CHANGES IN COMPUTER PERFORMANCE

a historical view

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The first 20 years of the computer industry have been hectic ones. Great strides have been taken to provide reliable and inexpensive computation capability. To obtain a clearer picture we will explore our past to see where we have been and how fast we have had to move to get to where we are today. From our analysis of the first 20 years of the computing industry, we have arrived at four fascinating observations that we will discuss in this paper.

1. We generate a performance description for 225 general-purpose computer systems. The performance description estimates the over-all capabilities of each computer system based upon its hardware features and basic elementary operations. We obtain estimates of the performance capabilities for both scientific and commercial computation for 225 different computer systems introduced between 1944 and 1963.
2. Using the performance descriptions for the computers introduced in any one year, we generate a technology curve for that year. The technology curve describes the theoretical performance that can be purchased for different monthly rental expenditures.
3. Grosch's law is upheld. For any one year we find the relation between computing power and system cost to be approximately as follows: Computing power $= (C \approx \text{system cost})^2$; $C =$ constant.
4. Improvement in number of operations per dollar between 1950 and 1962 has been at an average rate of 81% per year for scientific computation and 87% per year for commercial computation.

functional description of gp computers

The capability of each system to perform its computing tasks represents the functional description (or evaluation) of that system. For our purposes we will only look at two aspects of computer performance: 1) Computing power, indicated by the number of standard operations performed per second (P); 2) Cost of the computing equipment, which equals the number of seconds of system operations per dollar of equipment cost (C).

Computing power (P) evaluates the rate at which the system performs information processing, the number of operations performed per second. Two machines solve a specific problem with different internal operations because of their individual equipment features. (P) will, therefore, describe operations of equivalent problem solving value to provide the desired measure of a computer's performance. We will estimate (P) from structure. In order to do this, we first must understand which structural factors influence computing capability. Then we determine the manner in which the structural factors interact to develop the functional model through the use of detailed study of the operation of computing equipment and the problems performed. (P) consists of three main components: 1) the internal calculating speed of the computer's central processor (t_c); 2) the time the central processor is idle and waiting for information input or output ($t_{I/O}$); and 3) the memory capacity of the computer (M). These factors are the important performance measures needed to determine (P). We define t_c as the time (in microseconds) needed to perform 1 million operations, and ($t_{I/O}$) as the

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on-overlapped input-output time (microseconds) necesary for these 1 million operations. Therefore, the computr performs $\frac{10^{12}}{t_c + t_{I/O}}$ operations per second. The comuter's memory has a strong influence on (P). We found at the memory factor interacts with internal operating me to determine computing power as follows:

$$M \times \frac{10^{12}}{t_c+t_{I/O}} = P$$ [1]

The internal speed of the central processor, t_c, is the time ken by the computer to perform its information processg tasks. The speed equals the internal operation times f each computer, multiplied by the frequency with hich each operation is used. To determine the internal eed, therefore, it is necessary to measure the frequency ith which the various operations are performed in a pical problem. For scientific computation we considered pproximately 15,000,000 operations of an IBM 704 and n IBM 7090, from a mix of over 100 problems. In the nalysis of the operations used in this "problem mix" e instructions were grouped into five categories:

1. Fixed add (and subtract) and compare instructions performed.
2. Floating add (and subtract) instructions required.
3. Multiply instructions.
4. Divide instructions.
5. Other manipulation and logic instructions—this category combines a large number of branch, shift, logic, and load-register instructions.

The relative frequency with which each of the five ypes were used in the scientific programs we traced is resented in Fig. 1. (p. 42).

To determine the frequency with which the different perations were used in commercial computation, nine rograms were analyzed in detail (two inventory control, ree general accounting, one billing, one payroll, and two roduction planning). All nine problems were run on an BM 705, representing over one million operations. We nalyzed the nine programs using the same five instrucion categories selected for scientific computation. The elative frequency with which each of the five types of structions were used in commercial computation is preented in Fig. 1. The time the central processor stands lle waiting for information input or output, $t_{I/O}$ is a unction of the amount of information that must be taken to the computer, the amount of information that must e sent out of the computer, the rate at which informaion is transferred in and out of the computer, and the legree to which input and/or output can take place while he central processor is operating.

When we studied the input-output requirements we vere unable to count the actual number of pieces (or umber of words) read or written. Instead, the time he computer's central processing unit (1) operated alone, 2) operated concurrently with I/O, and (3) idled, waitng for information input-output to take place, was measred. From the actual input-output times, and published nput-output rates, it was possible to estimate the number f words read and written. The following computing sysems were studied to estimate $t_{I/O}$: IBM 704, 705, 650, '070, 7090, and 1401; Philco 211; and Bendix G15. The igures for the 7090 were accurately obtained, using the ystem's clock for single channel I/O, double channel I/O, and double channel I/O with program interrupt. The other figures were obtained by less precise counting methods. The results obtained from the precise 7090 measures, and from the other systems, were very similar and are presented in Fig. 1.

The memory capacity M of a computing system greatly influences its computing ability. Increased memory markedly improves the processing of very large problems which would otherwise be split into subproblems. There are also advantages to larger memories when performing smaller problems because they allow the use of compiling routines, subroutines, etc. Recently, with the advent of multiple input-output capability, and multiple program operation with executive and interrupt routines, larger memories provide additional advantages for all sizes and types of problems.

We were unable to find a feasible means to measure analytically the influence which memory has upon a computer's performance capability. Our best alternative was to obtain the opinions of the individuals who were most familiar with computers. A total of 43 engineers, programmers, and other knowledgeable people were contacted and asked to evaluate the influence of computing memory upon performance. While their opinions varied, their answers were analogous enough to construct the functional model that estimates the effect memory has upon computer performance. The results of our inquiry are presented in Fig. 1.

machines covered

The two characteristics of the functional description for each computer which this study considers are calculated for the general purpose computers (up to the 1963 cut-off date) in the United States known to the author. The list of computers introduced between 1944 and 1963 was obtained through a detailed search of the computing literature. All the systems that did not have structural elements to satisfy the functional model (specifically P), the special purpose computers, were deleted from the list. Computers which are not in the class of functionally similar products defined by the functional model are those that were built and used to perform a set of specialized information processing tasks. As a result these systems contained limited and specialized input-output equipment or limited internal arithmetic capabilities and are not included in our sample.

Most of the recent general purpose computers have been manufactured in quantities from tens to thousands. With quantity production the manufacturers have offered a large number of alternative system configurations. For these computers one functional description does not fully describe the computer. Many of the computers offer over eight memory sizes, three input-output systems, four input-output channel configurations, and four arithmetic and control extras. This represents over (8 x 3 x 4 x 4) 384 different computing systems. Although only a few configurations eventually are produced, the modern systems potentially consist of several hundred alternatives. It would be impossible to calculate (P) for even a few alternatives of each system. We must therefore settle on one configuration for each computer.

There appears to be a good method for selecting the configurations, and that is to consider the most typical configuration of the computer. Where structural changes have been made, we have used the equipment which was available when the system was first introduced. In a few cases where important modifications have been introduced at a later date, these modifications are considered as separate computers, and are treated as such in the study. The calculations of P and C for both scientific

(Text cont'd on p. 45, Fig. 1 on p. 42)

A more detailed description of the development of the functional model presented in K. E. Knight's *A Study of Technological Innovation — The volution of Digital Computers,* an unpublished Ph.D. Dissertation, arnegie Institute of Technology, 1963.

Fig. 1—Functional Model-Algorithm to Calculate P for any Computer System

$$P = \frac{\dfrac{[(L\text{-}7)\ (T)\ (WF)]^{i}}{10^{12}\ [32{,}000\ (36\text{-}7)]^{i}}}{t_{C} + t_{I/O}}$$

$$t_C = 10^4\ [C_1A_{Fi} + C_2A_{FL} + C_3M + C_4D + C_5L]$$

$$t_I/_O = P \times OL_1\ [10^6\ (W_{I1} \times B \times 1/K_{I1}) + (W_{01} \times B \times 1/K_{01}) + N(S_1 + H_1)]\ R_1 + (1\text{-}P)\ OL_2\ [10^6\ (W_{I2} \times B \times 1/K_{I2}) + (W_{02} \times B \times 1/K_{02}) + N(S_2 + H_2)]$$

VARIABLES—ATTRIBUTES OF EACH COMPUTING SYSTEM

P = the computing power of the n^{th} computing system
L = the word lengths (in bits)
T = the total number of words in memory
t_c = the time for the Central Processing Unit to perform 1 million operations
$t_{I/O}$ = the time the Central Processing Unit stands idle waiting for I/O to take place
A_{Fi} = the time for the Central Processing Unit to perform 1 fixed point addition
A_{Fl} = the time for the Central Processing Unit to perform 1 floating point addition
M = the time for the Central Processing Unit to perform 1 multiply
D = the time for the Central Processing Unit to perform 1 divide
L = the time for the Central Processing Unit to perform 1 logic operation
B = the number of characters of I/O in each word
K_{I1} = the Input transfer rate (characters per second) of the primary I/O system
K_{O1} = the Output transfer rate (characters per second) of the primary I/O system
K_{I2} = the Input transfer rate (characters per second) of the secondary I/O system
K_{O2} = the Output transfer rate (characters per second) of the secondary I/O system
S_1 = the start time of the primary I/O system not overlapped with compute
H_1 = the stop time of the primary I/O system not overlapped with compute
S_2 = the start time of the secondary I/O system not overlapped with compute
H_2 = the stop time of the secondary I/O system not overlapped with compute
R_1 = 1 + the fraction of the useful primary I/O time that is required for non-overlap rewind time

	SEMI CONSTANT FACTORS	VALUES	
SYMBOL	DESCRIPTION	SCIENTIFIC COMPUTATION	COMMERCIAL COMPUTATION
WF	the word factor		
	a. fixed word length memory	1	1
	b. variable word length memory	2	2
C_1	weighting factor representing the percentage of the fixed add operations		
	a. computers without index registers or indirect addressing	10	25
	b. computers with index registers or indirect addressing	25	45
C_2	weighting factor that indicates the percentage of floating additions	10	0
C_3	weighting factor that indicates the percentage of multiply operations	6	1
C_4	weighting factor that indicates the percentage of divide operations	2	0
C_5	weighting factor that indicates the percentage of logic operations	72	74
P	percentage of the I/O that uses the primary I/O system		
	a. systems with only a primary I/O system	1.0	1.0
	b. systems with a primary and secondary I/O system	variable	variable
W_{I1}	number of input words per million internal operations using the primary I/O system		
	a. magnetic tape I/O system	20,000	100,000
	b. other I/O systems	2,000	10,000
W_{O1}	number of output words per million internal operations using the primary I/O system	the values are the same as those give above for W_{I1}	
W_{I2}/ W_{O2}	number of input/output words per million internal operations using the secondary I/O system	the values are the same as those given above for W_{I1}	
N	number of times separate data is read into or out of the computer per million operations	4	20
OL_1	overlap factor 1—the fraction of the primary I/O system's time not overlapped with compute		
	a. no overlap—no buffer	1	1
	b. read or write with compute—single buffer	.85	.85
	c. read, write and compute—single buffer	.7	.7
	d. multiple read, write and compute—several buffers	.60	.60
	e. multiple read, write and compute with program interrupt—several buffers	.55	.55
OL_2	overlap factor 2— the fraction of the secondary I/O system's time not overlapped with compute	values are the same as those given above for OL_1, a through e	
i	the exponential memory weighting factor	.5	.333

and commercial computation for the 225 computers we consider are presented in Table 1 (below).

Table I also contains date of introduction for each of the 225 computers we consider. For our study we define the date of introduction as the delivery date of an operating system to the first user. Where the computer is manufactured and used by the same organization, the date of introduction is defined as that when the completed computer passes a minimal acceptance test.

Technology curves

Since the functional descriptions consist of two attributes, we can display them on a two-dimensional graph. Fig. 2 (p. 47) contains points obtained when operations/second (P) is plotted against seconds/dollar (C) for computers performing scientific computation. Because of the tremendous range of P and C, Fig. 2 is drawn on log-log graph paper. The number next to each point identifies the corresponding computer as listed in Table I.

From an initial glance at Fig. 2, it is apparent that the early systems generally fall on the lower left portion of the graph, and the newer ones on the upper right. The graph shows how much computing power is obtained at different costs; there are high cost systems (few seconds per dollar) and low cost ones (many seconds per dollar). In any year, an expensive computer has greater computing power (higher number of operations per second) than a less expensive one. It is also apparent from Fig. 2 that for a constant C we obtain greater P over time.

A curve that connects the functional descriptions of the computers in a single year describes the computing tech-

(Cont. p. 49)

Table 1

COMPUTING SYSTEMS

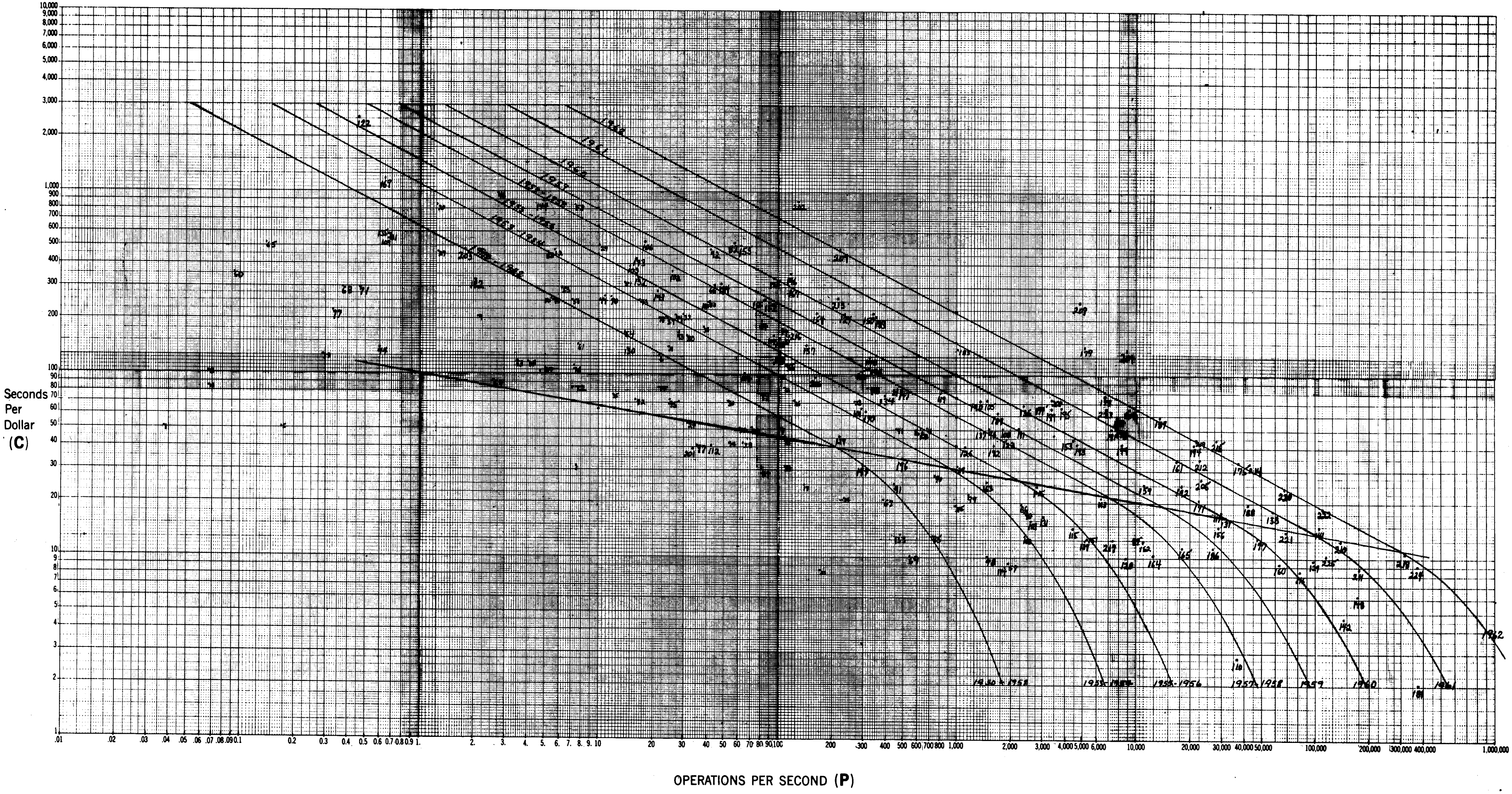
Computer No.	Name	Date Introduced	Scientific Computation P Ops/Sec	Scientific Computation C Secs/$	Commercial Computation P Ops/Sec	Commercial Computation C Secs/$
1	Harvard Mark I	1944	.0379	50.94	0.406	50.94
2	Bell Lab Computer Model IV	March 1945	.0068	509.4	0.035	509.4
3	Eniac	1946	7.448	31.81	44.65	31.81
4	Bell Computer Model V	Late 1947	.0674	84.83	0.296	84.83
5	Harvard Mark II	Sept. 1948	.1712	50.94	0.774	50.94
6	Binac	Aug. 1949	21.75	127.2	11.70	127.2
7	IBM CPC	1949	2.126	207.8	14.37	207.8
8	Bell Computer Model III	1949	.0674	102.2	0.296	102.2
9	SEAC	May 1950	102.8	50.94	253.8	50.94
10	Whirlwind I	Dec. 1950	110.7	31.81	45.57	31.18
11	Univac 1101 Era 1101	Dec. 1950	682.5	50.94	301.8	50.94
12	IBM 607	1950	5.666	479.6	34.06	479.6
13	Avdiac	1950	108.5	84.83	51.20	84.83
14	Adec	Jan. 1951	54.26	42.42	57.16	42.42
15	Burroughs Lab Calculator	Jan. 1951	5.605	254.5	7.718	254.5
16	SWAC	March 1951	632.2	50.94	324.7	50.94
17	Univac I	March 1951	140.1	24.94	271.4	24.94
18	ONR Relay Computer	May 1951	.2937	127.2	1.050	127.2
19	Fairchild Computer	June 1951	2.000	127.2	4.539	127.2
20	National 102	Jan. 1952	1.260	848.3	2.998	848.3
21	IAS	March 1952	467.0	84.83	305.0	84.83
22	Maniac I	March 1952	302.7	101.9	163.4	101.9
23	Ordvac	March 1952	268.8	72.76	127.8	72.76
24	Edvac	April 1952	31.56	54.22	14.86	54.22
25	Teleregister Special Purpose Digital Data Handling	June 1952	12.16	78.93	26.43	78.93
26	Illiac	Sept. 1952	123.1	72.76	50.43	72.76
27	Elcom 100	Dec. 1952	1.278	424.2	3.241	424.2
28	Harvard Mark IV	1952	63.99	42.42	64.95	42.42
29	Alwac II	Feb. 1953	10.17	509.4	12.08	509.4
30	Logistics Era	March 1953	52.85	72.00	39.01	72.0
31	Oarac	April 1953	24.38	141.4	35.71	141.4
32	ABC	May 1953	29.88	212.1	11.66	212.1
33	Raydac	July 1953	171.3	8.483	244.6	8.483
34	Whirlwind II	July 1953	233.4	21.21	95.96	21.21
35	National 102A	Summer 1953	4.089	116.5	8.400	116.5
36	Consolidated Eng. Corp. Model 36-101	Summer 1953	38.31	181.8	21.07	181.8
37	Jaincomp C	Aug. 1953	4.745	103.9	3.375	103.9
38	Flac	Sept. 1953	61.55	50.94	107.9	50.94
39	Oracle	Sept. 1953	1002.	31.81	563.4	31.81
40	Univac 1103	Sept. 1953	749.0	28.34	666.2	28.34
41	Univac 1102	Dec. 1953	460.3	50.94	240.0	50.94
42	Udec I	Dec.1953	16.38	72.67	21.93	72.67
43	NCR 107	1953	16.99	254.5	34.44	254.5
44	Miniac	Dec. 1953	10.91	267.6	9.545	267.6
45	IBM 701	1953	992.7	18.34	615.7	18.34
46	IBM 604	1953	2.766	974.2	20.19	974.3
47	AN/UJQ-2(YA-1)	1953	21.48	84.83	56.16	84.83
48	Johnniac	March 1954	319.2	84.83	284.9	84.83
49	Dyseac	April 1954	72.18	50.90	172.4	50.90
50	Elcom 120	May 1954	5.471	261.9	6.456	262.0
51	Circle	June 1954	14.04	318.1	10.59	318.1
52	Burroughs 204 & 205	July 1954	80.84	77.94	187.3	77.94
53	Modac 5014	July 1954	6.238	299.8	10.09	299.8
54	Ordfiac	July 1954	2.607	92.51	6.011	92.51
55	Datatron	Aug. 1954	113.7	113.2	243.1	113.2
56	Modac 404	Sept. 1954	7.116	254.5	15.29	254.5
57	Lincoln Memory Test	Dec. 1954	1925.	9.285	768.7	9.285
58	TIM II	Dec. 1954	7.414	848.3	7.439	848.3
59	Caldic	1954	23.99	203.8	41.34	203.8
60	Univac 60 & 120	Nov. 1954	.0924	356.3	1.473	356.3
61	IBM 650	Nov. 1954	110.8	155.9	291.1	155.9
62	WISC	1954	7.736	145.7	6.413	145.7
63	NCR 303	1954	3.491	117.6	8.281	117.6
64	Mellon Inst. Digital Computer	1954	14.23	169.9	10.55	169.9
65	IBM 610	1954	.1408	519.6	0.437	519.6
66	Alwac III	1954	44.80	302.7	91.42	302.7
67	IBM 702	Feb. 1955	394.4	20.78	1063.	20.78
68	Monrobot III	Feb. 1955	.3743	299.8	1.188	299.8
69	Norc	Feb. 1955	545.8	10.17	268.2	10.17
70	Miniac II	March 1955	11.76	267.6	17.44	267.6
71	Monrobot V	March 1955	.4678	295.5	1.607	295.5
72	Udec II	Oct. 1955	7.244	84.83	10.65	84.83
73	RCA BIZMAC I & II	Nov. 1955	285.6	5.668	967.9	5.668
74	Pennstac	Nov. 1955	26.75	212.1	22.98	212.1
75	Technitral 180	1955	110.0	46.19	190.1	46.19
76	National 102D	1955	7.317	112.3	14.20	112.3
77	Monrobot VI	1955	.3293	222.7	0.966	222.7
78	Modac 410	1955	24.18	203.8	51.84	169.9
79	Midac	1955	101.6	169.9	29.00	169.9
80	Elcom 125	1955	31.24	164.1	29.01	164.1
81	Burroughs E 101	1955	.6898	580.0	2.319	580.0
82	Bendix G15	Aug. 1955	57.34	419.9	30.25	419.9
83	Alwac III E	Nov. 1955	41.50	249.4	90.15	249.4
84	Readix	Feb. 1956	80.63	194.9	87.99	194.9
85	IBM 705, I, II	March 1956	734.0	13.27	2087.	13.27
86	Univac 1103 A	March 1956	2295.	19.49	1460.	19.49
87	AF CRC	April 1956	81.66	31.81	28.97	31.81
88	Guidance Function	April 1956	5.246	461.9	7.744	461.9
89	IBM 704	April 1956	10,670.	13.18	3,785.	13.18
90	IBM 701 (CORE)	1956	2378.	17.81	1807.	17.81
91	Narec	July 1956	444.8	25.45	190.6	25.45

(Table 1 cont. p. 46)

Table 1 (Cont.)

Computer		Date	Scientific Computation		Commercial Computation	
No.	Name	Introduced	P Ops/Sec	C Secs/$	P Ops/Sec	C Secs/$
92	LGP 30	Sept. 1956	41.94	479.6	32.75	479.6
93	Modac 414	Oct. 1956	28.26	169.9	42.94	169.9
94	Elecom 50	1956	.5990	139.2	1.776	1039.
95	Udec III	March 1957	25.11	72.76	20.85	72.76
96	George I	Sept. 1957	1538.	50.94	571.9	50.94
97	Univac File O	Sept. 1957	35.20	41.02	73.17	41.02
98	Lincoln TXO	Fall 1957	1,471.	10.19	359.6	10.19
99	Univac II	Nov. 1957	1,155.	22.27	2,363.	22.27
100	IBM 705 III	Late 1957	2,379.	13.27	7,473.	13.27
101	Teleregister Telefile	Late 1957	286.0	65.98	935.9	65.98
102	Recomp I	Late 1957	25.76	363.8	16.14	363.8
103	IBM 608	1957	15.21	389.7	60.69	389.7
104	Mistic	1957	64.28	101.9	24.50	101.9
105	Maniac II	1957	1,491.	72.84	1,421.	72.84
106	IBM 609	1957	18.19	530.7	75.21	530.7
107	IBM 305	Dec. 1957	94.47	163.0	96.47	163.0
108	Corbin	1957	1,794.	50.90	2407.	50.90
109	Burroughs E 103	1957	.6736	551.8	2.286	551.8
110	AN/FSQ 7 & 8	1957	36,730.	2.834	15,560.	2.834
111	Alwac 880	1957	2,198.	50.90	959.7	50.90
112	Univac File I	Jan. 1958	42.49	41.05	92.04	41.05
113	Lincoln CG24	May 1958	6,394	21.21	5,933.	21.21
114	IBM 709	Aug. 1958	1,869.	8.882	10,230.	8.882
115	Univac 1105	Sept. 1958	4,433.	14.50	5,527.	14.50
116	Lincoln TX2	Fall 1958	82,050.	8.483	34,000.	8.483
117	Philco 2000-210	Nov. 1958	29,970.	17.81	28,740.	17.81
118	Recomp II	Dec. 1958	41.36	249.4	28.03	249.4
119	Burroughs 220	Dec. 1958	810.2	79.94	1,616.	79.94
120	Mobidic	1958-1960	8741.	10.19	12,250.	10.19
121	Philco CXPO	1958	2,622.	15.91	1,576.	15.91
122	Monrobot IX	1958	.4598	2,545.	1.334	2,545.
123	GE 210	June 1959	1,884.	44.54	5,085.	44.54
124	Cyclone	July 1959	234.6	215.0	119.6	215.0
125	IBM 1620	Oct. 1959	94.79	331.7	47.20	331.7
126	NCR 304	Nov. 1959	1,136.	40.23	2,445.	40.23
127	IBM 7090	Nov. 1959	97,350.	9.742	45,470.	9.742
128	RCA 501	Nov. 1959	638.7	38.97	1,877.	38.97
129	RW 300	Nov. 1959	218.6	45.58	534.3	45.78
130	RPC 9000	1959	14.50	138.6	9.521.	138.6
131	Librascope Air Traffic	1959	3043.	16.94	6,130.	16.94
132	Jukebox	1959	16.56	338.9	18.66	338.9
133	Datamatic 1000	1959	480.8	13.44	1,455.	13.44
134	CCC Real Time	1959	393.8	77.17	280.3	77.17
135	Burroughs E 102	1959	.6670	580.0	1.847	580.0
136	Burroughs D 204	1959	2,354.	68.00	1,183.	68.00
137	AN/TYK 6V BASICPAC	1959	1,365.	50.90	493.0	50.90
138	CDC 1604	Jan. 1960	58,290.	18.34	20,390.	18.34
139	Librascope 3000	Jan. 1960	5,177.	12.47	25,320.	12.47
140	Univac Solid State 80/90 I	Jan. 1960	329.1	124.7	489.6	124.7
141	Philco 2000-211	March 1960	105,844.	14.845	55,740.	14.85
142	Univac Larc	May 1960	142,600.	4.619	40,450.	4.619
143	Libratrol 500	May 1960	21.07	286.0	20.38	286.0
144	Monrobot XI	May 1960	4.839	890.7	10.30	890.7
145	IBM 7070	June 1960	2,813.	23.98	5,139.	23.98
146	CDC 160	July 1960	119.3	354.3	49.63	354.2
147	IBM 1401 (Mag. Tape)	Sept. 1960	496.7	83.14	1,626.	83.14
148	AN/FSQ 31 & 32	Sept. 1960	172,200.	6.235.	48,360.	6.285
149	Merlin	Sept. 1960	8,306.	42.42	2,925.	42.42
150	IBM 1401 (Card)	Sept. 1960	340.9	215.0	967.8	215.0
151	Mobidic B	Fall 1960	5,251.	12.72	8,630.	12.72
152	RPC 4000	Nov. 1960	89.91	249.4	54.11	249.4
153	PDP-1 (M.T.)	Nov. 1960	4,455.	41.57	2,173.3	41.6
154	PDP-1 (P.T.)	Nov. 1960	166.6	215.	57.16	215.0
155	Packard Bell 250 (PT)	Dec. 1960	62.23	506.9	22.21	506.9
156	Honeywell 800	Dec. 1960	28,790.	14.85	23,760.	14.85
157	General Mills AD/ECW-57	Dec. 1960	143.9	141.7	44.03	141.7
158	Philco 3000	Late 1960	102.2	155.9	66.13	155.8
159	Maniac III	Late 1960	11,140.	25.45	4723.	25.45
160	Sylvania S9400	Late 1960	62,510.	9.306	49.550.	9.306
161	Target Intercept	Late 1960	16,800.	33.89	16,070.	33.89
162	Westinghouse Airbourne	1960	10,950.	12.47	4806.	12.47
163	RCA 300	1960	1,466.	25.98	687.7	25.98
164	Mobidic CD & 7A AN/MYK	1960	12,410.	10.39	15,430.	10.39
165	Litton C7000	1960	18,200.	11.34	5,323.	11.34
166	Libratrol 1000	1960	84.16	254.5	50.85	254.5
167	GE 312	1960	122.0	299.8	47.12	299.8
168	Diana	1960	102.1	127.2	48.85	127.2
169	DE 60	Feb. 1960	.6384	1,155.	1.855	1155.
170	Burroughs D107	1960	311.8	63.62	73.95	63.62
171	AN/USQ 20	1960	22,390.	20.78	23,670.	20.78
172	AN/TYK 4V Compac	1960	1,610.	41.57	616,1	41.57
173	General Mills Apsac	Jan. 1961	16.22	424.2	7.084	424.2
174	Univac Solid State 80/90 II	Jan. 1961	3,199.	69.28	3,044.	69.28
175	Bendix G20 & 21	Feb. 1961	37,260.	33.17	17,060.	33.17
176	RCA 301	Feb. 1961	323.0	113.4	1,055.	113.4
177	BRLESC	March 1961	47,240.	12.72	28,550.	12.72
178	GE 225	March 1961	6,566	77.94	7,131.	77.94
179	CCC-DDP 19 (Card)	May 1961	5,159.	138.6	3,027.	138.6
180	CCC-DDP 19 (MT)	May 1961	7,908.	59.38	8,073.	59.38
181	IBM Stretch (7030)	May 1961	371,700.	2.078	631,200.	2.078
182	NCR 390	May 1961	2.034	328.2	10.43	328.2
183	Honeywell 290	June 1961	354.3	207.8	182.8	207.8
184	Recomp III	June 1961	48.28	311.8	35.76	311.8
185	CDC 160A	July 1961	1,015.	138.6	1,780.	138.6
186	IBM 7080	Aug. 1961	27,090.	11.34	30,860.	11.34
187	RW 530	Aug. 1961	13,460.	59.38	5086.	59.38
188	IBM 7074	Nov. 1961	41,990.	19.49	31,650.	19.49
189	IBM 1410	Nov. 1961	1,673.	62.35	4,638.	62.35
190	Honeywell 400	Dec. 1961	1,354.	71.67	2,752.	71.67
191	Rice Univ.	Dec. 1961	7,295.	50.90	2378.	50.90
192	Univac 490	Dec. 1961	17,770.	24.94	15,050.	24.94
193	AN/TYK 7V	1961	4,713.	41.57	9,077.	41.57
194	Univac 1206	1961	20,990.	42.42	17,700.	42.42
195	Univac 1000 & 1020	1961	3,861.	66.33	3,292.	66.33
196	ITT Bank Loan Process	1961	492.6	34.64	1,916.	34.64
197	George II	1961	298.	31.81	675.1	31.81
198	Oklahoma Univ.	Early 1962	7,723.	50.90	2,616.	50.90
199	NCR 315	Jan. 1962	3,408.	65.63	11,460.	65.63
200	NCR 315 CRAM	Jan. 1962	3,364.	73.36	9,896.	73.36
201	Univac File II	Jan. 1962	33.46	38.97	94.49	38.97
202	HRB-Singer Sema	Jan. 1962	129.2	890.7	56.94	890.7
203	Univac 1004	Feb. 1962	1.789	479.6	25.29	479.6
204	ASI 210	April 1962	8,868.	135.5	4,114.	135.5
205	Univac III	June 1962	22,720.	27.11	22,790.	27.11
206	Burroughs B200 Series-B270 & 280	July 1962	163.3	95.93	615.3	95.93
207	SDS 910	Aug. 1962	4,841.	249.4	2,355.	249.4
208	SDS 920	Sept. 1962	9,244	65.63	4,964	65.63
209	PDP-4	Sept. 1962	220.2	479.6	75.97	479.6
210	Univac 1107	Oct. 1962	138,700.	12.47	76,050.	12.47
211	IBM 7094	Nov. 1962	175,900.	8.782	95,900.	8.781
212	IBM 7072	Nov. 1963	22,710.	34.64	8,694.	34.64
213	IBM 1620 MOD III	Dec. 1962	214.8	259.8	56.89	259.8
214	Burroughs B5000	Dec. 1962	43,000.	32.82	15,910.	32.82
215	ASI 420	Dec. 1962	27,790.	44.54	11,090.	44.54
216	Burroughs B200 Series—Card Sys	Dec. 1962	114.3	160.1	437.2	164.1
217	RW 400 (AN/FSQ 27)	1962	7,437.	12.47	11,240.	12.47
218	CDC 3600	June 1963	315,900.	11.34	74,900.	11.34
219	IBM 7040	April 1963	21,420.	44.54	90.79	44.54
220	IBM 7044	July 1963	67,660.	23.98	23,420.	23.98
221	RCA 601	Jan. 1963	68,690.	13.86	58,880.	13.86
222	Honeywell 1800	Nov. 1963	110,600.	17.81	57,750.	17.81
223	Philco 1000 Transac S1000	June 1963	6,811.	65.63	10,440.	65.63
224	Philco 2000-212	Feb. 1963	369,800.	9.169	84,230.	9.169
225	Librascope L 3055	Dec. 1963	114,000.	10.39	30,620	10.39

Fig. 2 Functional Descriptions for Scientific Computers (The numbers in this graph to identify each computer correspond to the numbers in Table I.)



nology for that year. Improved performance consists of a continual shift over time, enabling an increased number of operations per second to be performed for a given cost.

We now wish to use our data to develop the technology curves. Unfortunately, the points for a particular year in Fig. 2 do not form smooth parallel curves. For any one year considerable scattering occurs because (1) not all systems are equally technically advanced, and (2) there are errors in the estimates of P and C.

The first reason for the scatter of points needs little explanation. In the computing industry, there have been many systems introduced, and these have resulted in a wide range of performance from improved to poorer. Some systems will make significant improvements and fall far to the right of the other points. Alternatively, many systems will not match the capabilities of existing computers and will lie in the range of the industry's previous know-how.

The second reason for the scatter is the expected variance in the estimates of the functional descriptions. P was obtained by means of the functional model, which estimated each system's actual performance. There are differences both in the pricing policies of the manufacturers and in our ability to determine what equipment constitutes each particular system that creates a variance in C. In the calculations we performed, many small errors could have crept into the estimates of P and C to produce random error, even if all the systems came from an identical level of technological knowledge.

Recognizing that variance exists, it is necessary to use a curve-fitting technique to estimate the desired technology lines. For this study we have used least square regression analysis. From a visual analysis of Fig. 2 it appears that the technology curves for the different years are approximately the same in shape, with a shift to the right over time. Thus, the data were fitted to the following equation:

$$\ln(C) = \alpha_0 + \alpha_1 \ln(P) + \alpha_2[\ln(P)]^2 + \beta_1 S_1 + \beta_2 S_2 + \ldots + \beta_7 S_7 \qquad \text{Eq. 1}$$

The α's and β's represent the regression coefficients to be determined by the least squares analysis. The $S_1, \ldots, S_7$ represent dummy variables (or shift parameters) for the different years considered. To fit the curve, the data were grouped into eight time periods (i.e., 1962, 1961, 1960, 1959, 1957-58, 1955-56, 1953-54, and 1950-51-52). The earlier years were combined because of the small number of systems introduced in each of these years. The dummy variables were used in the following manner: for 1962, $S_1, \ldots, S_2$ were all set equal to 0; for 1961, $S_1 = 1$ and $S_2 = S_3 = \ldots = S_7 = 0$; for 1960, $S_2 = 1$ and $S_1 = S_3 = \ldots = S_7 = 0$; . . . and finally for 1950-51-52, $S_7 = 1$ and $S_1 - S_2 = \ldots = S_6 = 0$. (P) and $[\ln(P)]^2$ were both initially included in the equation since a visual analysis of the lines made them appear curved.

After the initial regression estimate, all points that were more than ½ a standard deviation below and to the left of the curve for their year were omitted. Eliminating points in this manner provides a distinct procedure for determining which points we will include in the final determination of technology curves, and forces the technology curves to the right to provide a more accurate picture of the performance limits.

The regression analysis, using the data for computer performance in scientific computation with Equation 1, showed that $[\ln(P)]^2$ term was not significant. The least squares technique was then used to fit Equation 2 to the data.

$$\ln(C) = \alpha_0 + \alpha_1 \ln(P) + \beta_1 S_1 + \beta_2 S_2 + \ldots + \beta_7 S_7 \qquad \text{Eq. 2}$$

For the linear equation, all the terms were significant and the correlation coefficient was $r = +.9569$. Since the correlation coefficient equaled only +.9596 with Equation 1, it appeared most reasonable to use the simpler linear equation to plot the technology curves. In the calculation of both the polynomial and the linear equations, over 120 observations were used so that the sample sizes would be adequate. The equation for the scientific computation technology curves is as follows:

$\ln(C) = 8.9704 - .51934\,[\ln(P)]$ Eq. 3
$-.3650$ (1961] -1.6639 (1955-56)
$-.7874$ (1960) -1.9859 (1953-54)
-1.0724 (1959) -2.5013 (1950-51-52)
-1.3028 (1957-58)

The eight curves described by Equation 3 are drawn in Fig. 2.

We now perform a similar analysis for commercial computation. The results of the calculation of the technology curves for systems performing commercial computation are shown in Equation 4.[3]

$\ln(C) = 8.1672 - .459\,[\ln(P)]$ Eq. 4
$-.3643$ (1961) -1.187 (1955-56)
$-.6294$ (1960) -1.454 (1953-54)
$-.8561$ (1959) -2.164 (1950-51-52)
$-.9011$ (1957-58)

The eight curves drawn from Equation 4 are shown in Figure 3 (p. 51).

grosch's law upheld

We analyze the meaning of the technology curves by first restating the general equation for the curves:

$$C = (\alpha_0]\,(P)^{\alpha 1}\,(e^{\beta 1})\quad(e^{\beta 2})\ \ldots\ (e^{\beta 7}) \qquad \text{Eq. 5}$$

where $\ln \alpha_0 = \alpha_0$

and $\alpha_0, \alpha_1, \beta^1, \beta_2, \ldots, \beta_7$ are the values calculated with the least square regression analysis.

From Equation 5 we obtain the following:

$$\text{seconds/dollar} = k\left(\begin{matrix}\text{Shift parameter to}\\ \text{adust for year}\end{matrix}\right)(\text{operations/sec})^{\alpha 1} \qquad \text{Eq. 6}$$

For any particular year we can combine the constant, k, and the shift parameter into a new constant C [(k)] x (shift parameter) = C. If we, therefore, set $a_1 = -\alpha_1$, Equation 6 now becomes:

$$\text{dollars/second} = \frac{1}{C}\,(\text{operations/sec})^{\alpha 1}. \qquad \text{Eq. 7}$$

For scientific computation the value for $\alpha 1 = -.519$ so that a_1 equals .519. For commercial computation $\alpha_1 = -.459$ so that a_1 equals .459. We can therefore assume that a_1 is

(Cont. p. 54)

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[2] For this equation the following list contains the standard error and the test of significance (student's t test) for each regression coefficient.

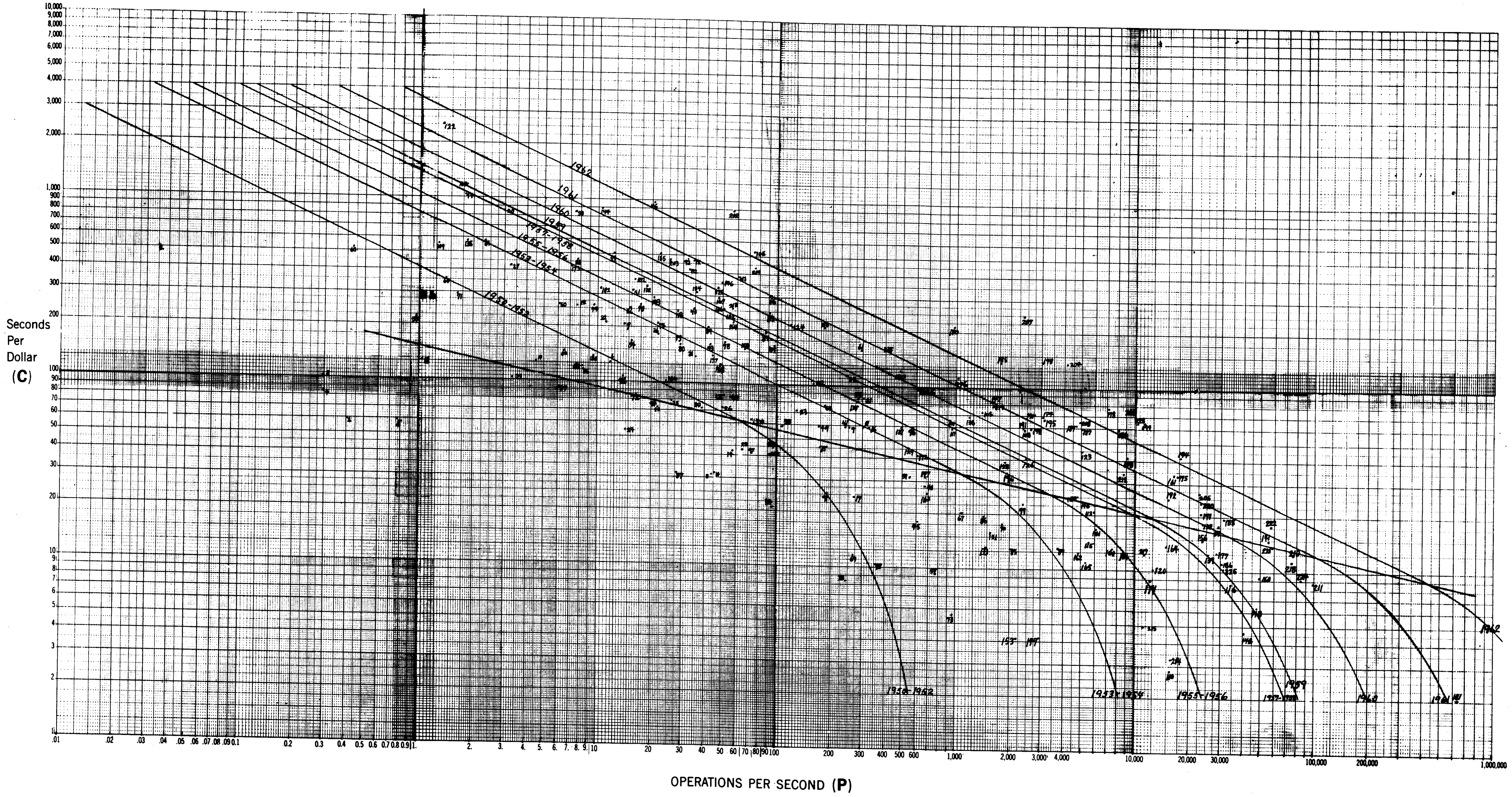
Regression Coefficient	Standard Error	t value
$\alpha 1$	.0171	−30.41
		−2.112
β_2	.1608	−4.897
β_3	.1887	−5.682
β_4	.1687	−7.723
β_5	.1992	−8.349
β_6	.1750	−11.34
β_7	.1943	−12.87

[3] The correlation coefficient, r, for the linear equation (Equation 2) was +.8543. The curves using Equation 1 and Equation 2 were similar and the correlation coefficients almost equal so that the simple linear equation was used to construct the technology curves.

[4] The following list contains the standard error and the test of significance (students' t test) for each regression coefficient in this equation.

Regression Coefficient	Standard Error	t value
α_1	.02983	−15.39
β_1	.2589	−1.407
β_2	.2758	−2.282
β_3	.2895	−2.957
β_4	.2537	−3.551
β_5	3029	−3.917
β_6	.2789	−5.214
β_7	.3109	−6.901

Fig. 3 Functional Descriptions for Commercial Computation (The numbers used in this graph to identify each computer correspond to the numbers in Table I.)



approximately equal to .5 and rewrite Equation 7 as follows:

$$\text{System Cost} = \frac{1}{C}\sqrt[2]{\text{Computing Power}} \qquad \text{Eq. 8}$$

This represents a very interesting result because it indicates that within the limits of the computing technology one can construct four times as powerful a computer at only twice the cost.

$$\text{Computing power} = (C \text{ system cost})^2 \qquad \text{Eq. 9}$$

That computing power increases as the square of cost was proposed in the late 1940's by Herb Grosch. Since that time the relationship expressed in Equation 9 has been referred to as Grosch's Law. We have seen the industry develop a sense of humor over its 20-year life with frequent jokes being made in reference to Grosch's Law. In a recent article by Charles W. Adams, "Grosch's Law Repealed," the author proposes to "replace the square (Grosch's Law) by the square root."[5] Grosch's Law has received much attention because of its implications about economies of scale, yet has never been supported with adequate quantitative data. We still need to question whether the Law (Computing power=Constant $(\text{Cost})^2$) is true, or if it is the artifact of the computer companies' pricing policy. The popularity of the Law and the difficulty in setting prices leads us to suspect the possibility of some bias in our data.

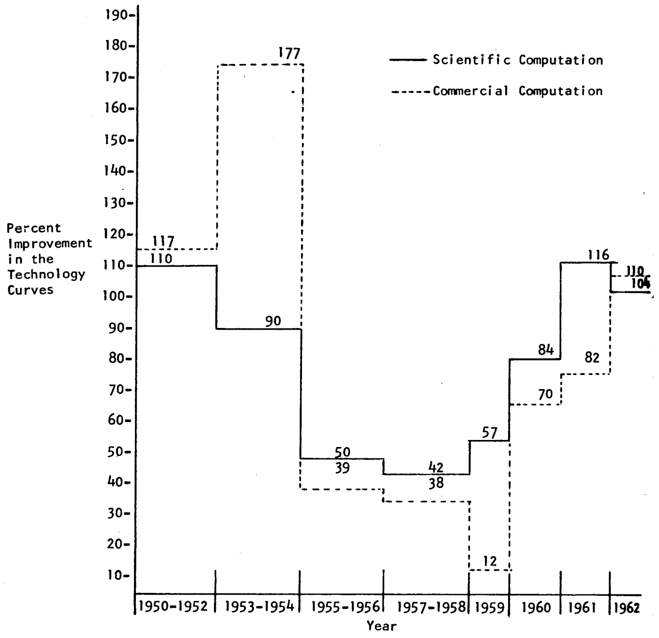
We must express another word of caution before we attach too much significance to Grosch's Law. In calculating the technology curves we were able to use the systems actually built. The equations derived are, therefore, applicable within the limited range of computers studied. Special consideration has to be given to the fact that there are definite limits to the maximum computing power that can be obtained at any one time. As the bounds of technological knowledge are reached, additional computing power is purchased at a very high price. For high value of P the technology curve will not remain a straight line but will curve downward to show an ever increasing negative slope. The reason that this did not show up in the regression analysis is that only a few computers came close to the maximum limits of computing power. Three noticeable ones are the AN/FSQ 7 and 8 (the Sage computers), the Univac Larc and the IBM Stretch. These computers each obtained a new high evaluation for absolute computing power, but at considerably lower number of operations/dollar. Grosch's Law did not hold for these three machines because the increases in power were obtained at less than the squared, or even a 1 to 1, relationship with Cost—the slope of the curve, or α^1, is less than -1. We cannot build larger and larger computers at reasonable costs since at any point in time there are absolute limits to the size and speed obtainable. This fact needs to be kept in mind when talking about Equation 8. The most powerful computing systems we could possibly build today or tomorrow would not be the most economical.

In order to estimate where the turning point occurs we use the computers that have had, at one time, the maximum absolute efficiency. For scientific computation there are eight systems, and for commercial computation ten. We add to Fig. 2 and 3 lines of maximum efficiency through these points. The point where the line crosses the technology curves for each year provides an estimate of where the technology curves start to slope downward to yield diminishing marginal returns for systems with greater computing power. The latter curves are drawn freehand on Fig. 2 and 3 to show their approximate shape.

performance improvements from 1950 to 1962

The continual stream of performance improvements appears to result from the dynamic nature of the industry itself. Most people in the computing field search conscientiously for faster and more economical machines. However, most of these individuals have a limited idea of what has happened over the past 20 years. For instance, they greatly underestimate the number of innovative systems produced and the amount of performance improvement which actually has taken place. The shift in the technology curves illustrates the performance advances. From 1950 through 1962 the technology curves have an average improvement of 81% per year for scientific computation and 87% per year for commercial computation. It is seen from Fig. 4 that there has been some variance in yearly percent improvement. The improvements in both scientific and commercial computation have been fairly similar, with the first five years, 1950-1954, and the last three years, 1960-1962, showing greater improvement than the years, 1955-1959. The large commercial computation improvement in 1953-1954 that we mentioned earlier as being significantly above the mean, can be explained by the great increase in speed and the number of machines using magnetic tape units. Since commercial computation relies more upon input-output capability than does scientific computation, the improvements and increased utilization of magnetic tapes aided this category more than they did the other. No simple explanation has been found for the other variations shown in Fig. 4.

Figure 4.—Average Yearly Shift of the Technology Curves



As a result of tremendous improvement from year to year, a computer has been marketable for from 3 to 6 years. With the great rate of improvement, by the time most users get around to purchasing a system it is greatly inferior to the newer ones being introduced. This illustrates the tremendous obsolescence problem the industry must face if the present rate of improvement continues. The problem will become especially acute if purchasers try to order machines now being designed for delivery one or two years away, rather than take an existing machine. Most computers already in production require from six months to two years for delivery. ■

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[5] C. W. Adams, "Grosch's Law Repealed," *Datamation* (July 1962), pp. 38-39.