$[T_n = 001]$ = 01.83 ⟶ AR +
83		00	01	0	00	00		$[T_n = 001]$
85	u	87	87	0	28	21		Modified Pickup Comm. = AR ⟶ 21.02
87		89	91	3	25	28		$x_1 = ID_1 \xrightarrow{-} AR$
91		93	94	1	21	29		$x = 21.01 \xrightarrow{+} AR +$
94		96	96	0	22	31		$T_1 \cdot AR \longrightarrow$ Test
96		98	61	0	28	27		$[x \geq x_1]$ $x - x_1 = AR \longrightarrow$ Test ⟶
97		96	u1	0	17	31		$[x < x_1]$ Ring Bell
u1		u3	67	0	16	31		Halt
61	u	63	67	0	25	28		$[x = x_i]$ $y = y_i = ID_0 \rightarrow AR$ ⟵ FR.P.5
62		64	66	4	25	20		$[x > x_i]$ $y_i, x_i = ID_{0,1} \rightarrow 21.00, 01$
66	u	68	69	0	28	21		$x - x_i = AR \longrightarrow 21.03$
69	u	71	71	0	21	28		Pickup Comm. = 21.02 ⟶ AR
71	u	73	73	0	01	29		[T = 002] = 01.72 ⟶ AR +
72		02	00	0	00	00		[T = 002]
73	u	75	75	0	28	21		Modified Pickup Comm. = AR ⟶ 21.02
(75		77	77	0	31	31)		NC From AR
(77) AR		$T+2_i$	82	4	L	25		$y_{i+1}, x_{i+1} = L_{T+2i, T+2i+1} \longrightarrow ID_{0,1}$
82	u	84	84	3	25	28		$x_{i+1} = ID_1 \xrightarrow{-} AR$
84	u	86	86	1	21	29		$x = 21.01 \xrightarrow{+} AR +$
86		88	88	0	22	31		$T_1 . AR \longrightarrow$ Test

FORM BCD 108-5.0

Bendix Computer Los Angeles 45, California

Page 2 of 2

Date: 3-13-57

Line 01

G-15D

Prepared by D. Stein, S. H. Lewis #1204

PROGRAM PROBLEM: LINEAR INTERPOLATION

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
88	u	90	61	0	28	27		$x \geq x_{i+1}$ $x - x_{i+1}$ = AR $\rightarrow$ Test (To p.4)
89	u	92	92	0	25	20		$x < x_{i+1}$ y_{i+1}, x_{i+1} = $ID_{0,1}$ $\rightarrow$ 20.02,03
92	u	94	95	3	20	28		x_i = 20.01 $\xrightarrow{-}$ AR
95		97	98	0	23	31		Clear Even
98		98	u0	1	20	29		x_{i+1} = 20.02 $\xrightarrow{+}$ AR +
u0	u	u2	u2	0	28	25		$x_{i+1} - x_i$ = AR $\rightarrow$ ID_1
u2	u	u4	u5	0	21	26		$x - x_i$ = 21.03 $\rightarrow$ PN_1
u5		58	74	5	25	31		Divide
74		76	80	6	24	25		$\frac{x - x_i}{x_{i+1} - x_i}$ = MQ_0 $\xrightarrow{tva}$ ID_1
80		82	90	1	20	28		y_{i+1} = 20.02 $\xrightarrow{+}$ AR
90		92	93	3	20	29		y_i = 20.00 $\xrightarrow{-}$ AR +
93	u	95	99	1	28	28		$y_{i+1} - y_i$ = AR $\xrightarrow{+}$ AR
99		u1	u3	0	28	24		$y_{i+1} - y_i$ = AR $\rightarrow$ MQ_1
u3		56	55	0	24	31		Multiply
55	u	58	58	5	01	30		Round Bit = 01.56,57 $\xrightarrow{+}$ PN +
56		00	00	0	00	00		Round Bit = 2^{-29}
57		00	00	4	00	00		
58		60	60	0	26	28		$y - y_i$ = PN $\rightarrow$ AR
60		62	63	1	28	28		$y - y_i$ = AR $\xrightarrow{+}$ AR
63		65	65	1	20	29		y_i = 20.00 $\xrightarrow{+}$ AR +
65		67	67	1	28	28		y = AR $\xrightarrow{+}$ AR
(67)								Return Command

FORM BCD 108-5.0

SPECIFICATIONS

Type:. . . . . .Subroutine for G-15D

Mode of Operation:. . . . . Fixed point, single precision

Execution:. . . . . Command Line 1

Entry:
- (a) Multiplication. . . . . . . .At word time 040
- (b) Division. . . . . .At word time 035

Exit:. . . . . .Normal or Overflow Ret. Comm. from AR at word time 038

Scaling:. . . . . Input and output at decimal scale 10^{-2}

Data Input:. . . . .
$10^{-2}d \longrightarrow 22.00$
$10^{-2}c \longrightarrow 22.01$
$10^{-2}b \longrightarrow 22.02$
$10^{-2}a \longrightarrow 22.03$

Ret. Comm. for normal return to Line 0 ⟶ 21.02
Ret. Comm. for return to Line 0 after overflow ⟶ 22.03

Data Output:. . . . .
$10^{-2}y$ = 20.00
$10^{-2}x$ = 20.01

Normal Ret. Comm. = 21.02
Overflow Ret. Comm. = 21.03

Input data in Line 22 is preserved after multiplication only.

Execution Time:
- (a) Multiplication. . . . . . . .116 msec (4 rev.)
- (b) Division. . . . . .261 msec (9 rev.)

Error Stops:. . . . . None

REMARKS

Programmer must reset overflow before entering routine. Since Line 22 is used, magnetic tape write must not be in process during execution of routine.

METHOD

Multiplication:

$$(a + i\, b)\ (c + i\, d) = X_1 + i\, y_1$$

where:

$$10^{-2} X_1 = 2^7 \cdot (10^2 \cdot 2^{-7}) \left[(10^{-2} \cdot a)(10^{-2} \cdot c) - (10^2 \cdot b)(10^{-2} \cdot d) \right]$$

$$10^{-2} y_1 = 2^7 \cdot (10^2 \cdot 2^{-7}) \left[(10^{-2} \cdot a)(10^{-2} \cdot c) + (10^2 \cdot b)(10^{-2} \cdot d) \right]$$

Division:

$$\frac{a + i\, b}{c + i\, d} = X_2 + i\, y_2$$

For $|c| \geq |d|$

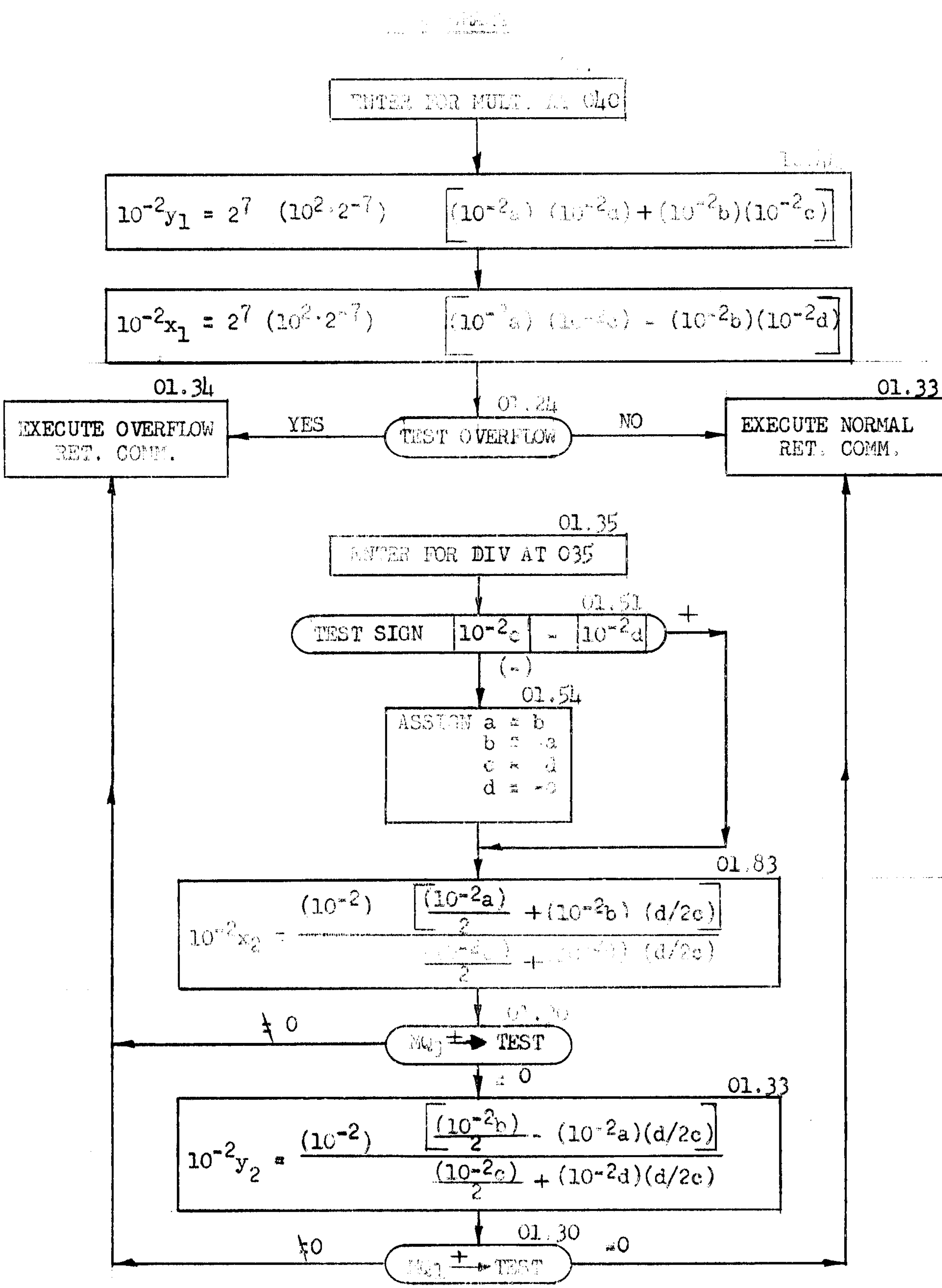
$$10^{-2} X_2 = \frac{(10^{-2}) \left[\frac{(10^{-2} \cdot a)}{2} + (10^{-2} \cdot b) \frac{d}{2c} \right]}{\frac{(10^{-2} \cdot c)}{2} + (10^{-2} \cdot c) \frac{d}{2c}}$$

$$10^{-2} y_2 = \frac{(10^{-2}) \left[\frac{(10^{-2} \cdot b)}{2} - (10^{-2} \cdot a) \frac{d}{2c} \right]}{\frac{(10^{-2} \cdot c)}{2} + (10^{-2} \cdot d) \frac{d}{2c}}$$

For $|c| < |d|$

$$10^{-2} X_2 = \frac{(10^{-2}) \left[\frac{(10^{-2} \cdot b)}{2} + (-10^{-2} \cdot a) \frac{-c}{2d} \right]}{\frac{(10^{-2} \cdot d)}{2} + (-10^{-2} \cdot c) \frac{-c}{2d}}$$

$$10^{-2} y_2 = \frac{(10^{-2}) \left[\frac{(-10^{-2} \cdot a)}{2} - (10^{-2} \cdot b) \frac{-c}{2d} \right]}{\frac{(10^{-2} \cdot d)}{2} + (-10^{-2} \cdot c) \frac{-c}{2d}}$$



FORM BCD 108-7.0

Bendix Computer Los Angeles 45, California

G-15D

PROGRAM PROBLEM: COMPLEX MULTIPLICATION AND DIVISION

Prepared by D. Stein #1205

 Date: 3-13-57 Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
40		42	44	6	22	25		$10^{-2}b = (22.02) \xrightarrow{tva} ID_1$
44	u	46	47	0	22	24		$10^{-2}c = (22.01) \longrightarrow MQ_1$
47		56	u4	0	24	31		Mult.
u4		u6	02	4	26	20		$10^{-4}bc = (PN_{0,1}) \longrightarrow 20.02,03$
02		04	06	6	22	25		$10^{-2}d = (22.00) \xrightarrow{tva} ID_1$
06	u	08	09	0	22	24		$10^{-2}a = (22.03) \longrightarrow MQ_1$
09		56	66	0	24	31		Mult.
66		68	70	4	26	21		$10^{-4}ad = (PN_{0,1}) \longrightarrow 21.00,01$
70		72	76	5	21	26		$10^{-4}ad = (21.00,01) \xrightarrow{+} PN_{0,1}$
76		78	80	5	20	26		$10^{-4}bc = (20.02,03) \xrightarrow{+} PN_{0,1}+$
80		82	84	5	26	20		$10^{-4}(ad+bc) = (PN_{0,1}) \xrightarrow{\pm} 20.02,03$
84		86	88	4	20	25		$10^{-4}(ad+bc) = (20.02,03) \rightarrow ID_{0,1}$
88	u	90	91	0	01	24		$10^2 \cdot 2^{-7} = (01.89) \longrightarrow MQ_1$
89								$10^2 \cdot 2^{-7} =$ w800000
91		10	01	0	24	31		Mult.
01		16	18	[illegible]	[illegible]	[illegible]		$2^7 \cdot \lvert PN \rvert \longrightarrow PN_{0,1}+$
18	u	21	22	2	26	20		$10^{-2}y = (PN_1) \xrightarrow{tva} 20.00$
22		24	26	0	23	31		Clear (Even)
26	u	28	28	0	22	25		$10^{-2}a = (22.03) \longrightarrow ID_1$
28	u	30	31	0	22	24		$10^{-2}c = (22.01) \longrightarrow MQ_1$
31		56	92	0	24	31		Mult.
92		94	96	4	26	20		$10^{-4}ac = (PN_{0,1}) \longrightarrow 20.02,03$
96		98	u0	6	22	25		$10^{-2}b = (22.02) \xrightarrow{tva} ID_1$
u0		u4	05	0	22	24		$10^{-2}d = (22.00) \xrightarrow{tva} MQ_1$
05		56	62	0	24	31		Mult.
62		64	68	0	26	21		$10^{-4}bd = (PN_{0,1}) \longrightarrow 21.00,01$

FORM BCD 108-5.0

Bendix Computer

Los Angeles 45, California

Prepared by D. Stein
S. H. Lewis #1205

Date: 3-13-57

Line 01

G-15D

PROGRAM PROBLEM: COMPLEX MULTIPLICATION AND DIVISION

0	1	2	3	L	P	T or L_k	N	C	S	D	BP	NOTES
4	5	6	7	68		70	74	5	20	26		$10^{-4}ac$ = (20.02,03) $\xrightarrow{+}$ $PN_{0,1}$
8	9	10	11	74		76	78	7	21	30		$10^{-4}bd$ = (21.00,01) $\xrightarrow{-}$ $PN_{0,1}$+
12	13	14	15	78		80	82	5	26	21		$10^{-4}(ac - bd)$ = $(PN_{0,1})$ $\xrightarrow{+}$ 21.00, 01
16	17	18	19	82		84	86	4	21	25		$10^{-4}(ac-bd)$-(21.00,01) $\rightarrow$ $ID_{0,1}$
20	21	22	23	86		89	93	0	01	24		$10^2 \cdot 2^{-7}$ = (01.89) $\longrightarrow$ MQ_1
24	25	26	27	93		10	03	0	24	31		Mult.
28	29	30	31	03		18	20	6	26	30		$2^7 \lvert PN_{0,1} \rvert$ $\longrightarrow$ $PN_{0,1}$+
32	33	34	35	20	u	22	24	0	26	20		$10^{-2}x$ = (PN_1) $\longrightarrow$ 20.01
36	37	38	39	24		26	33	0	29	31		Overflow $\longrightarrow$ Test
40	41	42	43	33	u	35	36	0	21	28		NORMAL {Comm.} = (21.02) $\longrightarrow$ AR
44	45	46	47	34	u	36	36	0	21	28		OVERFLOW {Comm.} = (21.03) $\longrightarrow$ AR
48	49	50	51	36		38	38	0	31	31		Execute AR (Exit)
52	53	54	55	35		38	38	0	23	31		Clear
56	57	58	59	38		40	41	2	22	28		$\lvert 10^{-2}d \rvert$ = 21.00 $\longrightarrow$ AR
60	61	62	63	41	u	43	43	0	28	20		$\lvert 10^{-2}d \rvert$ = (AR) $\longrightarrow$ 20.02
64	65	66	67	43		45	48	2	22	28		$\lvert 10^{-2}c \rvert$ = $\lvert$ 22.01 $\rvert$ $\longrightarrow$ AR
68	69	70	71	48		50	51	3	20	29		$\lvert 10^{-2}d \rvert$ = (20.02) $\xrightarrow{-}$ AR+
72	73	74	75	51		53	54	0	22	31		$T_1 \cdot$ AR $\longrightarrow$ Test
76	77	78	79	54		57	83	0	22	25		$\lvert c \geq d \rvert$ $10^{-2}c$ = (22.01) $\longrightarrow$ ID_1
80	81	82	83	55	u	61	63	2	22	22		$\lvert c < d \rvert$ (22.00,01,02,03) $\xrightarrow{tva}$ (22.01, 02,03,00)
84	85	86	87	63	u	65	65	3	22	28		$10^{-2}a$ = (22.00) $\xrightarrow{-}$ AR
88	89	90	91	65	u	67	69	3	22	30		$10^{-2}c$ = (22.02) $\xrightarrow{-}$ PN_0+
92	93	94	95	69	u	71	72	1	28	22		$-10^{-2}a$ = (AR) $\xrightarrow{+}$ 22.02
96	97	98	99	72		76	77	1	26	22		$-10^{-2}c$ = (PN_0) $\xrightarrow{+}$ 22.00
u0	u1	u2	u3	77		81	63	0	22	25		$10^{-2}d$ = (22.01) $\longrightarrow$ ID_1
u4	u5	u6		83		84	87	6	22	26		$10^{-2}d$ / $10^{-2}c$ = (22.00) $\xrightarrow{tva}$ PN_1

FORM BCD 108-5.0

Bendix Computer Los Angeles 45, California

G-15D

PROGRAM PROBLEM: COMPLEX MULTIPLICATION AND DIVISION

Prepared by D. Stein #1205

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Date: 3-13-57

Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
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52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
87		56	39	5	25	31		Divide
39	u	41	45	0	24	21		$\left.\begin{matrix} d/2c \\ -c/2d \end{matrix}\right] = (MQ_0) \longrightarrow 21.00$
45		48	50	6	21	25		$(21.00) \xrightarrow{tva} ID_1$
50		52	61	6	22	24		$\left.\begin{matrix} 10^{-2}d \\ -10^{-2}c \end{matrix}\right\} = (22.00) \xrightarrow{tva} MQ_1$
61		56	10	0	24	31		Mult.
10		12	16	4	26	20		$\left.\begin{matrix} 10^{-2}d^2/2c \\ 10^{-2}c^2/2d \end{matrix}\right\} = (PN_{0,1}) \longrightarrow 20.00,01$
16		19	19	0	23	31		Clear
19		21	23	0	22	25		$\left.\begin{matrix} 10^{-2}c \\ 10^{-2}d \end{matrix}\right\} = (22.01) \longrightarrow ID_1$
23		02	29	0	26	31		Shift
29		32	37	4	25	20		$\left.\begin{matrix} 10^{-2}c/2 \\ 10^{-2}d/2 \end{matrix}\right\} = (ID_{0,1}) \longrightarrow 20.02,03$
37	u	42	42	5	20	30		$\left.\begin{matrix} 10^{-2}(c/2+d^2/2c) \\ 10^{-2}(d/2+c^2/2d) \end{matrix}\right\} = (20.02,03) + (20.00,01)$
								$\xrightarrow{+} PN_{0,1}+$
42		44	46	5	26	22		Den. $= (PN_{0,1}) \xrightarrow{+} 22.00,01$
46		48	52	6	21	25		$\left.\begin{matrix} d/2c \\ -c/2d \end{matrix}\right\} = (21.00) \xrightarrow{tva} ID_1$
52		54	57	6	22	24		$\left.\begin{matrix} 10^{-2}b \\ -10^{-2}a \end{matrix}\right\} = (22.02) \xrightarrow{tva} MQ_1$
57		56	07	0	24	31		Mult.
07		10	11	4	26	20		$\left.\begin{matrix} 10^{-2}bd/2c \\ 10^{-2}ac/2d \end{matrix}\right\} = (PN_{0,1}) \longrightarrow 20.00,01$
11		14	14	0	23	31		Clear
14	u	16	17	0	22	25		$\left.\begin{matrix} 10^{-2}a \\ 10^{-2}b \end{matrix}\right\} = (22.03) \longrightarrow ID_1$
17		02	21	0	26	31		Shift
21	u	24	25	4	25	20		$\left.\begin{matrix} 10^{-2}a/2 \\ 10^{-2}b/2 \end{matrix}\right\} = (ID_{0,1}) \longrightarrow 20.01,03$
25	u	30	32	5	20	20		$\left.\begin{matrix} 10^{-2}(a/2+bd/2c) \\ 10^{-2}(b/2+ac/2d) \end{matrix}\right\} =$
								$(20.02,03)+(20.00,01) \xrightarrow{+} PN_{0,1}+$
032		34	49	5	26	20		$(PN_{0,1}) \xrightarrow{+} 20.02,03$
49		52	58	4	20	25		$(20.02,03) \longrightarrow ID_{0,1}$
58		31	99	1	01	21		$\overline{10^{-2}} = (01.88) \xrightarrow{+} MQ_1$

FORM BCD 108-5.0

Bendix Computer Los Angeles 45, California

G-15D

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Prepared by D. Stein #1205

Date: 3-13-57

PROGRAM PROBLEM: COMPLEX MULTIPLICATION AND DIVISION

Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
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32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
81								10^{-2} = zx70u3y-
99		58	53	0	24	31		Mult.
52	u	56	[illegible]	1	[illegible]	20		$10^{-4}(a/2+bd/2c)$, $10^{-4}(-b/2+ac/2d)$ = $PN_{0,1}$ → 20.02, 03
56		60	64	4	22	25		Den. = (22.00,01) → $ID_{0,1}$
64		65	15	4	20	26		(20.02,03) → $PN_{0,1}$
15		58	90	5	25	31		Divide
90		92	94	1	24	27		(MQ_1) —+→ Test
94		96	98	6	24	20		NORMAL $10^{-2}x$ = (MQ_0) —tva→ 20.01
95		99	36	0	21	28		OVERFLOW {Comm.} = (21.03) → AR
98		u0	00	6	21	25		d/2c, -c/2d = (21.00) —tva→ ID_1
00		03	13	0	22	24		$10^{-2}a$, $10^{-2}b$ = (22.03) → MQ_1
13		56	79	0	24	31		Mult.
79		82	85	4	26	20		$10^{-2}ad/2c$, $-10^{-2}bc/2d$ = ($PN_{0,1}$) → 20.02,03
85		88	97	6	22	25		$10^{-2}b$, $-10^{-2}a$ = (22.02) —tva→ ID_1
97		02	u1	0	26	31		Shift
u1	u	u4	04	4	25	22		$10^{-2}b/2$, $-10^{-2}a/2$ = ($ID_{0,1}$) → 22.02,03
04		06	08	5	22	26		$10^{-2}b/2$, $-10^{-2}a/2$ = (22.02,03) —+→ $PN_{0,1}$
08		10	12	7	20	30		$10^{-2}ad/2c$, $-10^{-2}bc/2d$ = (20.02,03) —−→ $PN_{0,1}$+
12		16	59	5	26	21		$10^{-2}(b/2-ad/2c)$, $-10^{-2}(-a/2+bc/2d)$ = ($PN_{0,1}$) → 21.00, 01
59		60	73	4	21	25		(21.00,01) → $ID_{0,1}$
73		81	u3	1	01	24		$\overline{10^{-2}}$ = (1_{31}) —+→ MQ_1
u3		58	60	0	24	31		Mult.
60		64	67	4	26	21		$10^{-4}(b/2-ad/2c)$, $10^{-4}(-a/2+bc/2d)$ = ($PN_{0,1}$) → 21.01
67	u	70	71	4	22	25		Den. = (22.00,01) → $ID_{0,1}$
71	u	74	75	4	21	26		No. = (21.00,01) → $PN_{0,1}$
75		58	27	5	25	31		Divide

FORM BCD 108-5.0

Bendix Computer

Los Angeles 45, California

G-15D

PROGRAM PROBLEM: COMPLEX MULTIPLICATION AND DIVISION

Prepared by D. Stein
S. H. Lewis

#1205

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Date: 3-13-57

Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
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64	65	66	67
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72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
27	u	29	30	0	24	20		10^{-2} y = $(MQ_0) \longrightarrow 20.00$
30		32	33	1	24	27		$(MQ_1) \longrightarrow$ Test

FORM BCD 108-5.0

SPECIFICATIONS

Type:. . . . . Subroutine for G-15D

Mode of Operation:. . . .Fixed point, single precision

Execution:. . . . . .From Command Line 1

Entry:. . . . . .First command location = 073

Exit:. . . . . .At word time 054

Scaling:. . . . . .x and a must be given with equal scale factors.
The exponent "m" is to be scaled at 2^{-6}. However, it can be changed to another scaling by changing the increment in 074 to the same scale factor. x^m is obtained at the scale factor with which a^m is entered.

Data Input:. . . . . a^m stored in L21 words 000 and 001
m stored in L21 word 002
a stored in L21 word 003
x stored in AR

Return stored in L1, word 054

Data Output:. . . . . . . .x^m stored in L21, word 001

Execution Time:. . . . . 3 drum revolutions per iteration. Total time requirement is dependent upon the degree of precision desired before termination of calculation, and the choice of input constants.

Error Stops:. . . . . . .None

REMARKS

The choice of the constant "a" is made with consideration for the ease in solving a set of numbers to the same exponent and the need for as few iterations as possible in getting the desired answer. The exponent "m" will usually be an exponent of nonintegral value.

METHOD

Using the Taylor expansion:

$$x^m = a^m + \dot{a}(x-a) + \frac{\ddot{a}}{2!}(x-a)^2 + \dots + \frac{a^{(n)}}{n!}(x-a)^n + \dots$$

where:

$$\dot{a} = ma^{m-1}$$

$$\ddot{a} = m(m-1)a^{m-2}$$

\- - - - - - - - -

$$a^{(n)} = m(m-1)\dots(m-n+1)a^{m-n}$$

Let T_n be the (n+1) term in the above series, i.e.

$$x^m = a^m + \sum_{n=1}^{\infty} T_n$$

T_n can be expressed as:

$$T_n = m(m-1)\dots(m-n+1)a^{m-n}\frac{(x-a)^n}{n!}$$

Since:

$$T_{n-1} = m(m-1)\dots(m-n+2)a^{m-n+1}\frac{(x-a)^{n-1}}{(n-1)!}$$

T_n can be written as:

$$T_n = \frac{(m-n+1)}{n}\ \frac{(x-a)}{a}\ T_{n-1}$$

For n=1, $T_{n-1} = a^m$

Thus, successive terms in the series can be generated by increasing "n" in the quantity:

$$\frac{(m-n+1)}{n}$$

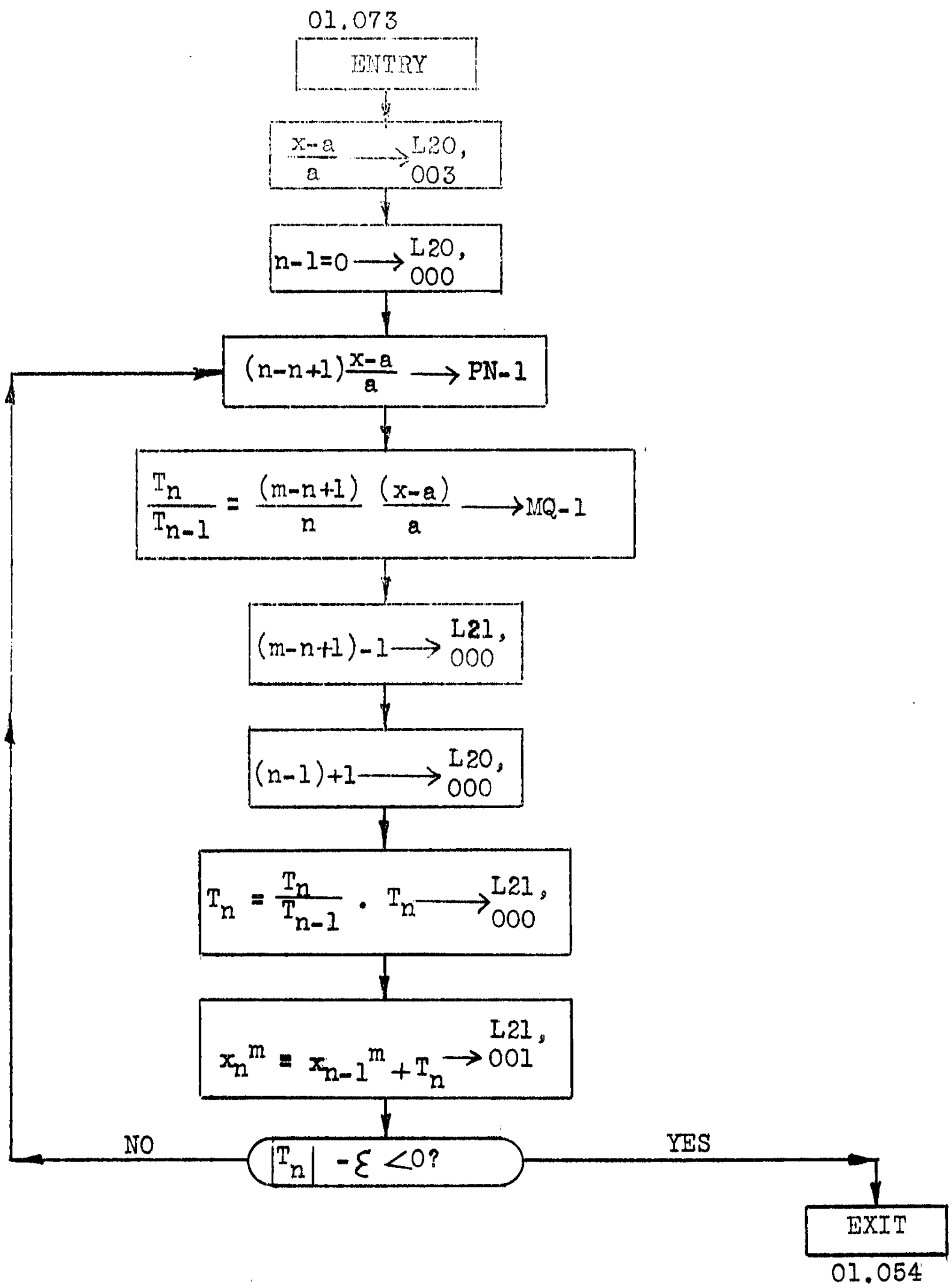
and multiplying by previously obtained quantities. These terms are generated until the T_n obtained is sufficiently small.

FLOW CHART

Preliminary: x ⟶ AR
a^m ⟶ L20 words 000,001
m ⟶ L21 word 002
a ⟶ L21 word 003
Return Link ⟶ 01.054

$$x^m = a^m + \sum_{n=1}^{\infty} T_n$$

$$T_n = \frac{(m-n+1)}{n} \frac{(x-a)}{a}$$



Bendix Computer

Los Angeles 45, California

G-15D

PROGRAM PROBLEM: $x^{m^{th}}$

Prepared by D. Hasselli / Shig Ochi #1206

Page 1 of 2

Date: 3-13-57

Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
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64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
73	u	76	78	4	23	31		Clear
78		80	82	0	21	25		a to ID - 1
82	u	84	84	3	21	29		x - a in AR
84	u	86	86	1	28	28		$\pm\lvert x-a \rvert$ in AR
86	u	88	91	0	28	26		$\pm\lvert x-a \rvert$ to PN - 1
91		58	45	5	25	31		Divide: $\frac{x-a}{a}$ in MQ - 0
45	u	49	53	6	24	20		$\frac{x-a}{a}$ to 20.003; (n-1) to 20.000
53		56	58	0	23	31		Clear
58		62	64	6	21	25		(m-n+1) to ID - 1
64		66	70	1	21	28		(m-n+1) to AR
70	u	72	72	0	20	24		$\frac{x-a}{a}$ to MQ - 1
72		74	75	3	01	29		[(m-n+1)-1] in AR
75		56	25	0	24	31		Mult: (m-n+1) $\frac{x-a}{a}$ in PN - 1
25	u	27	27	1	28	21		[(m-n+1)-1] to 21.002
27	u	29	34	0	20	28		(n-1) to AR
34	u	36	39	1	01	29		n in AR
39	u	41	48	0	28	20		n to 20.000
48	u	50	51	1	28	25		n to ID -1
51		v6	60	5	25	31		Divide: $\frac{T_n}{T_{n-1}}$ in MQ - 1
60	u	62	63	0	24	28		$\frac{T_n}{T_{n-1}}$ to AR
63		65	67	0	28	25		$\frac{T_n}{T_{n-1}}$ to ID - 1
67	u	70	71	2	21	24		T_{n-1} to MQ - 1
71		56	22	0	24	31		Mult: T_n in PN - 1
22	u	24	24	0	26	28		$\pm\lvert T_n \rvert$ to AR
24	u	26	31	1	28	28		T_n in AR
31	u	33	36	1	23	21		$\pm\lvert T_n \rvert$ to 21.000

FORM BCD 108-5.0

Bendix Computer

Los Angeles 45, California

Prepared by D. Hassell / Shig Ochi #1206

Date: 3-13-57

G-15 D

PROGRAM PROBLEM: x^{m}th

Line 01

L	P	T or L_k	N	C	S	D	BP	NOTES
36	u	38	40	4	21	29		$x_n^m = x_{n-1}^m + T_n$ in AR
40	u	42	42	1	28	21		x_n^m to 21.001
42	u	44	47	3	01	28		$-\varepsilon$ to AR
47	u	49	50	2	21	29		$\lvert T_n\rvert - \varepsilon$ in AR
50	u	52	53	0	22	31		$\lvert T_n\rvert - \varepsilon < 0$?

If test is not set, go back to 033 and calculate next T_n.

If test is set, 054 has return link to main routine.

CONSTANTS:

L	T	NOTES
35	0400000	1×2^{-6}
43	0000004	ε
74	0400000	1×2^{-6}

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 u0 u1 u2 u3 u4 u5 u6

SPECIFICATIONS

Type:	Subroutine for G-15D
Mode of Operation:	Fixed point, single or double precision
Execution:	From Line 01
Entry:	Single precision: Word time 00
	Double precision: Word time 02
Exit:	Ret. comm. from line 01, word time 01
Scaling:	Binary scaling on x such that $x \cdot 2^{\alpha} < 1$
	(α negative if $x > 1$)
	α at 2^{-28}
Data Input:	Single precision: $x \longrightarrow$ 20.00
	Double precision: $x \longrightarrow$ 20.00 and 20.01
	Binary scale factor (α) $\longrightarrow$ 20.01 (necessary for single precision only)
Data Output:	$\log_e x \cdot 2^{-5} \longrightarrow$ 20.01
	$\log_{10} x \cdot 2^{-5} \longrightarrow$ 20.03
	Ret. Comm. $\longrightarrow$ 01.01
Execution Time:	149 msec (5 rev.)
Error Stops:	If $x \leq 0$, routine will ring bell and halt.
Short Lines Used:	20.00, 20.01, 20.02, and 20.03
Registers Used:	$MQ_{0,1}$, $ID_{0,1}$, $PN_{0,1}$, and AR

METHOD

Transform x to a variable y where: $1/\sqrt{2} \leq y \leq \sqrt{2}$

as follows:

Let $x = w \cdot 2^{\alpha}$ where: $\frac{1}{2} \leq w < 1$ (1)

and $y = \sqrt{2} \cdot w$

therefore, $\log_2 y = \log_2 \sqrt{2} + \log_2 w$ (2)

Use series

$$\log_2 y = c_1 \,(y-1/y+1) + c_3 \,(y-1/y+1)^3 + c_5 \,(y-1/y+1)^5$$

Obtain $\log_2 w$

from eq (2) $\log_2 w = \log_2 y - \log_2 \sqrt{2}$

Transform $\log_2 w \longrightarrow \log_2 x$

from eq (1) $\log_2 x = \log_2 w + \alpha \log_2 2$

$$\log_2 x = \log_2 w + \alpha$$

Finally $\log_e x = \log_2 x / \log_2 e = \log_2 x \;(.69315)$

and $\log_{10} x = \log_e x / \log_e 10 = \log_e x \;(.4342944)$

Ref: Approximations for Digital Computers by Cecil Hastings, Jr.
(A research study by the Rand Corporation)
Princeton University Press, 1955, page 165

PRELIMINARY:

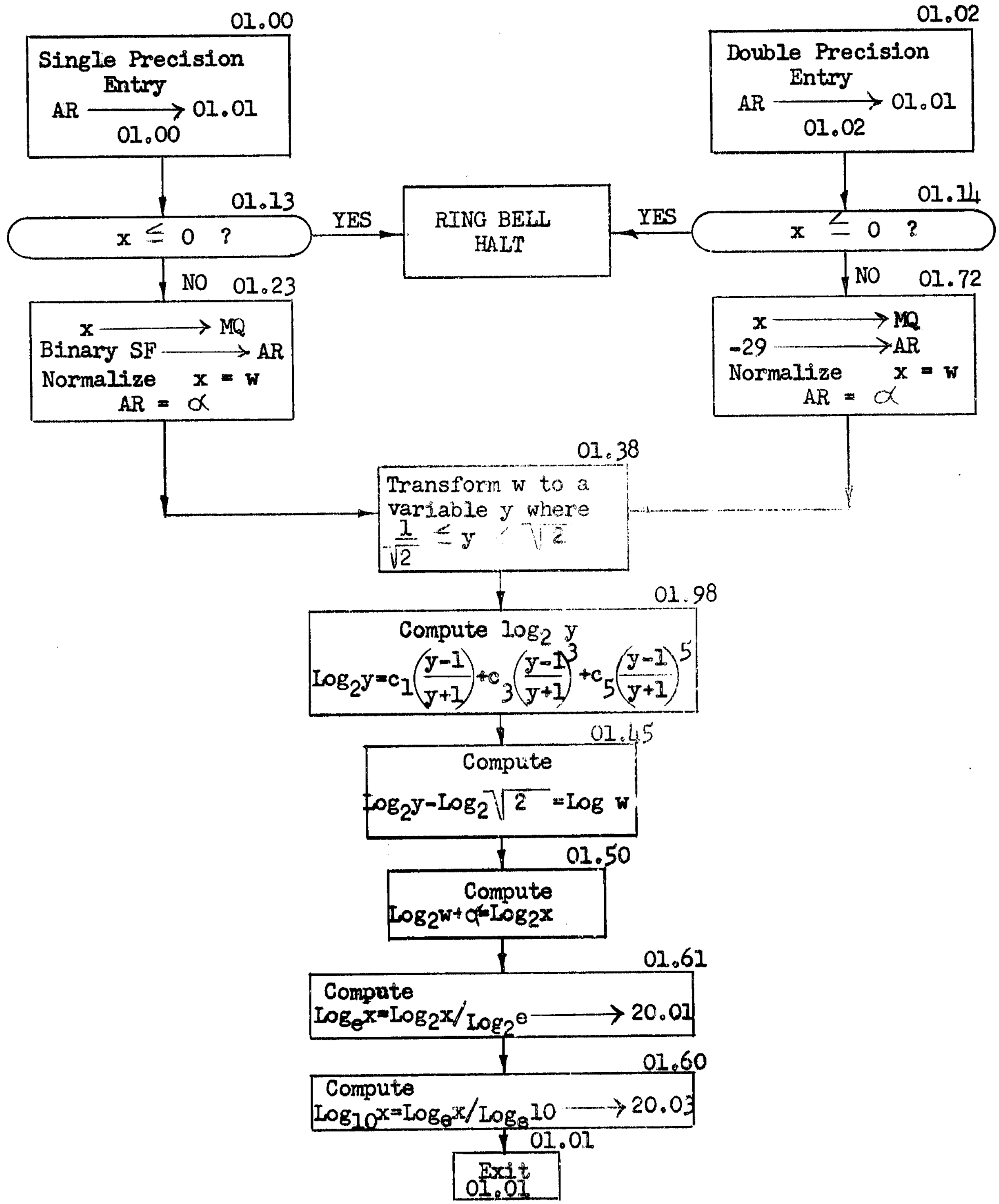
SINGLE PRECISION

X ⟶ 20.00
Binary Scale Factor ⟶ 20.01

DOUBLE PRECISION

X ⟶ 20.00,20.01

Return Link ⟶ AR



Bendix Computer Los Angeles 45, California

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G-15D

PROGRAM PROBLEM: LOG X

Prepared by D. Hassell #1207

Date: 3-13-57

Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
00		01	04	0	28	01		Single Precision Link
04		07	08	0	23	31		Clear 2wd registers
08		12	13	6	20	24		x ⟶ MQ
13		16	17	1	20	28		x ⟶ AR
17		20	21	0	22	31		AR < 0 ?
21		24	22	0	20	27		No 20.00 = 0 ?
22		21	26	0	17	31		YES Ring Bell
26		28	29	0	16	31		Halt
23		25	27	1	20	28		NO 20.01=(BSF exponent) ⟶ AR
02		01	03	0	28	01		Double precision link
03		06	09	0	23	31		Clear 2 word register
09		12	14	4	20	24		2000/2001 ⟶ $MQ_{0,1}$
14		17	24	0	20	27		2001 = 0 ?
24		28	80	0	20	27		Yes 2000 = 0 ?
25		28	33	0	20	28		No 2000 ⟶ AR
80		00	22	0	01	01		Yes (Time link)
81		00	25	0	01	01		No (Time link)
33		70	72	0	22	31		AR < 0 ?
72		74	27	1	01	28		No $(-29 \cdot 2^{-28})$=01.74 ⟶ AR
73		00	22	0	01	01		(Time link)
(74)								$-29 \cdot 2^{-28}$
27		u4	35	0	27	31		Normalize MQ = w
35		37	38	1	28	20		(BSF)=AR ⟶ 20.01
38		39	41	0	01	25		$\sqrt{2}$ ⟶ ID
(39)								$\sqrt{2} \cdot 2^{-2}$
41		56	98	0	24	31		$\sqrt{2} \cdot w = y$

FORM BCD 108-5.0

Bendix Computer

Los Angeles 45, California

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Prepared by D. Hassell #1207 Date: 3-13-57

G-15 D
PROGRAM PROBLEM: LOG X

Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
98		99	u0	0	26	20		y → 20.03
u0		u3	u4	1	20	28		y → AR
u4		12	16	0	01	20		$1 \cdot 2^{-2}$ → 20.00
(12)								$1 \cdot 2^{-2}$
16		20	29	1	20	29		y + 1
29		31	32	0	28	25		y + 1 → ID_1
32		35	36	1	20	28		y → AR
36		40	42	3	20	29		y - 1 → AR
42		43	44	1	28	28		AR → AR
44		45	47	0	28	26		y - 1 → AR
47		57	u5	1	25	31		$\frac{y-1}{y+1} = \phi$
u5		00	07	0	24	20		ϕ → 20.00
07		10	19	0	23	31		Clear registers
19		20	28	6	20	25		ϕ → ID_1
28		32	37	6	20	24		ϕ → MQ_1
37		56	94	0	24	31		ϕ^2
94		95	96	0	26	20		ϕ^2 → 20.03
96		99	u1	0	20	25		ϕ^2 → ID_1
u1		u3	31	0	01	24		c_5 → MQ_1
(u3)								$c_5 \cdot 2^{-5}$
31		56	88	0	24	31		$\phi^2 \cdot c_5$
88		89	92	0	26	28		$\phi^2 c_5$ → AR
92		93	95	1	01	29		$\phi^2 c_5 + c_3$
(93)								$c_3 \cdot 2^{-5}$
95		97	99	0	28	25		$\phi^2 c_5 + c_3$ → ID_1
99		u3	53	0	20	24		ϕ^2 → MQ_1

FORM BCD 108-5.0

Bendix Computer

Los Angeles 45, California

G-15D

Prepared by D. Hassell #1207 Date: 3-13-57

PROGRAM PROBLEM: LOG X Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
53		56	06	0	24	31		$\phi^2 (\phi^2 c_5 + c_3)$
06		07	30	0	26	28		$\phi^2 [\phi^2 c_5 + c_3]$
30		34	40	1	01	29		$\phi^2 [\phi^2 c_5 + c_3] + c_1$
(34)								$c_1 \cdot 2^{-5}$
40		45	46	0	28	25		$\phi^2 [0^2 c_5 + c_3] + c_1 \longrightarrow ID$
46		48	49	6	20	24		$\phi \longrightarrow MQ_1$
49		56	u6	0	24	31		$c_1\phi + c_3\phi^3 + c_5\phi^5 = Log_2 y$
u6		07	10	0	26	28		$Log_2 y \longrightarrow AR$
10		13	43	0	23	31		Clear register
43		44	45	1	28	28		$AR \longrightarrow AR$
45		51	52	3	01	29		$Log\ y - Log\ 2\sqrt{2} = Log\ w$
(51)								$Log_2 \sqrt{2} \cdot 2^{-5}$
52		55	56	1	28	20		$Log_2 w \longrightarrow 20.03$
56		57	59	0	20	24		$\alpha \longrightarrow MQ_1$
59		46	48	0	26	31		Shift 23 bits
48		49	50	0	24	20		$\alpha \longrightarrow 20.01$
50		51	54	1	20	28		$Log_2 w \longrightarrow AR$
54		57	58	3	20	29		$Log_2 w + \alpha = Log_2 x$
58		60	61	1	28	20		$Log_2 x \longrightarrow 20.00$
61		64	66	6	20	25		$Log_2 x \longrightarrow ID_1$
66		67	69	0	01	24		$f_1 \longrightarrow MQ_1$
69		56	55	0	24	31		$Log_2 x \cdot f_1 = Log_e x$
(67)								f_1 (69314)
55		57	60	0	26	20		$Log_e x \longrightarrow 20.01$
60		61	62	0	20	25		$Log_e x \longrightarrow ID_1$
62		65	71	0	01	24		$f_2 \longrightarrow MQ$

Bendix Computer

Los Angeles 45, California

G-15D

PROGRAM PROBLEM: LOG X

Prepared by D. Hassell #1207

Date: 3-13-57

Line 01

L	P	T or L_k	N	C	S	D	BP	NOTES
(65)								(f_2)
71		56	57	0	24	31		$Log_e x \circ f_2 = Log_{10} x$
57		59	01	0	26	20		$Log_{10} x \longrightarrow 20.03$
u7		[-y2wzx84]						Converts check sum to index
								number = 102 - D

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 u0 u1 u2 u3 u4 u5 u6

FORM BCD 108-5.0

SPECIFICATION

Type. . . . . Subroutine for G-15D

Mode of Operation . . . . . . Fixed point, single precision

Execution . . . . From Line 01

Entry . . . . At memory location 01.00

Exit. . . . . Return command from 01.01

Scaling . . . . Input $|x| < 19.35$

Output $2^{-28} < e^x < 2^{29}$

Input . . . . $x \cdot 2^{-5} \longrightarrow 20.00$

. . . . Return command $\longrightarrow$ AR

Output. . . . . e^x (Integer part) $\cdot 2^{-29} \longrightarrow MQ_1$

(Fractional part) $\cdot 2^0 \longrightarrow MQ_0$

Return Command = 01.01

Execution Time . . . . 319 msec (11 rev)

Error Stops . . . . If $|x| > 19.35$ routine will ring bell and halt.

Short Lines Used. . . . . 20.00 thru 20.03

REMARKS

If $x < 0$ the output can be thought of as being in MQ_0 scaled 2^0.

METHOD

Convert x to an exponent z so that $10^z = e^x$ as follows:

$$\text{Let } x \cdot \text{Log}_{10} e = z \qquad (1)$$

$$\text{then } \quad 10^z = e^x \qquad (2)$$

Let $z = I + f$ where I = Integer and f = fraction (3)

use series $10^f = \left[1 + a_1 f + a_2 f^2 + \ldots. + a_7 f^7\right]^2$

a_1 = 1.1512,9277,603
a_2 = .6627,3088,429
a_3 = .2543,9357,484
a_4 = .0729,5173,666
a_5 = .0174,2111,988
a_6 = .0025,5491,796
a_7 = .0009,3264,267

10^I is stored in routine where $0 \leq I \leq 8$

from eq. (2) $10^z = 10^I \cdot 10^f$ (4)

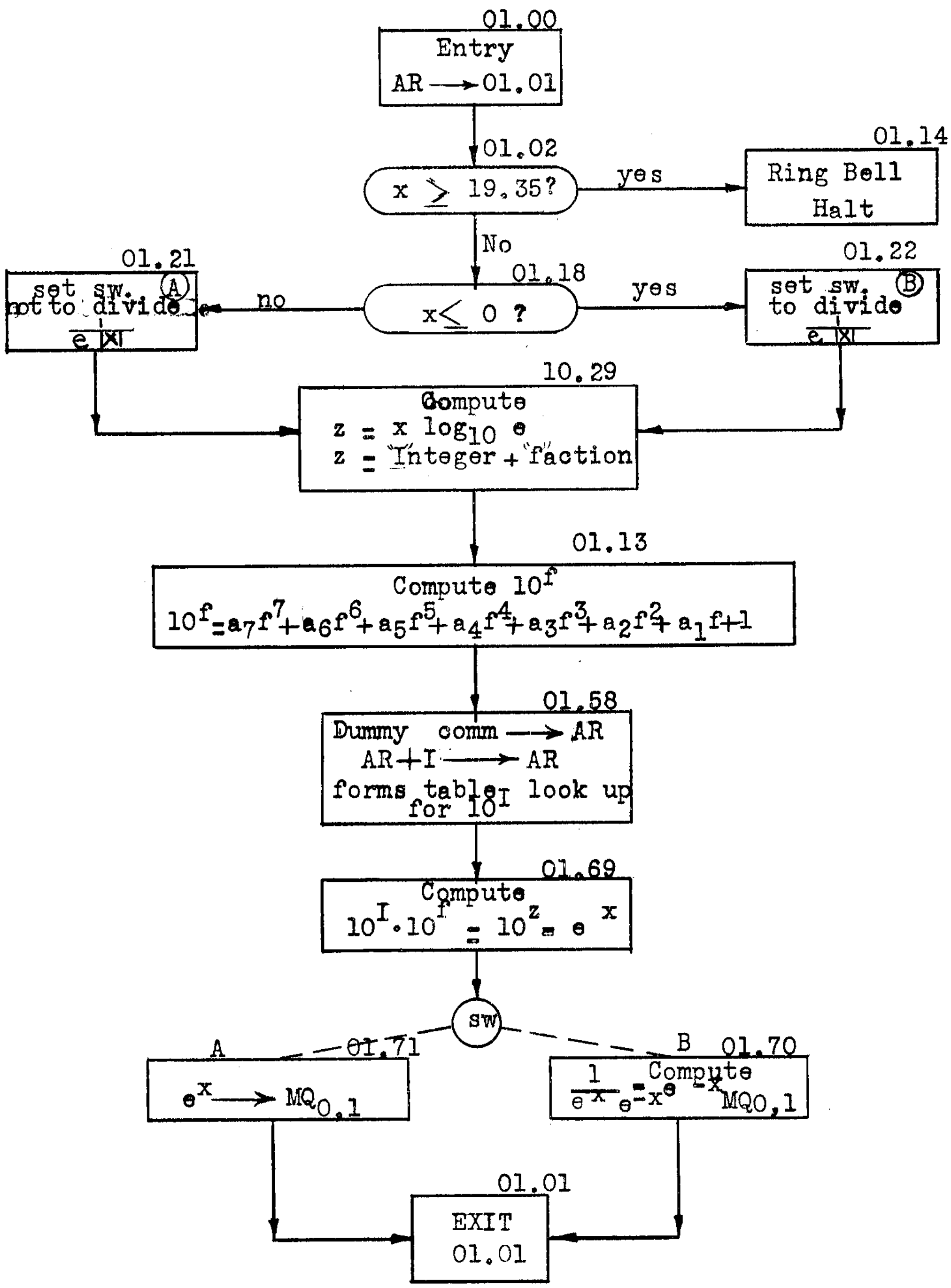
and finally from eq. (2) and eq. (4) $10^I \cdot 10^f = e^x$

Ref: Approximations for Digital Computers by Cecil Hastings
(a research study by the Rand Corporation)
Princeton University Press, 1955, page 144

FLOW CHART

Preliminary: $x \cdot 2^{-5} \longrightarrow 20.00$

Return Link $\longrightarrow$ AR



Bendix Computer

Los Angeles 45, California

Prepared by D. Hassell #1208 Date: 3-13-57

G-15D
PROGRAM PROBLEM: e^x (G-15D) Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
00		01	02	0	28	01		Link
02		04	06	2	20	28		$\lvert x \rvert \rightarrow$ AR 2^{-5}
06		07	08	3	01	29		$\lvert x \rvert - 19.35 \rightarrow$ AR
08		10	14	0	22	31		AR < 0 ? yes
15		16	18	0	20	28		$x \rightarrow$ AR
14		13	20	0	17	31		Ring bell
20		17	22	0	16	31		Halt
18		20	21	0	22	31		AR < 0 ?
21		23	28	0	01	28		set dummy no div. no
22		24	26	3	28	20		set dummy to $\frac{1}{e^{\lvert x \rvert}}$ yes
26		27	28	0	01	28		set Link comm.
28		69	29	0	28	01		
29		30	31	6	01	25		$\text{Log}_{10}\, e \rightarrow ID_{1,0}$
31		32	35	6	20	24		$x \rightarrow MQ_1$ 2^{-5}
35		56	98	0	24	31		$\text{Log}_{10}\, e \cdot x = z$ 2^{-5}
98		99	u0	0	26	28		$z \rightarrow$ AR
u0		u2	u3	0	28	24		
u3		10	09	0	26	31		Shift 10 wts.
09		10	11	0	24	20		$f \cdot 2^0 \rightarrow 20.02$
11		02	13	0	26	31		$I \cdot 2^{-28}\ \Sigma\ MQ_1$
13		16	17	6	01	25		$a_7 \rightarrow ID_1$
17		19	24	0	24	20		$I \rightarrow 20.03$
24		26	33	6	20	24		f
33		56	99	0	24	31		$a_7 \cdot f$
99		u1	u2	1	01	30		$a_7 \cdot f + a_6$
u2		u3	u4	0	26	25		

FORM BCD 108-5.0

Bendix Computer

Los Angeles 45, California

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G-15D

PROGRAM PROBLEM : e^x (G-15D)

Prepared by D. Hassell #1208

Date: 13-3-57

Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
u4		u6	03	6	20	24		f_1
03		56	60	0	24	31		$(a_7 f + a_6)\ f$
60		61	62	1	01	30		$(a_7 f + a_6)\ f + a_5$
62		63	64	0	26	25		
64		66	67	6	20	24		f
67		56	32	0	24	31		$[(a_7 f + a_6)\ f + a_5]\ f$
32		37	38	1	01	30		$a_7 f_3 + a_6 f^2 + a_5 f + a_4$
38		39	40	0	26	25		
40		42	43	6	20	24		f
43		56	u5	0	24	31		$a_7 f^4 + a_6 f^3 + a_5 f^2 + a_4 f$
u5		05	10	1	01	30		$a_7 f^4 + a_6 f^3 + a_5 f^1 + a_4 f + a_3$
10		11	34	0	26	25		
34		38	39	6	20	24		f
39		56	u6	0	24	31		$a_7 f^5 + a_6 f^4 + a_5 f^3 + a_4 f^2 + a_3 f$
u6		41	42	1	01	30		$a_7 f^5 + a_6 f^4 + a_5 f^3 + a_4 f^2 + a_3 f + a_2$
42		43	44	0	26	25		
44		46	47	6	20	24		f
47		56	04	0	24	31		$a_7 f^6 + a_6 f^5 + a_5 f^4 + a_4 f^3 + a_2 f^2 + a_2 f$
04		45	48	1	01	30		$a_7 f^6 + a_6 f^5 + a_5 f^4 + a_4 f^3 + a_3 f^2 + a_2 f + a_1$
48		49	50	0	26	25		
50		54	55	6	20	24		f
55		56	49	0	24	31		$a_7 f^7 + a_6 f^6 + a_5 f^5 + a_4 f^4 + a_3 f^3 + a_2 f^2 + a_1 f$
49		51	52	1	01	30		$a_7 f^7 + a_6 f^6 + a_5 f^5 + a_4 f^4 + a_3 f^3 + a_2 f^2 + a_1 f + 1$
52		53	54	0	26	25		
54		55	57	0	25	24		
57		56	53	0	24	31		$10^f \rightarrow$ PN

FORM BCD 108-5.0

Bendix Computer Los Angeles 45, California

G-15D PROGRAM PROBLEM: e^x (G-15D)

Prepared by D. Hassell #1208

Date: 3-13-57

Line 01

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63
64	65	66	67
68	69	70	71
72	73	74	75
76	77	78	79
80	81	82	83
84	85	86	87
88	89	90	91
92	93	94	95
96	97	98	99
u0	u1	u2	u3
u4	u5	u6	

L	P	T or L_k	N	C	S	D	BP	NOTES
53		55	56	0	26	24		
56		59	63	1	20	28		I ⟶ AR
63	u	85	58	1	28	29		2 I
58		59	65	0	01	29		
65		67	67	0	31	31		N.C. ⟵ AR
69		56	66/70	4	24	31		Multiply
66		68	71	4	26	24		$e^x \longrightarrow MQ_{0,1}$ Ⓐ
71		04	01	0	26	31		Shift e^x (exit) - 29
70		72	74	4	26	25		$e^{\lvert x \rvert} \longrightarrow ID_{0,1}$ Ⓑ
74		76	79	4	01	26		1 ⟶ PN
79		57	01	5	25	31		$\frac{1}{e^{\lvert x \rvert}} = e^{-x}$ (exit)
80-1								$10^0 \cdot 2^{-27}$
82-3								$10^1 \cdot 2^{-27}$
84-5								$10^2 \cdot 2^{-27}$
86-7								$10^3 \cdot 2^{-27}$
88-9								$10^4 \cdot 2^{-27}$
90-1								$10^5 \cdot 2^{-27}$
92-3								$10^6 \cdot 2^{-27}$
94-5								$10^9 \cdot 2^{-27}$
96-7								$10^8 \cdot 2^{-27}$
76-7								$1 \quad 2.^{31}$

FORM BCD 108-5.0

Bendix Computer

Los Angeles 45, California

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G-15D

PROGRAM PROBLEM: e^x (G-15D)

Prepared by D. Hassell #1208

Date: 3-13-57

Line 01

0 1 2 3	L	P	T or L_k	N	C	S	D	BP	NOTES
4 5 6 7	(07)								$19.35 \cdot 2^{-5}$
8 9 10 11	(23)		56	26	4	24	31		dummy
12 13 14 15	(27)		56	70	4	24	31		dummy
16 17 18 19	(30)								$\log_{10} e$
20 21 22 23	(16)								a_7 = (.0009326426)
24 25 26 27	(u1)								a_6 = (.00255491796)
28 29 30 31	(61)								a_5 = (.0174211198)
32 33 34 35	(37)								a_4 = (.07295173666)
36 37 38 39	(05)								a_3 = (.25439357484)
40 41 42 43	(41)								a_2 = (.6627,3088,429)
44 45 46 47	(45)								a_1 = (1.1512,92177603)
48 49 50 51	(51)								(1)
52 53 54 55	(59)	w	80	69	4	01	25		$10^I \rightarrow ID_{0,1}$ clear $PN_{0,1}$
56 57 58 59									
60 61 62 63									
64 65 66 67	(u7)								a constant used to set up index number
68 69 70 71									
72 73 74 75									
76 77 78 79									
80 81 82 83									
84 85 86 87									
88 89 90 91									
92 93 94 95									
96 97 98 99									
u0 u1 u2 u3									
u4 u5 u6									

FORM BCD 108-5.0

APPROXIMATIONS USED:

Function: $\sin \pi/2\, x$

Range: $-1 \leq x \leq 1$

Approx: $\sin \pi/2\, x^* = C_1 x + C_3 x^3 + C_5 x^5 + C_7 x^7$

C_1	1.570794852
C_3	-.645920978
C_5	.079487663
C_7	-.004362476

Max. Relative Error: .000001

Function: Arcsine $x = \pi/2 - \sqrt{1-x}\ \psi(x)$

Range: $0 \leq x \leq 1$

Approx: $\psi^*(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$

a_0	1.570795207
a_1	-.214512362
a_2	.087876311
a_3	-.044958884
a_4	.019349939
a_5	-.004337769

Use is made of trig. identities:

$\cos \pi/2\, x = \sin \pi/2\, (x + 1)$ for cos routine

APPROXIMATIONS USED: Continued

$$\text{Sin } \theta = \frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}}$$ for arctan routine

Greater accuracy of the routines may be achieved by inserting the constants via an intercom program prior to punching the routine on tape. For example, to obtain C_1, use program:

Clear and add	51.15707
Add	46.94852
Store in Loc	036

If it is known that the angle (of which the sine or cosine is to be obtained) lies within a certain 180° range, the routine may be speeded up considerably by dividing by appropriate constant ($\pi/2$ or 90) after putting angle in principal value range ($0 \leq \cos\theta \leq 180$ or $-90 \leq \sin\theta \leq 90$), then entering at 015 for cos and 017 for sin.

ENTRIES

sin θ	degrees	000
	radius	001
cos θ	degrees	003
	radius	004
arctan X		037
arcsin X		044

Input: θ or x to A register
Output: x or θ in A register

sin ent = 0,1
cos ent = 3,4
arctan ent = 37
arcsin ent = 44

Reference: Hastings Approx. for Digital Computers.

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G-15 INTERCOM ORDERS

Prepared by William Krause

Date: 3-13-57

PROBLEM: INTERCOM SUBROUTINE Sine

Line

NOTES	LOCATION	OPER. CODE	ADDRESS	k	(A)
Multiply by $\pi/180$	0	67	028	0	
Divide by $\pi/2$	1	4z	29	0	
Transfer to (6)	2	19	06	0	
Multiply by $\pi/180$	3	67	28	0	
Divide by $\pi/2$	4	4z	29	0	
Add 1	5	5v	30	0	
Add 1	6	5v	30	0	
Transfer if non-neg to (10)	7	10	10	0	
Add 4	8	5v	32	0	
Transfer if neg to (8)	9	0v	08	0	
Subtract 4	10	59	32	0	
Transfer if non-neg to (10)	11	10	10	0	
Add 2	12	5v	31	0	
Negate	13	29	00	0	
Transfer if neg to (16)	14	0v	16	0	
Negate	15	29	00	0	
Add 1	16	5v	30	0	
Store x	17	5x	77	0	
Multiply by x	18	67	77	0	
Store x^2	19	5x	78	0	
Multiply by C_7	20	67	33	0	
Add C_5	21	5v	34	0	
Multiply by x^2	22	67	78	0	
Add C_3	23	5v	35	0	
Multiply by x^2	24	67	78	0	
Add C_1	25	5v	36	0	

FORM BCD 108-4.0

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G-15 INTERCOM ORDERS

Prepared by William Krause

Date: 3-13-57

PROBLEM: INTERCOM SUBROUTINE Sine

Line

NOTES	LOCATION	OPER. CODE	ADDRESS	k	(A)
Multiply by $\lvert x \rvert$	52	67	078	0	
Add a_4	53	5v	72	0	
Multiply by $\lvert x \rvert$	54	67	78	0	
Add a_3	55	5v	73	0	
Multiply by $\lvert x \rvert$	56	67	78	0	
Add a_2	57	5v	74	0	
Multiply by $\lvert x \rvert$	58	67	78	0	
Add a_1	59	5v	75	0	
Multiply by $\lvert x \rvert$	60	67	78	0	
Add a_o	61	5v	76	0	
Multiply $\sqrt{1 - \lvert x \rvert}$	62	67	79	0	
Subtract $\pi/2$	63	59	29	0	
Store $\pm$ θ	64	5x	79	0	
Clear and add x	65	4v	77	0	
Transfer if neg. to 69	66	0v	69	0	
Clear and Subtract - θ	67	49	79	0	
Return Transfer	68	44	00	0	
Clear and Add θ	69	4v	79	0	
Return Transfer	70	44	00	0	
a_5	71		-48.43378		
a_4	72		49.19350		
a_3	73		-49.44959		
a_2	74		49.87876		
a_1	75		-50.21451		
a_o	76		51.15708		

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G-15 INTERCOM ORDERS

Prepared by William Krause

Date: 3-17-57

PROBLEM: INTERCOM SUBROUTINE Sine

Line

NOTES	LOCATION	OPER. CODE	ADDRESS	k	(A)
Multiply by x	26	67	077	0	
Return Transfer	27	44	00	0	
$\pi/180$	28		49.17453		
$\pi/2$	29		51.15708		
1	30		51.10000		
2	31		51.20000		
4	32		51.40000		
C_7	33		-48.43625		
C_5	34		49.79488		
C_3	35		-50.64592		
C_1	36		51.15708		
Store y	37	5x	077	0	
Square y	38	67	77	0	
Add 1	39	5v	30	0	
Sq. Rt.	40	07	00	0	
Store $\sqrt{1 + y^2}$	41	5x	78	0	
Clear and Add y	42	4v	77	0	
Divide by $\sqrt{1 + y^2}$	43	4z	78	0	
Store x	44	5x	77	0	
Take abs. value of x	45	13	00	0	
Store $\|x\|$	46	5x	78	0	
Negate	47	29	00	0	
Add 1	48	5v	30	0	
Square Rt.	49	07	00	0	
Store $\sqrt{1 - \|x\|}$	50	5x	79	0	
Clear and add a_5	51	4v	71	0	

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G-15 INTERCOM ORDERS

Prepared by William Krause

Date: 3-13-57

PROBLEM: INTERCOM SUBROUTINE Sine

Line

NOTES	LOCATION	OPER. CODE	ADDRESS	k	(A)
	77				
Storage	78				
	79				

FORM BCD 108-4.0